# Definition of Derivative - Classwork

Let us do a bit of reviewing. For the function $f(x) = x^2 - 3x + 1$, find

a) the slope of the tangent line at $x = 2$

b) the slope of the tangent line at $x = 0$.

c) the slope of the tangent line at $x = -1$.

Obviously we have duplicated our efforts a great deal. The process is the same - only the point at which we find the slope of the tangent changes. With that in mind, we are ready to introduce a basic concept of calculus.

The **derivative** of a function is a formula for the slope of the tangent line to that function at any point $x$. The process of taking derivatives is called **differentiation**.

We now define the derivative of a function $f(x)$ as $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. This mimics the procedure used above but it calculates the slope of the tangent line at any generic point $x$. Notice that we can now talk about this process in terms of a limit. When you do your cancellation, you are essentially performing the limit procedures we did in the last section.

Example 1) For the function $f(x) = x^2 - 3x + 1$, find its derivative and evaluate the derivative at $x = 2$, 0, and -1.

We now need to specify some notation for the derivative. When the function is defined as $f(x)$, the derivative will be written as $f'(x)$ or $f'$. When the function is written in the form of $y =$, the derivative is written as $y'$ or $\frac{dy}{dx}$. The latter looks like a fraction but for now will be one entity, the derivative of $y$ and it is pronounced "$dy$ $dx$."

Example 2) $f(x) = 4x$, find $f'(x)$

Example 3) $f(x) = x^2 + x$, find $f'(x)$

Example 4) $f(x) = 2x^2 - 5x + 6$, find $f'(x)$ and $f'(3)$

Example 5) $y = \frac{4}{x}$, find $\frac{dy}{dx}$

For the following functions, find their derivative and evaluate at $x = 2$, $-4$, $0$ and $\pi$. Use proper notation.

1. $y = 2x$

Answers: $y' = 2$

$y'(2) = 2 \quad y'(-4) = 2 \quad y'(0) = 2 \quad y'(\pi) = 2$

2. $y = x^2 - 5$

Answers: $y' = 2x$

$y'(2) = 4 \quad y'(-4) = -8 \quad y'(0) = 0 \quad y'(\pi) = 2\pi$

3. $f(x) = x^2 + 3x - 4$

Answers: $f'(x) = 2x + 3$

$y'(2) = 7 \quad y'(-4) = -5 \quad y'(0) = 3 \quad y'(\pi) = 2\pi + 3$

4. $f(x) = 4x^2 - 6x + 1$

Answers: $f'(x) = 8x - 6$

$y'(2) = 10 \quad y'(-4) = -38 \quad y'(0) = -6 \quad y'(\pi) = 8\pi - 6$

5. $f(x) = x^3 + 2x$

Answers: $f'(x) = 3x^2 + 2$

$y'(2) = 14 \quad y'(-4) = 50 \quad y'(0) = 2 \quad y'(\pi) = 3\pi^2 + 2$

6. $f(x) = \dfrac{5}{x} + 1$

Answers: $f'(x) = \dfrac{-5}{x^2}$

$y'(2) = \dfrac{-5}{4} \quad y'(-4) = \dfrac{-5}{16} \quad y'(0) = DNE \quad y'(\pi) = \dfrac{-5}{\pi^2}$

7. $f(x) = \dfrac{-1}{x^2}$

Answers: $f'(x) = \dfrac{2}{x^3}$

$y'(2) = \dfrac{1}{4} \quad y'(-4) = \dfrac{-1}{32} \quad y'(0) = DNE \quad y'(\pi) = \dfrac{2}{\pi^3}$

8. $f(x) = \sqrt{x}$

Answers: $f'(x) = \dfrac{1}{2\sqrt{x}}$

$y'(2) = \dfrac{1}{2\sqrt{2}} \quad y'(-4) = DNE \quad y'(0) = DNE \quad y'(\pi) = \dfrac{1}{2\sqrt{\pi}}$