Implicit Differentiation - Classwork

Suppose you were asked to find the slope of the tangent line to the curve $x^2 + y^2 = 25$ at the point (4, 3).

First, we can solve for y. ______________________________
Next, take the derivative: ______________________________
and simplify ______________________________
Now plug in the value of $x = 4$. ______________________________

Clearly, we have a problem. We have a ± in our derivative. Which one is it? ______________________________

The process we have gone through is actually called *explicit differentiation*. This process is accomplished by solving for y, then taking the derivative. But suppose we were asked to find the slope of the tangent line to the curve $x^2 + y^3 + y = 2$. Clearly, it is difficult or possibly impossible to solve for y. Another technique is needed. That technique is called *implicit differentiation*. In implicit differentiation we take the derivative without solving for y.

Let us do the original problem, finding the slope of the tangent line to the curve $x^2 + y^2 = 25$ at the point (4, 3) using implicit differentiation.

Original equation: $x^2 + y^2 = 25$
Take derivative (remember chain rule) ______________________________
Now plug in the point (4, 3) ______________________________

Notice that we did not actually solve for the derivative before we plugged in the point. But if we needed to find a general statement for $\frac{dy}{dx}$, it would be:

The key to doing implicit differentiation is to use your derivative rules on every term in the function but remembering to use the chain rule on that term. In differentiating the expression $x^2 + y^2 = 25$, we write: $2x\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$. The $\frac{dx}{dx}$ term equals one and is not necessary but it is best to put it down and then eliminate it. Frequent careless mistakes are made by forgetting this term.

Example 1) Find $\frac{dy}{dx}$ for $x^2 - y^2 = 16$ at $(5,-3)$

Example 2) Find $\frac{dy}{dx}$ for $xy + y = 8$ at $(2,3)$

Example 3) Find $\frac{dy}{dx}$ for $x^2y + xy^2 = 2x$ at $(1,1)$

Example 4) Find $\frac{dy}{dx}$ for $y + \sqrt{xy} = 2$ at $(2,2)$

Example 5) Find $\frac{dy}{dx}$ for $(x+y)^2 + y = 2$ at $(0,1)$

Example 6) Find $\frac{dy}{dx}$ for $x^2 + 4y^2 = 4$ at $(2,0)$

Example 7) Find $\frac{dy}{dx}$ for $\sin(xy) = 1$

Example 8) Given $x^2 + y^2 = 100$, find $\frac{d^2y}{dx^2}$

Example 9) Find the equation of the tangent line to

$$\sqrt{x} + \sqrt{y} - 1 = y \text{ at } (9,4)$$

Example 10) Find the points where the curve

$$25x^2 + 16y^2 + 200x - 160y + 400 = 0$$

has horizontal and vertical tangent lines.

Implicit Differentiation - Homework

1. Find $\dfrac{dy}{dx}$ for $xy = 4$ at $(-4,-1)$

2. Find $\dfrac{dy}{dx}$ for $x^2 - y^3 = 0$ at $(1,1)$

3. Find $\dfrac{dy}{dx}$ for $\sqrt{x} + \sqrt{y} = 9$ at $(16,25)$

4. Find $\dfrac{dy}{dx}$ for $x^3 - xy + y^2 = 4$ at $(0,-2)$

5. Find $\dfrac{dy}{dx}$ for $x^2 y - xy^2 = -6$ at $(2,-1)$

6. Find $\dfrac{dy}{dx}$ for $(x+y)^3 = x^3 + y^3$ at $(-1,1)$

7. Find $\dfrac{dy}{dx}$ for $\sqrt{xy} = x - 2y$ at $(4,1)$

8. Find $\dfrac{dy}{dx}$ for $x\cos y = 1$ at $\left(2,\dfrac{\pi}{3}\right)$

9. Find $\frac{dy}{dx}$ for $x^3 y - y = x$

10. Find $\frac{dy}{dx}$ for $\sin x + 2\cos 2y = 1$

11. Find $\frac{dy}{dx}$ for $2\sin x \cos y = 1$

12. Find $\frac{dy}{dx}$ for $\tan(x + y) = y$

13. Given $1 - xy = x - y$, find $\frac{d^2 y}{dx^2}$

14. Find the equations of the lines both tangent and normal to $x^3 + y^3 = 2xy$ at $(1, 1)$

15. Find the points at which the graph of $x^2 + 4y^2 - 4x + 16y + 4 = 0$ has a vertical and horizontal tangent line.