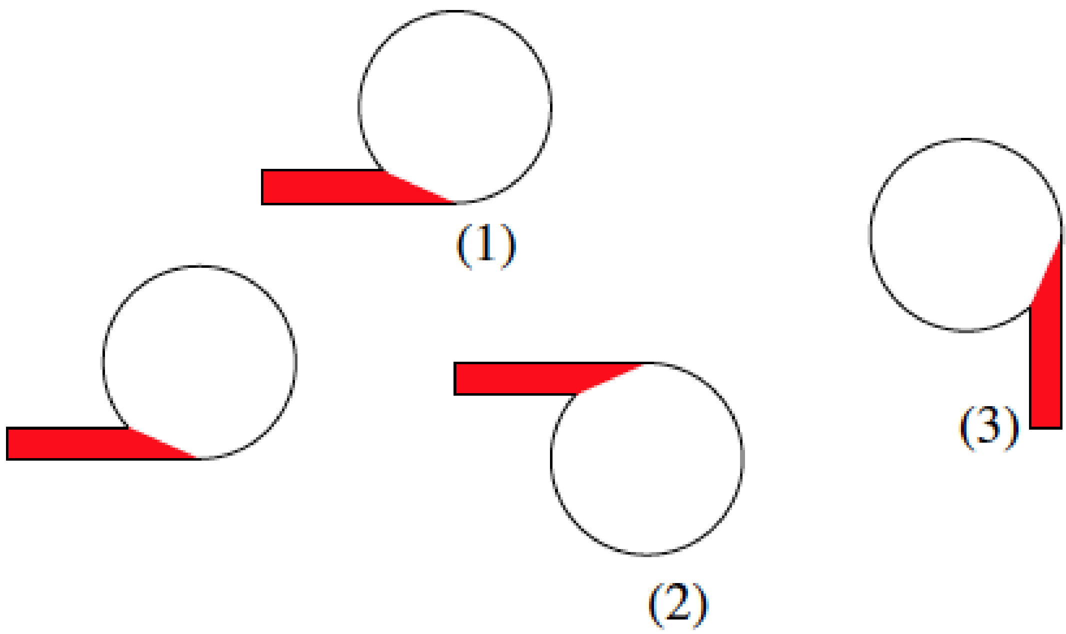
Lesson 1: Why Move Things Around?

Classwork

Exploratory Challenge

1. Describe, intuitively, what kind of transformation will be required to move the figure on the left to each of the figures (1)–(3) on the right. To help with this exercise, use a transparency to copy the figure on the left. Note: Begin by moving the left figure to each of the locations in (1), (2), and (3).



2. Given two segments AB and CD, which could be very far apart, how can we find out if they have the same length without measuring them individually? Do you think they have the same length? How do you check? In other words, why do you think we need to move things around on the plane?

Lesson Summary

A **transformation** of the plane, to be denoted by F, is a rule that assigns to each point P of the plane one and only one (unique) point which will be denoted by $F(P)$.

- So, by definition, the symbol $F(P)$ denotes a specific single point.
- The symbol $F(P)$ shows clearly that F moves P to $F(P)$.
- The point $F(P)$ will be called the image of P by F.
- We also say F maps P to $F(P)$.

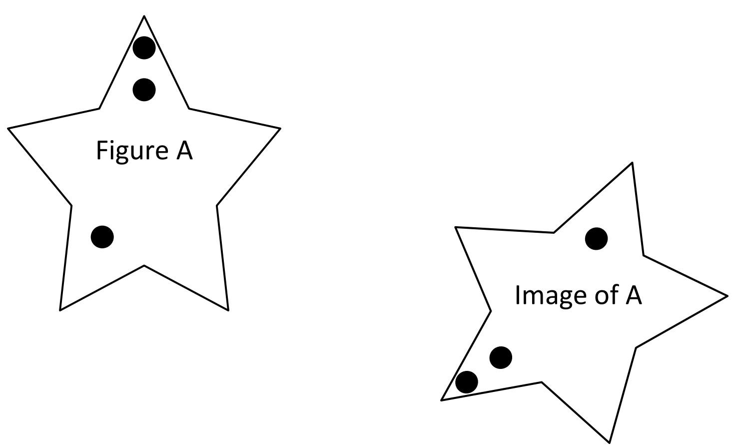
If given any two points P and Q, the distance between the images $F(P)$ and $F(Q)$ is the same as the distance between the original points P and Q, then the transformation F preserves distance, or is distance-preserving.

- A **distance-preserving** transformation is called a rigid motion (or an isometry), and the name suggests that it *moves* the points of the plane around in a *rigid* fashion.

Problem Set

1. Using as much of the new vocabulary as you can, try to describe what you see in the diagram below.

2. Describe, intuitively, what kind of transformation will be required to move Figure A on the left to its image on the right.

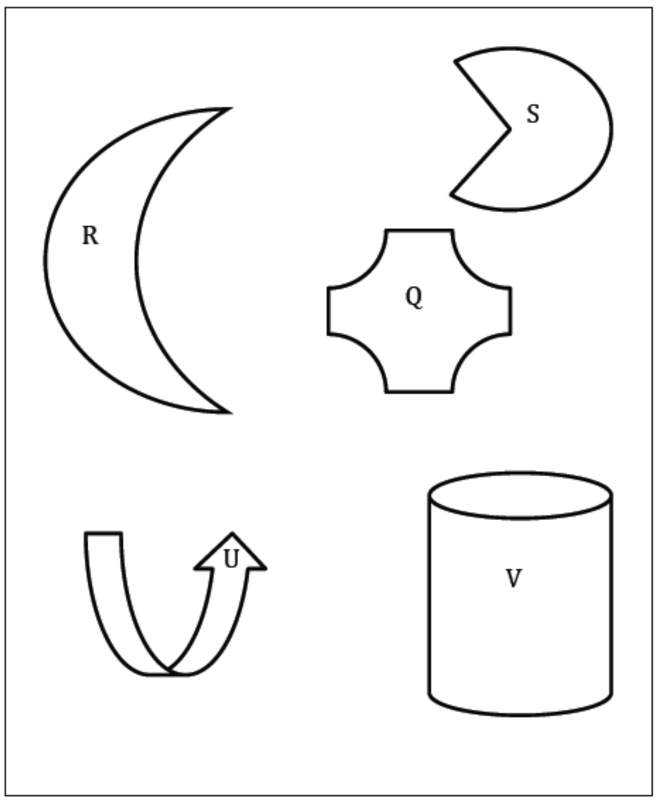


Lesson 2: Definition of Translation and Three Basic Properties

Classwork

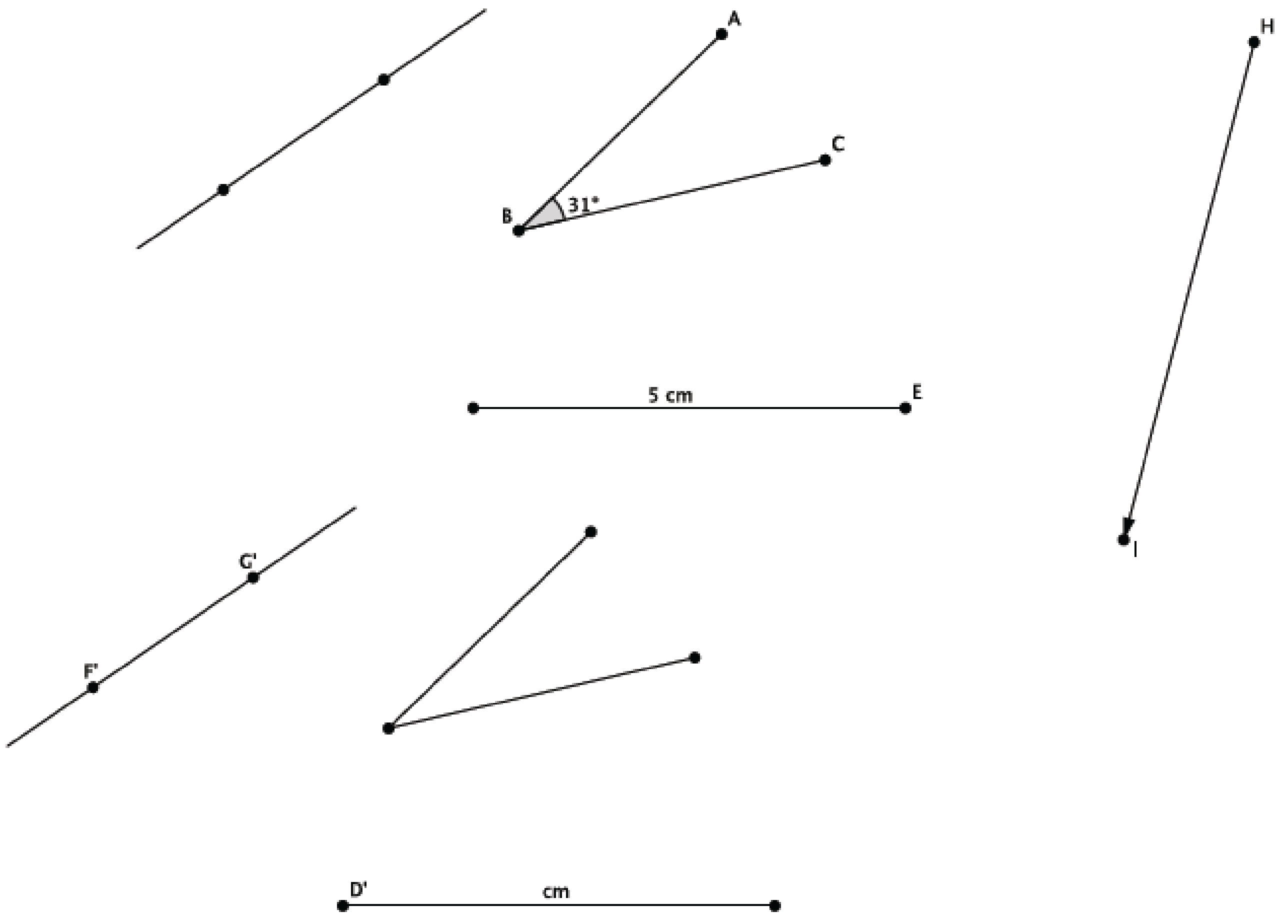
Exercise 1

Draw at least three different vectors, and show what a translation of the plane along each vector will look like. Describe what happens to the following figures under each translation using appropriate vocabulary and notation as needed.



Exercise 2

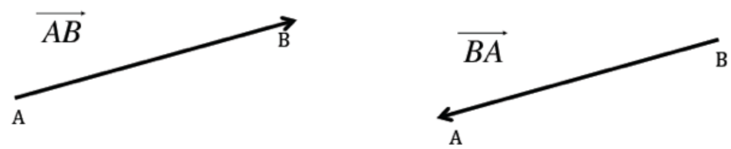
The diagram below shows figures and their images under a translation along $\overrightarrow{HI}$. Use the original figures and the translated images to fill in missing labels for points and measures.



Lesson Summary

Translation occurs along a given vector:

- A vector is a segment in the plane with direction. One of its two endpoints is known as a starting point; while the other is known simply as the endpoint.
- The length of a vector is, by definition, the length of its underlying segment.
- Pictorially note the starting and endpoints:



A translation of a plane along a given vector is a basic rigid motion of a plane.

The three basic properties of translation are as follows:

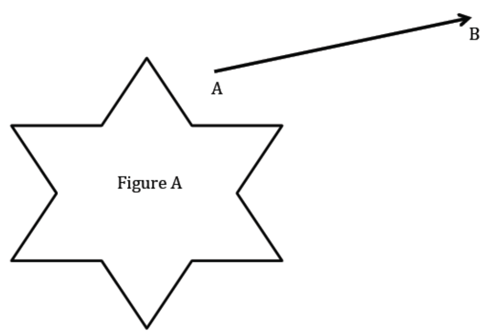
(Translation 1) A translation maps a line to a line, a ray to a ray, a segment to a segment, and an angle to an angle.

(Translation 2) A translation preserves lengths of segments.

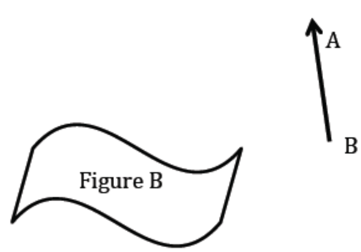
(Translation 3) A translation preserves measures of angles.

Problem Set

1. Translate the plane containing Figure A along $\overrightarrow{AB}$. Use your transparency to sketch the image of Figure A by this translation. Mark points on Figure A and label the image of Figure A accordingly.



2. Translate the plane containing Figure B along $\overrightarrow{BA}$. Use your transparency to sketch the image of Figure B by this translation. Mark points on Figure B and label the image of Figure B accordingly.



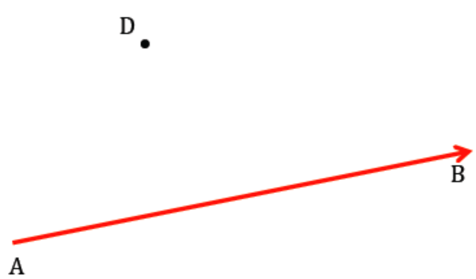
3. Draw an acute angle (your choice of degree), a segment with length 3 cm, a point, a circle with radius 1 in., and a vector (your choice of length, i.e., starting point and ending point). Label points and measures (measurements do not need to be precise, but your figure must be labeled correctly). Use your transparency to translate all of the figures you have drawn along the vector. Sketch the images of the translated figures and label them.

4. What is the length of the translated segment? How does this length compare to the length of the original segment? Explain.

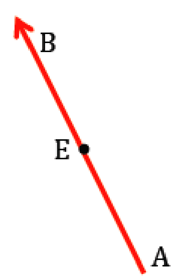
5. What is the length of the radius in the translated circle? How does this radius length compare to the radius of the original circle? Explain.

6. What is the degree of the translated angle? How does this degree compare to the degree of the original angle? Explain.

7. Translate point D along vector $\overrightarrow{AB}$ and label the image D'. What do you notice about the line containing vector $\overrightarrow{AB}$ and the line containing points D and D'? (Hint: Will the lines ever intersect?)



8. Translate point E along vector $\overrightarrow{AB}$ and label the image E'. What do you notice about the line containing vector $\overrightarrow{AB}$ and the line containing points E and E'?

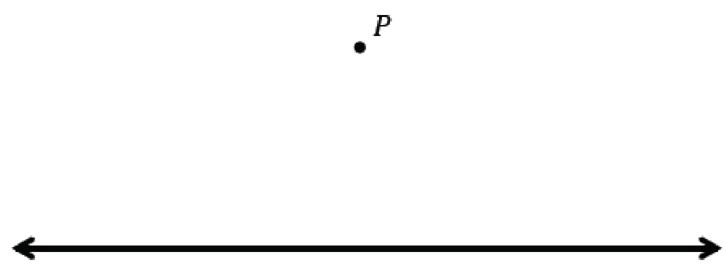


Lesson 3: Translating Lines

Classwork

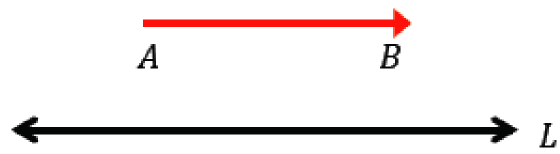
Exercises

1. Draw a line passing through point P that is parallel to line L. Draw a second line passing through point P that is parallel to line L, and that is distinct (i.e., different) from the first one. What do you notice?

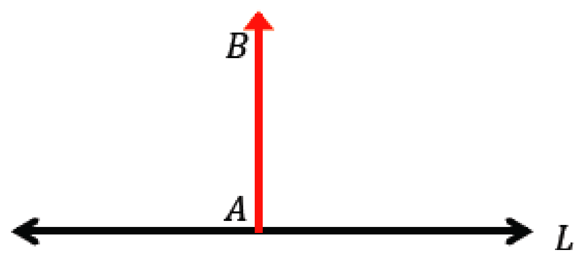


2. Translate line L along the vector $\overrightarrow{AB}$. What do you notice about L and its image L'?

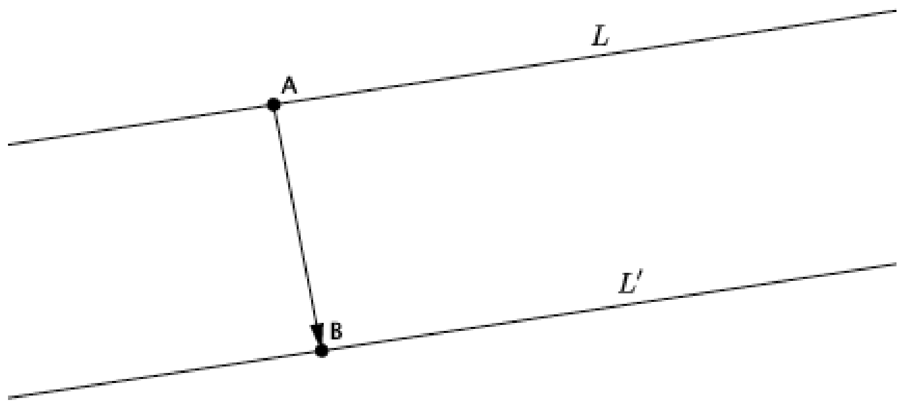
3. Line L is parallel to vector $\overrightarrow{AB}$. Translate line L along vector $\overrightarrow{AB}$. What do you notice about L and its image, L'?



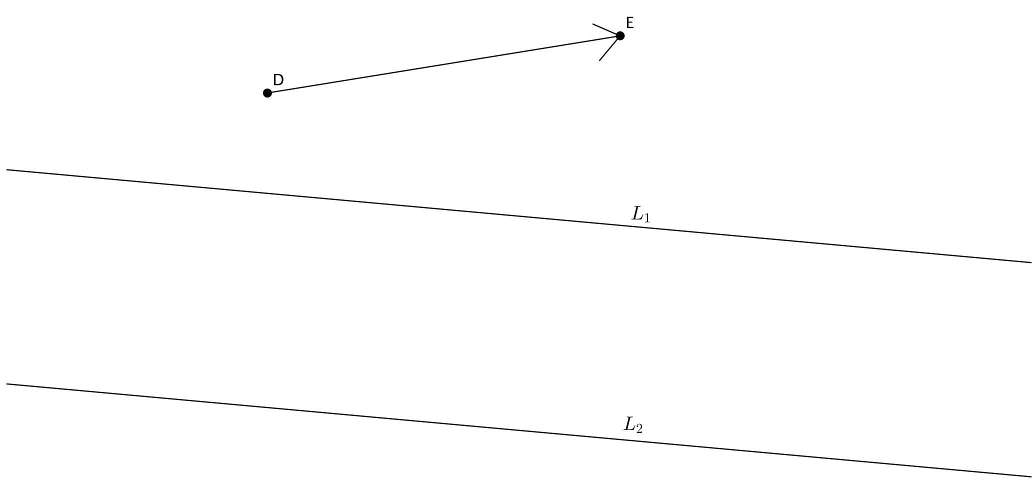
4. Translate line L along the vector $\overrightarrow{AB}$. What do you notice about L and its image, L'?



5. Line L has been translated along vector $\overrightarrow{AB}$ resulting in L'. What do you know about lines L and L'?



6. Translate L_1 and L_2 along vector $\overrightarrow{DE}$. Label the images of the lines. If lines L_1 and L_2 are parallel, what do you know about their translated images?

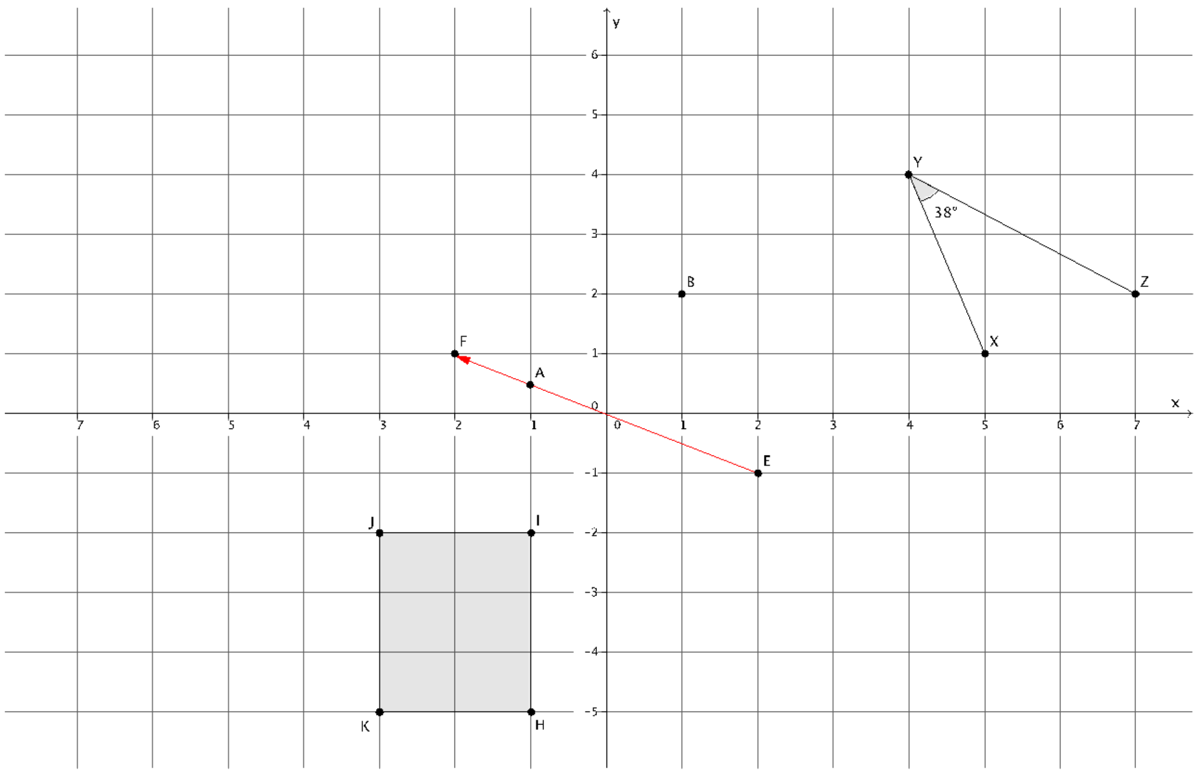


Lesson Summary

- Two lines are parallel if they do not intersect.
- Translations map parallel lines to parallel lines.
- Given a line L and a point P not lying on L, there is at most one line passing through P and parallel to L.

Problem Set

1. Translate $\angle XYZ$, point A, point B, and rectangle $HIJK$ along vector $\overrightarrow{EF}$. Sketch the images and label all points using prime notation.



2. What is the measure of the translated image of $\angle XYZ$. How do you know?

3. Connect B to B'. What do you know about the line formed by BB' and the line containing the vector $\overrightarrow{EF}$?

4. Connect A to A'. What do you know about the line formed by AA' and the line containing the vector $\overrightarrow{EF}$?

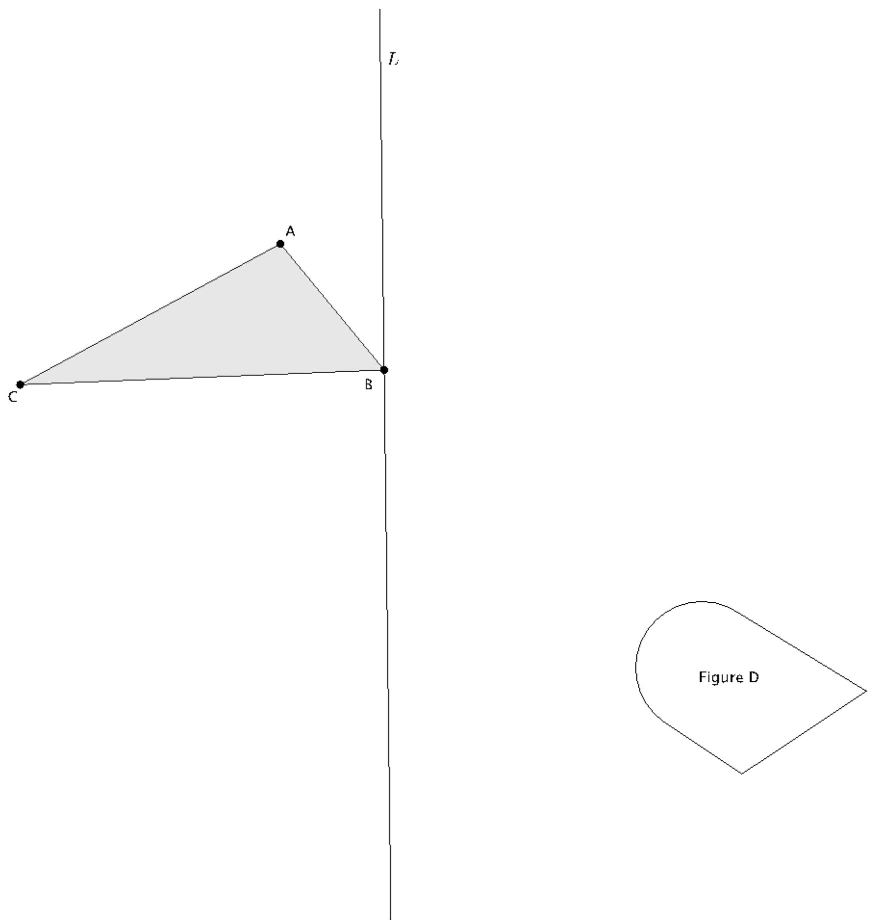
5. Given that figure $HIJK$ is a rectangle, what do you know about lines HI and JK and their translated images? Explain.

Lesson 4: Definition of Reflection and Basic Properties

Classwork

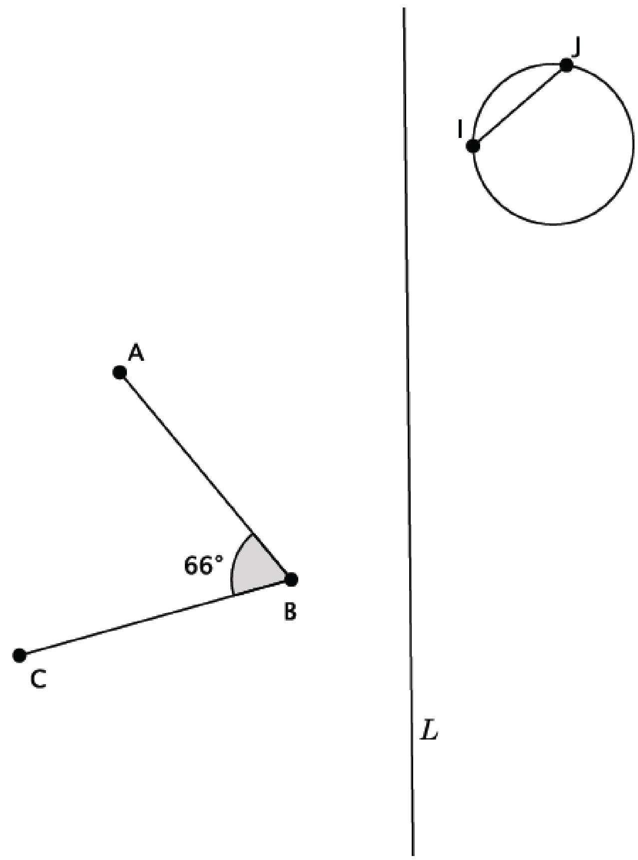
Exercises

1. Reflect $\triangle ABC$ and Figure D across line L. Label the reflected images.



2. Which figure(s) were not moved to a new location on the plane under this transformation?

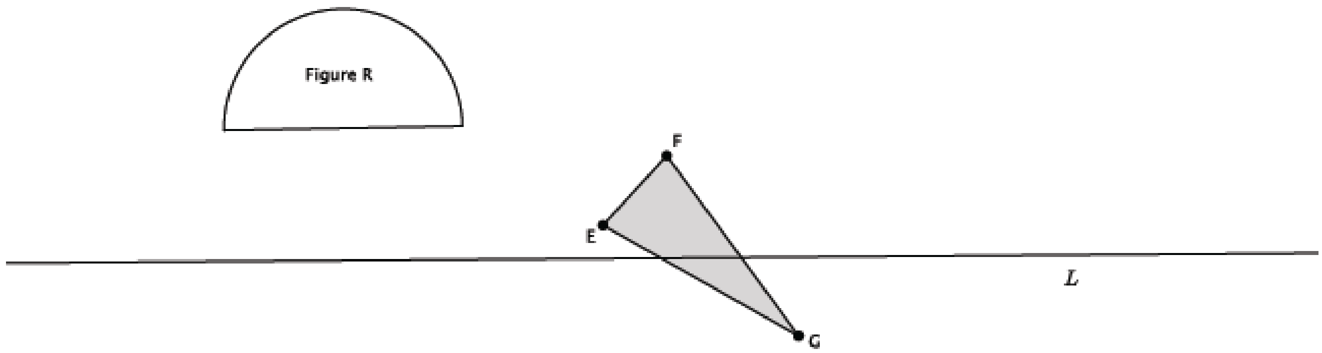
3. Reflect the images across line L. Label the reflected images.



4. Answer the questions about the image above.
   a. Use a protractor to measure the reflected $\angle ABC$. What do you notice?

   b. Use a ruler to measure the length of IJ and the length of the image of IJ after the reflection. What do you notice?

5. Reflect Figure R and ΔEFG across line L. Label the reflected images.



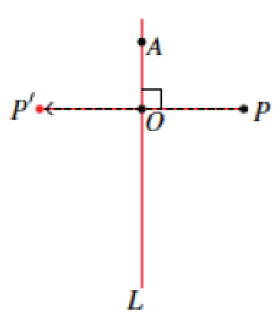
Basic Properties of Reflections:

(Reflection 1) A reflection maps a line to a line, a ray to a ray, a segment to a segment, and an angle to an angle.

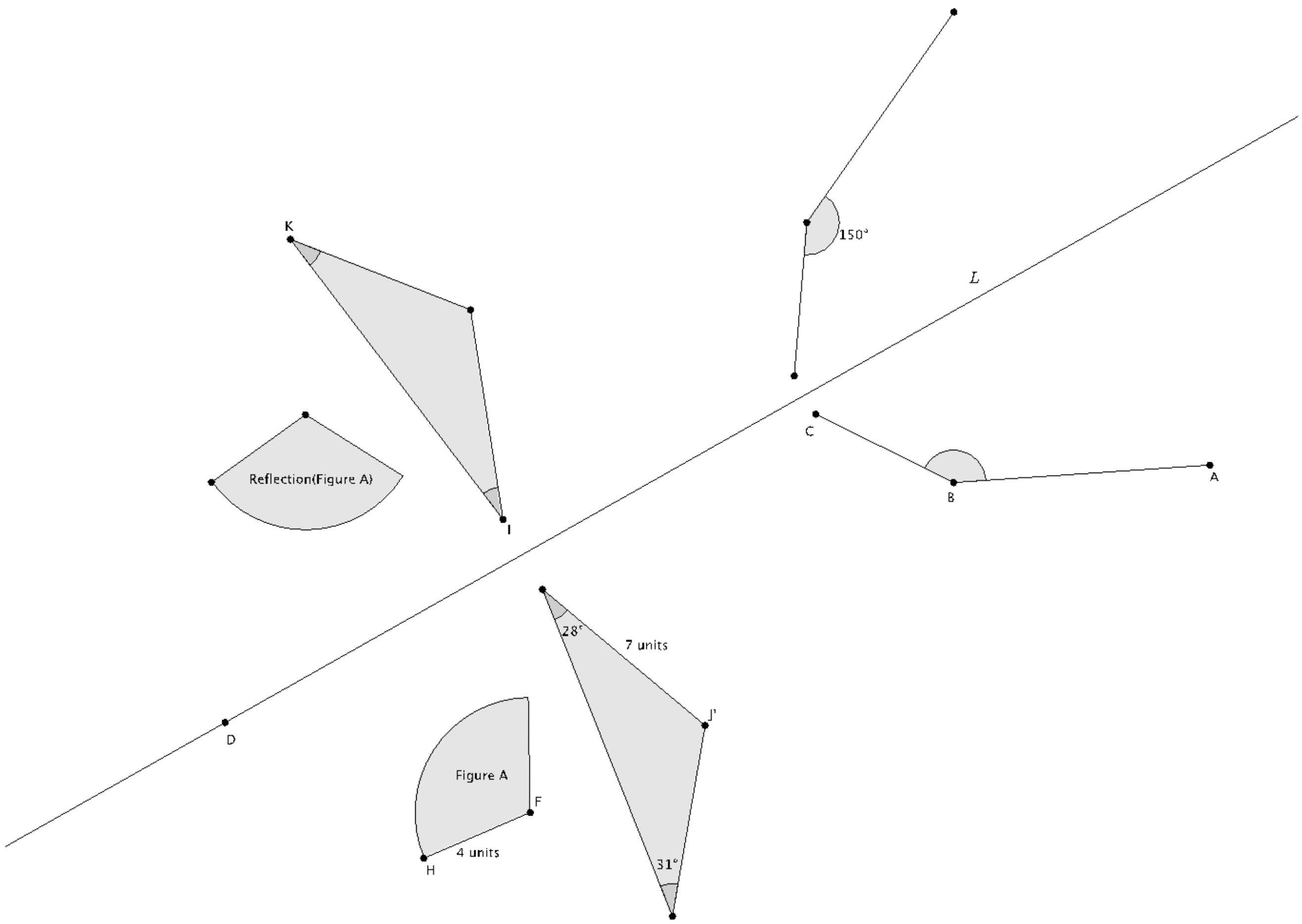
(Reflection 2) A reflection preserves lengths of segments.

(Reflection 3) A reflection preserves measures of angles.

If the reflection is across a line L and P is a point not on L, then L bisects the segment PP', joining P to its reflected image P'. That is, the lengths of OP and OP' are equal.



Use the picture below for Exercises 6–9.



6. Use the picture to label the unnamed points.

7. What is the measure of $\angle JKI$? $\angle KIJ$? $\angle ABC$? How do you know?

8. What is the length of segment $Reflection(FH)$? IJ? How do you know?

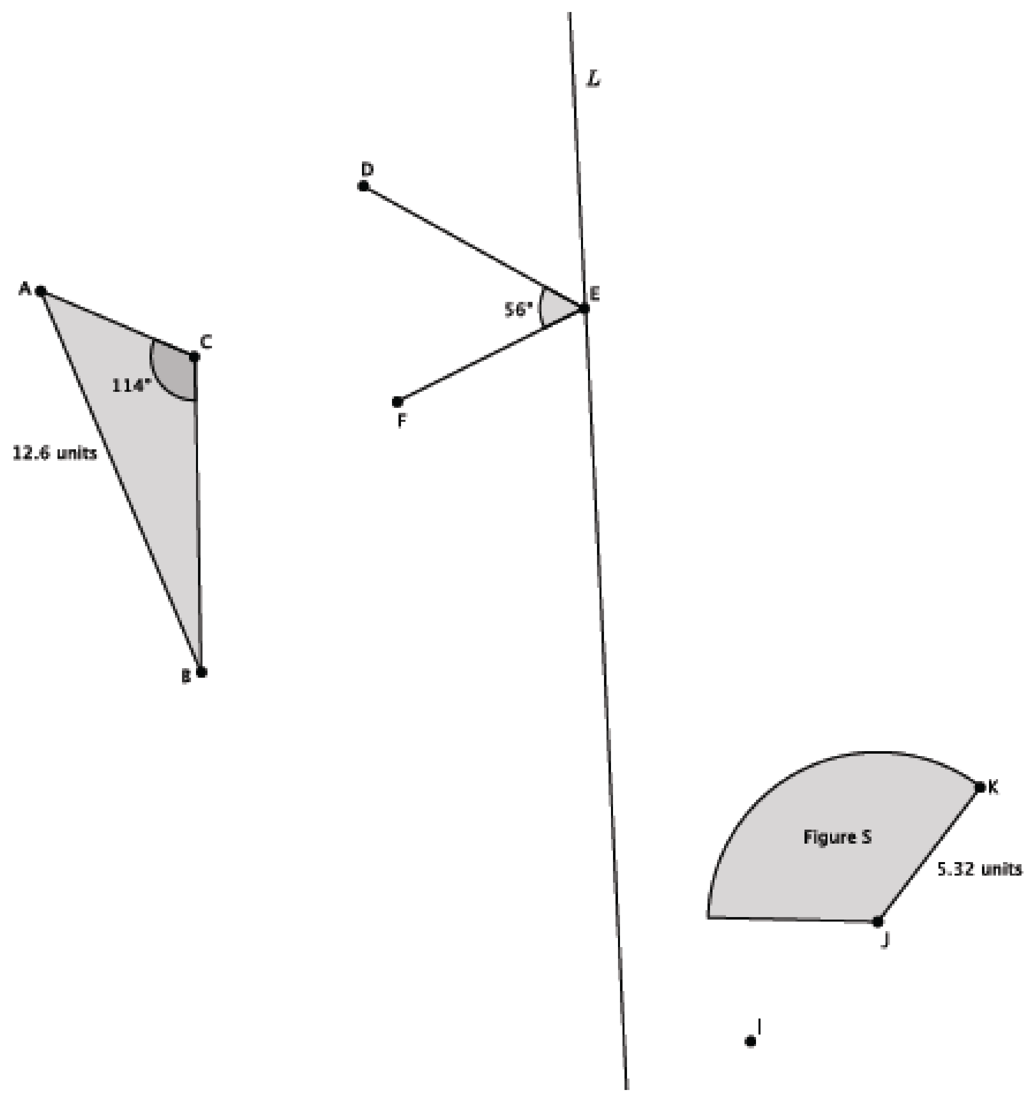
9. What is the location of $Reflection(D)$? Explain.

Lesson Summary

- A reflection is another type of basic rigid motion.
- Reflections occur across lines. The line that you reflect across is called the line of reflection.
- When a point, P, is joined to its reflection, P', the line of reflection bisects the segment, PP'.

Problem Set

1. In the picture below, $\angle DEF = 56°$, $\angle ACB = 114°$, $AB = 12.6$ units, $JK = 5.32$ units, point E is on line L, and point I is off of line L. Let there be a reflection across line L. Reflect and label each of the figures, and answer the questions that follow.



2. What is the measure of $Reflection(\angle DEF)$? Explain.

3. What is the length of $Reflection(JK)$? Explain.

4. What is the measure of $Reflection(\angle ACB)$?

5. What is the length of $Reflection(AB)$?

6. Two figures in the picture were not moved under the reflection. Name the two figures and explain why they were not moved.

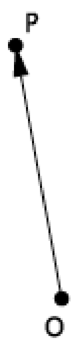
7. Connect points I and I'. Name the point of intersection of the segment with the line of reflection point Q. What do you know about the lengths of segments IQ and QI'?

Lesson 5: Definition of Rotation and Basic Properties

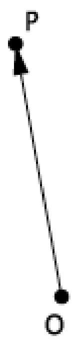
Classwork

Exercises

1. Let there be a rotation of d degrees around center O. Let P be a point other than O. Select d so that $d \geq 0$. Find P' (i.e., the rotation of point P) using a transparency.



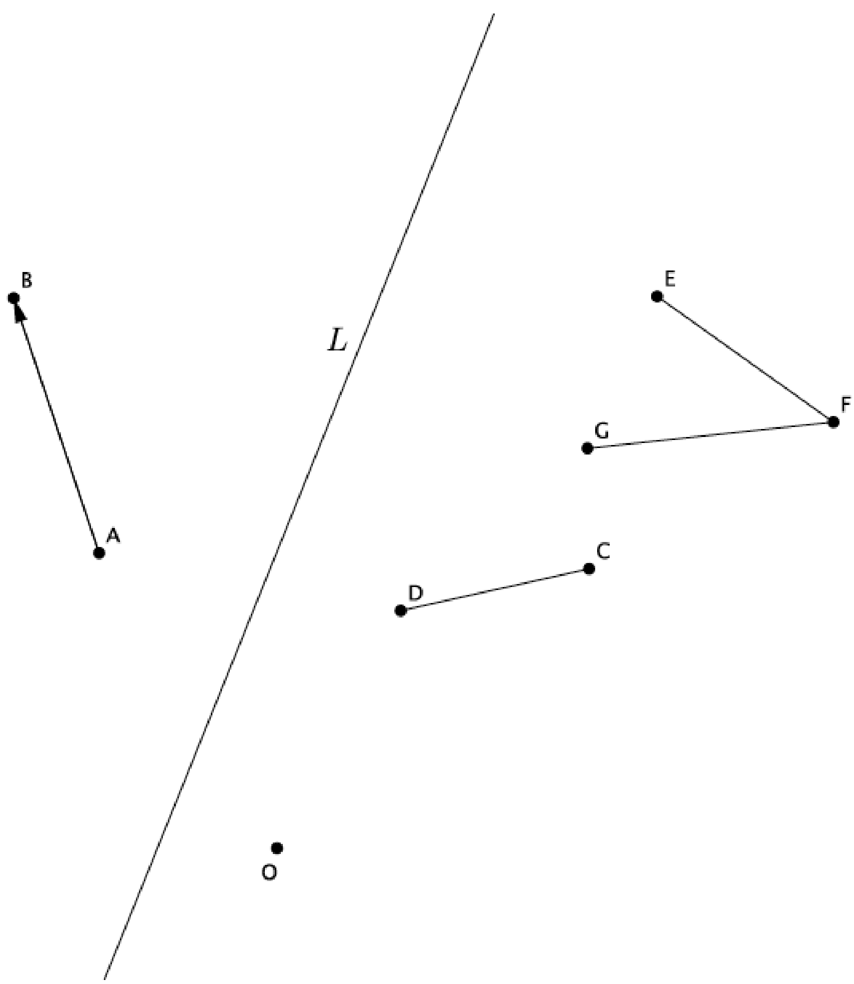
2. Let there be a rotation of d degrees around center O. Let P be a point other than O. Select d so that $d < 0$. Find P' (i.e., the rotation of point P) using a transparency.



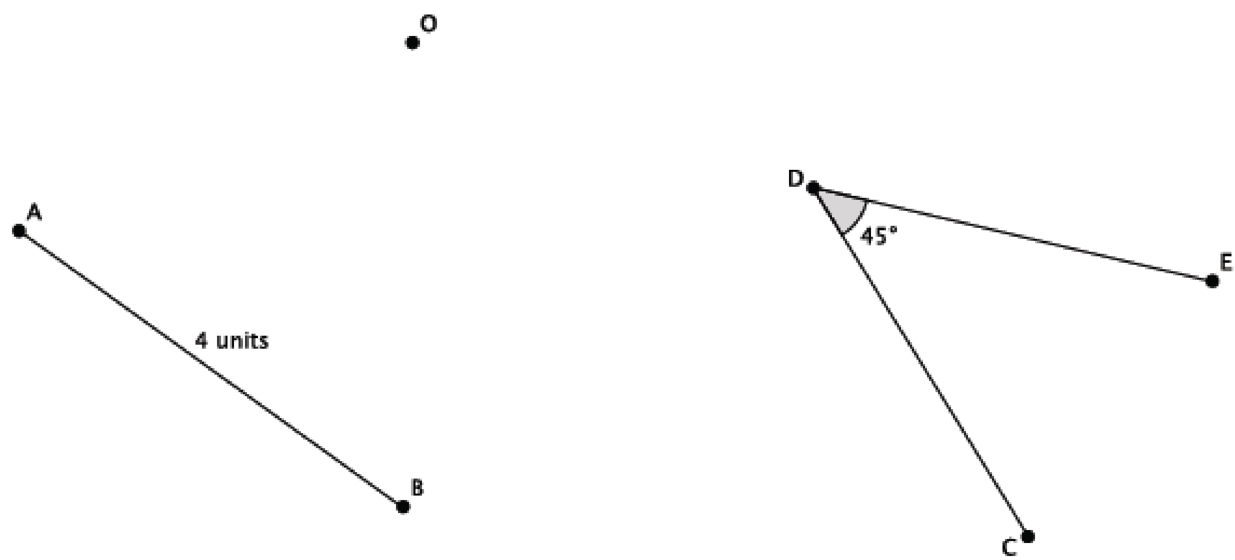
3. Which direction did the point P rotate when $d \geq 0$?

4. Which direction did the point P rotate when $d < 0$?

5. Let L be a line, $\overrightarrow{AB}$ be a ray, CD be a segment, and $\angle EFG$ be an angle, as shown. Let there be a rotation of d degrees around point O. Find the images of all figures when $d \geq 0$.



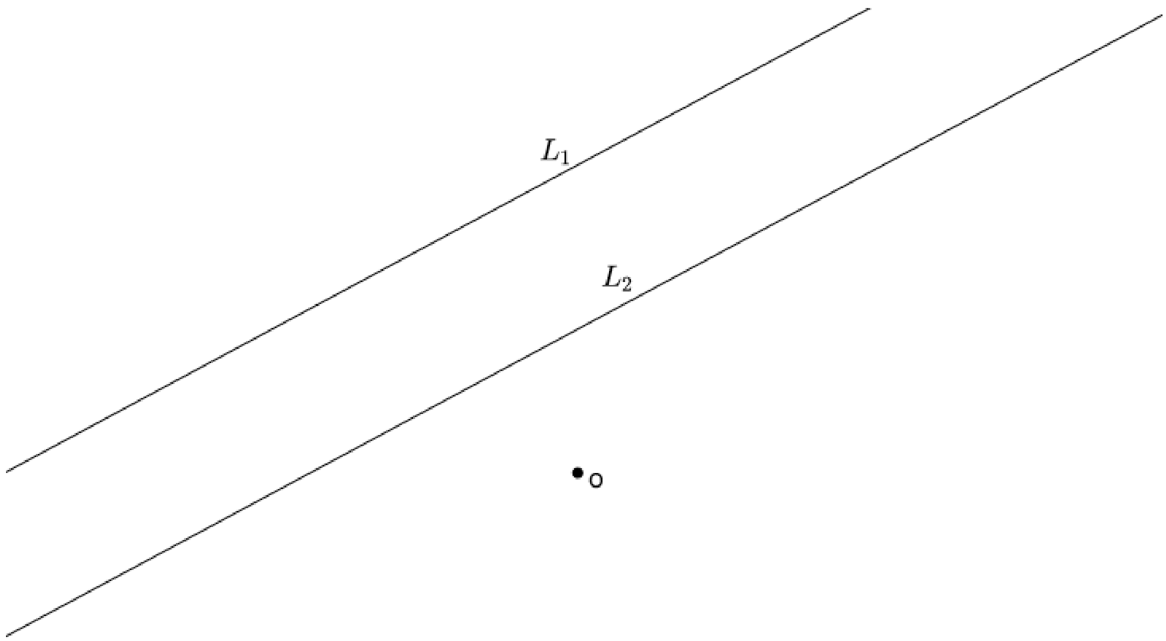
6. Let $\overline{AB}$ be a segment of length 4 units and $\angle CDE$ be an angle of size $45°$. Let there be a rotation by d degrees, where $d < 0$, about O. Find the images of the given figures. Answer the questions that follow.



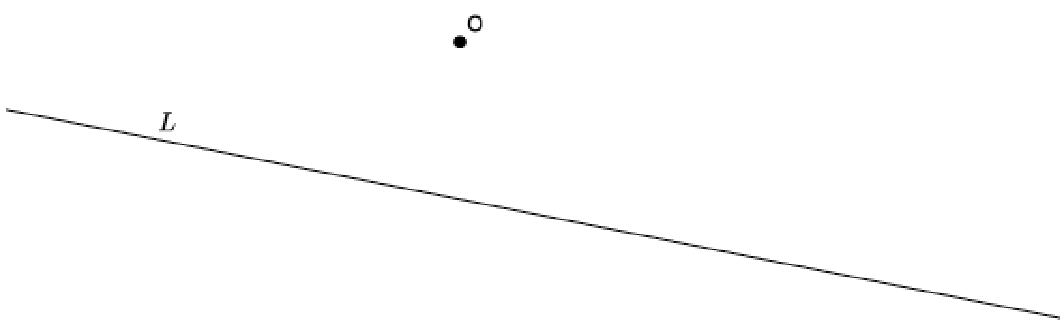
a. What is the length of the rotated segment $Rotation(AB)$?

b. What is the degree of the rotated angle $Rotation(\angle CDE)$?

7. Let L_1 and L_2 be parallel lines. Let there be a rotation by d degrees, where $-360 < d < 360$, about O. Is $(L_1)' \parallel (L_2)'$?



8. Let L be a line and O be the center of rotation. Let there be a rotation by d degrees, where $d \neq 180$ about O. Are the lines L and L' parallel?



Lesson Summary

Rotations require information about the center of rotation and the degree in which to rotate. Positive degrees of rotation move the figure in a counterclockwise direction. Negative degrees of rotation move the figure in a clockwise direction.

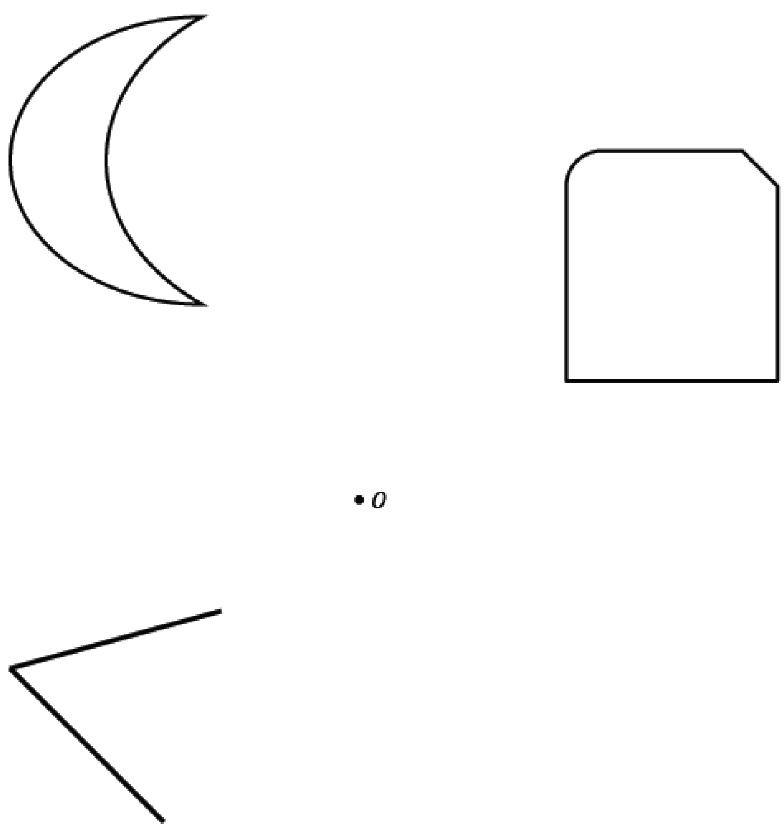
Basic Properties of Rotations:

- (Rotation 1) A rotation maps a line to a line, a ray to a ray, a segment to a segment, and an angle to an angle.
- (Rotation 2) A rotation preserves lengths of segments.
- (Rotation 3) A rotation preserves measures of angles.

When parallel lines are rotated, their images are also parallel. A line is only parallel to itself when rotated exactly $180°$.

Problem Set

1. Let there be a rotation by $-90°$ around the center O.



2. Explain why a rotation of 90 degrees around any point O never maps a line to a line parallel to itself.

3. A segment of length 94 cm has been rotated d degrees around a center O. What is the length of the rotated segment? How do you know?

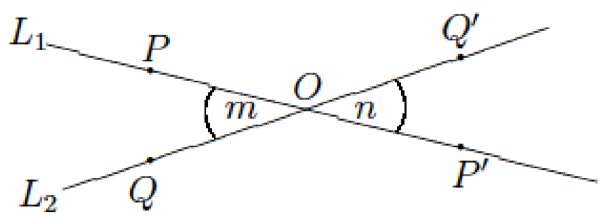
4. An angle of size $124°$ has been rotated d degrees around a center O. What is the size of the rotated angle? How do you know?

Lesson 6: Rotations of 180 Degrees

Classwork

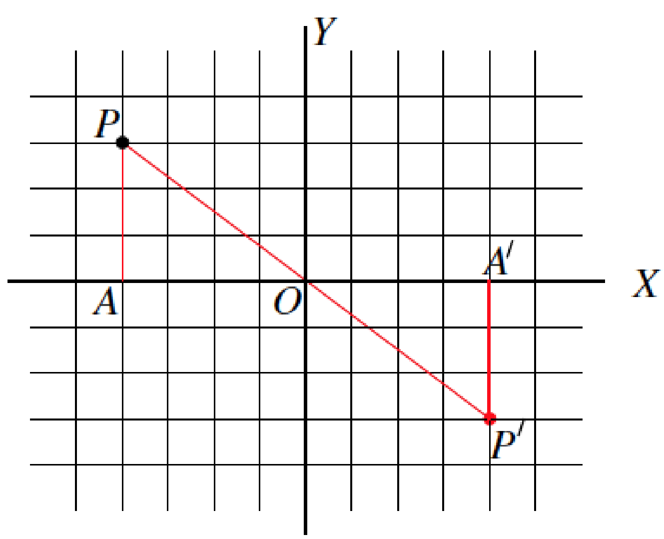
Example 1

The picture below shows what happens when there is a rotation of $180°$ around center O.



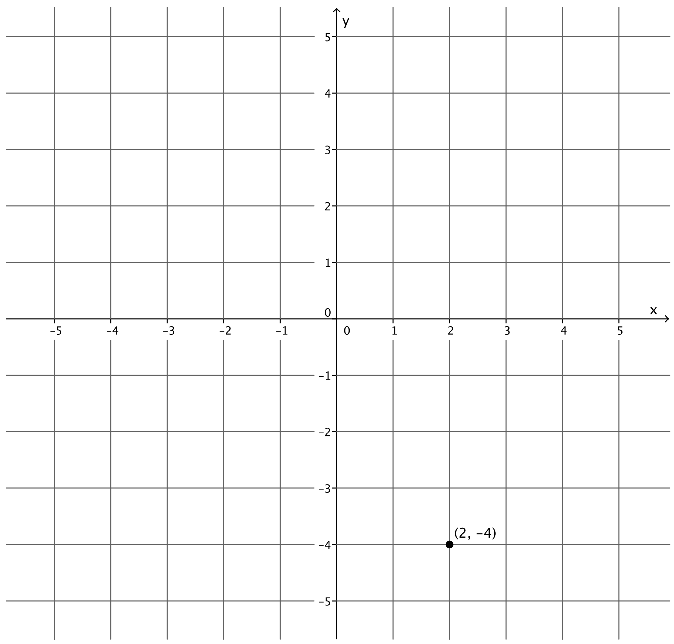
Example 2

The picture below shows what happens when there is a rotation of $180°$ around center O, the origin of the coordinate plane.

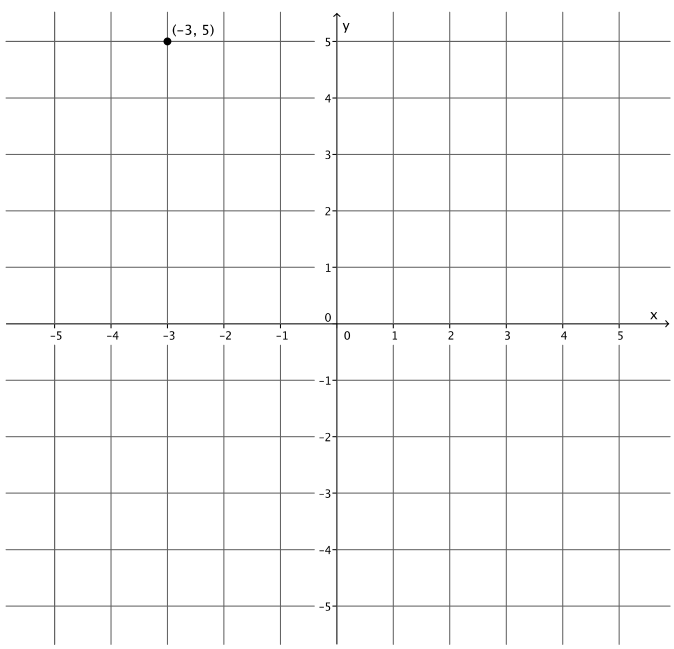


Exercises 1–9

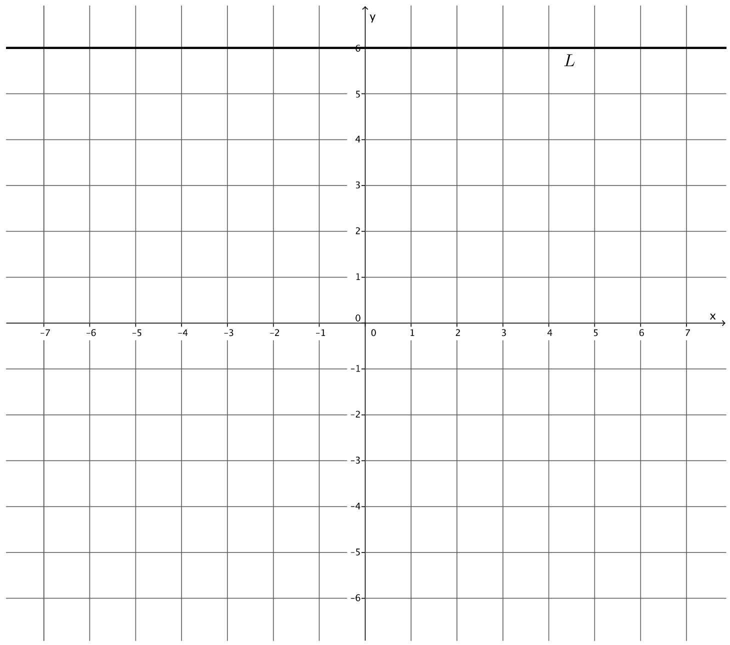
1. Using your transparency, rotate the plane 180 degrees, about the origin. Let this rotation be $Rotation_0$. What are the coordinates of $Rotation_0(2, -4)$?



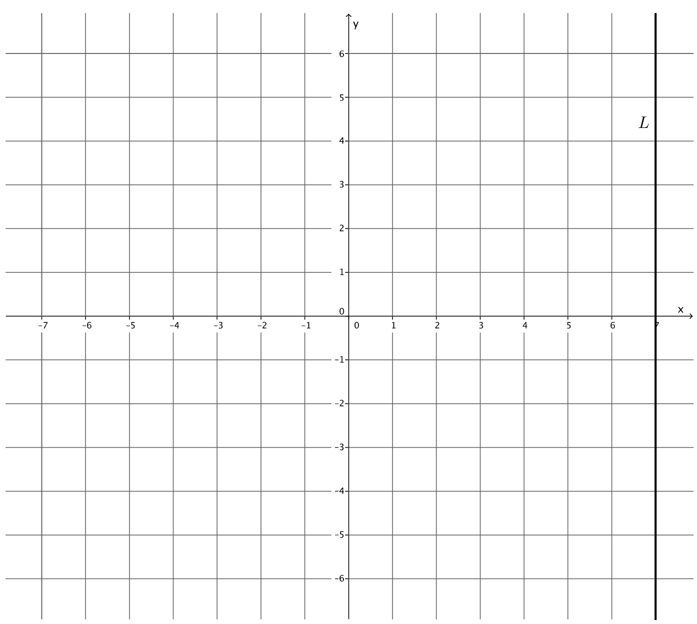
2. Let $Rotation_0$ be the rotation of the plane by 180 degrees, about the origin. Without using your transparency, find $Rotation_0(-3, 5)$.



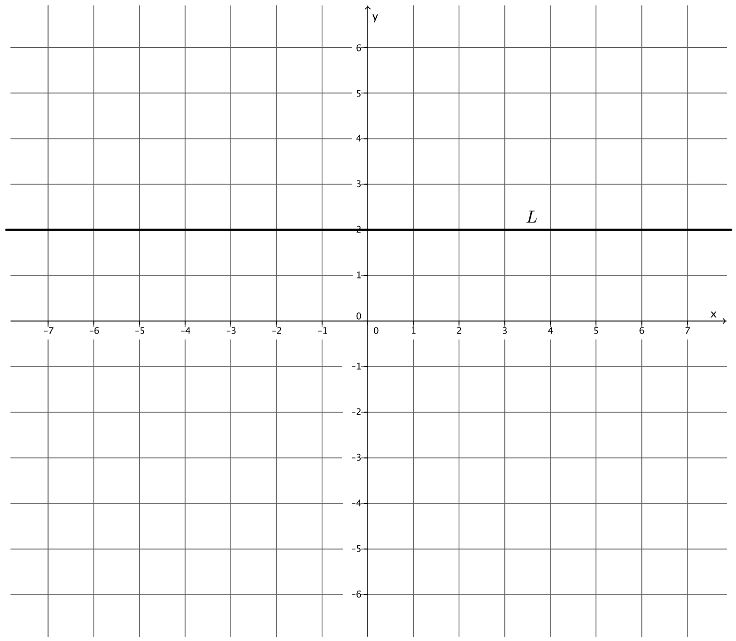
3. Let $Rotation_0$ be the rotation of 180 degrees around the origin. Let L be the line passing through $(-6,6)$ parallel to the x-axis. Find $Rotation_0(L)$. Use your transparency if needed.



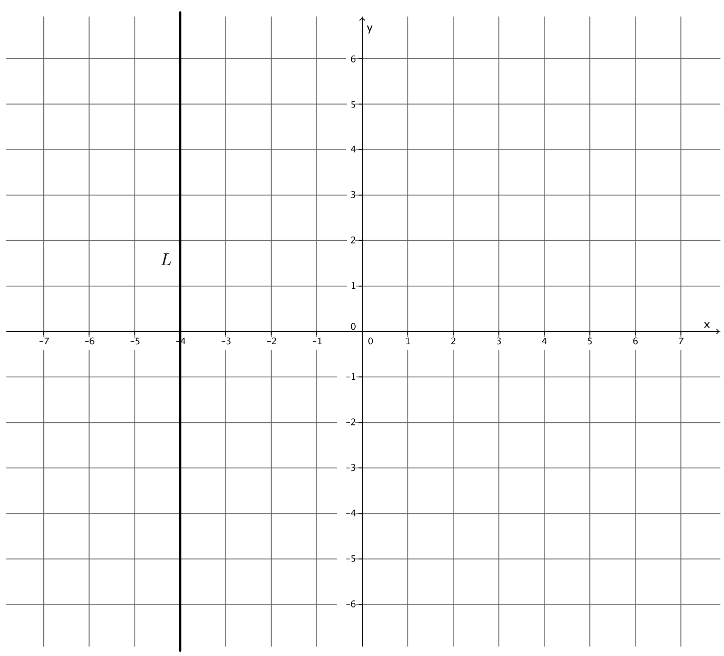
4. Let $Rotation_0$ be the rotation of 180 degrees around the origin. Let L be the line passing through $(7,0)$ parallel to the y-axis. Find $Rotation_0(L)$. Use your transparency if needed.



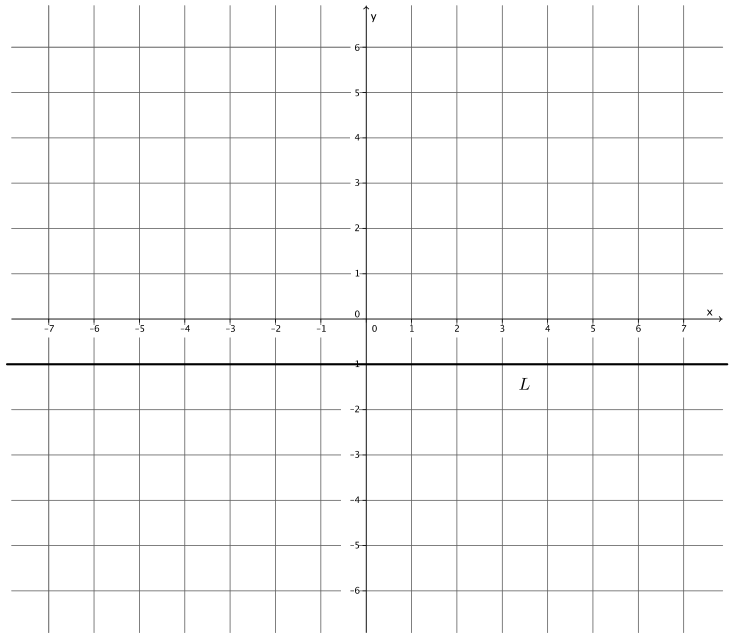
5. Let $Rotation_0$ be the rotation of 180 degrees around the origin. Let L be the line passing through $(0,2)$ parallel to the x-axis. Is L parallel to $Rotation_0(L)$?



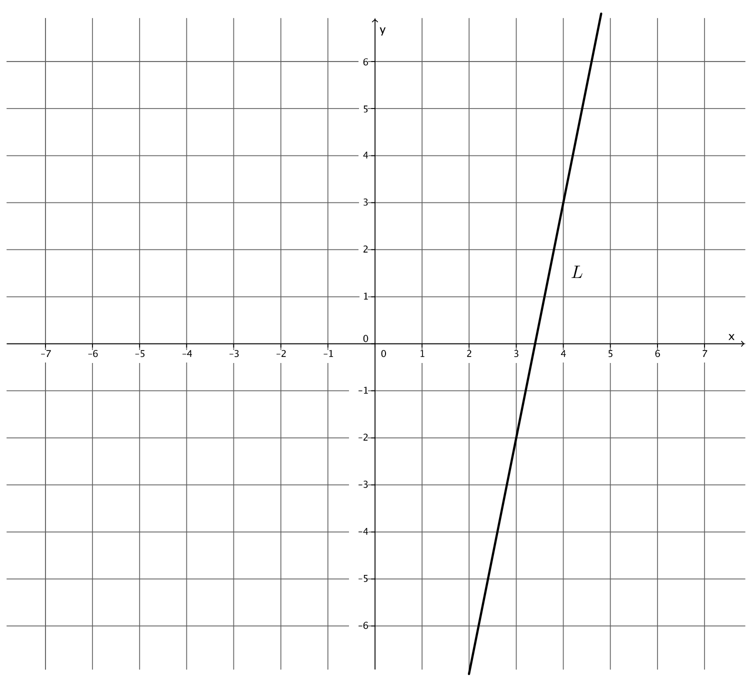
6. Let $Rotation_0$ be the rotation of 180 degrees around the origin. Let L be the line passing through $(4,0)$ parallel to the y-axis. Is L parallel to $Rotation_0(L)$?



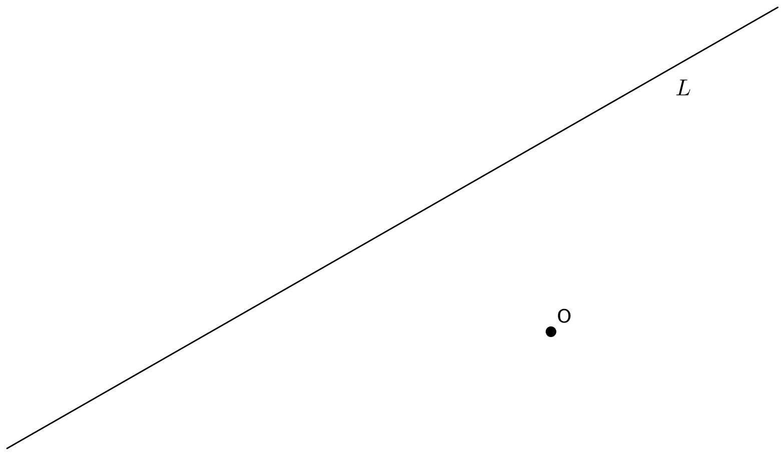
7. Let $Rotation_0$ be the rotation of 180 degrees around the origin. Let L be the line passing through $(0, -1)$ parallel to the x-axis. Is L parallel to $Rotation_0(L)$?



8. Let $Rotation_0$ be the rotation of 180 degrees around the origin. Is L parallel to $Rotation_0(L)$? Use your transparency if needed.



9. Let $Rotation_0$ be the rotation of 180 degrees around the origin. Is L parallel to $Rotation_0(L)$? Use your transparency if needed.



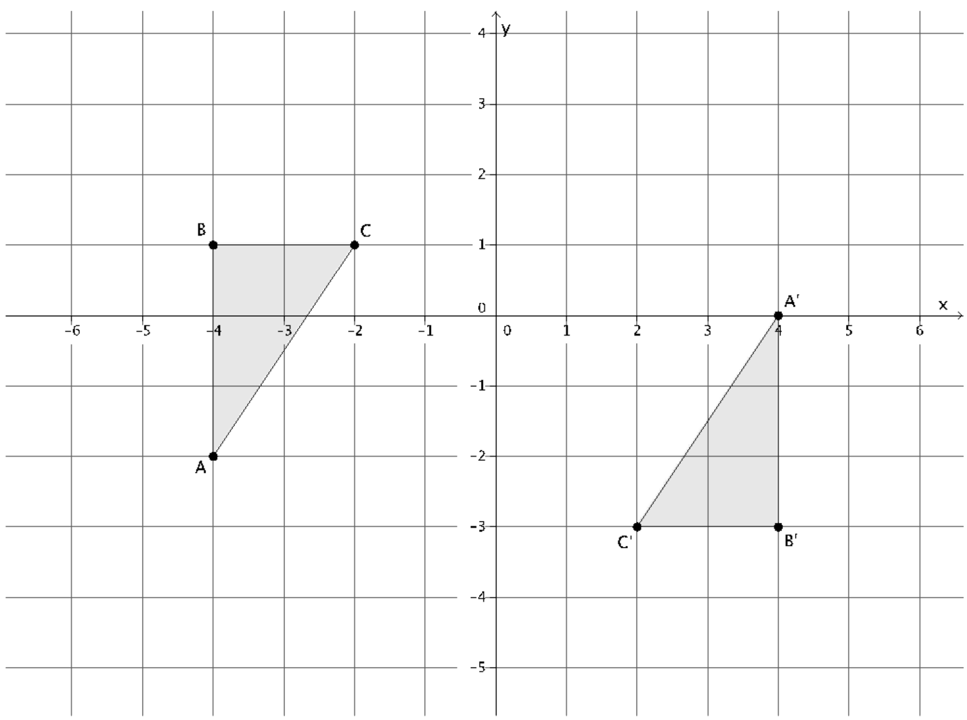
Lesson Summary

- A rotation of 180 degrees around O is the rigid motion so that if P is any point in the plane, P, O, and $Rotation(P)$ are *collinear* (i.e., lie on the same line).
- Given a 180-degree rotation, R_0 around the origin O of a coordinate system, and a point P with coordinates (a, b), it is generally said that $R_0(P)$ is the point with coordinates $(-a, -b)$.

Theorem: Let O be a point not lying on a given line L. Then, the 180-degree rotation around O maps L to a line parallel to L.

Problem Set

Use the following diagram for Problems 1–5. Use your transparency as needed.



1. Looking only at segment BC, is it possible that a 180° rotation would map BC onto $B'C'$? Why or why not?

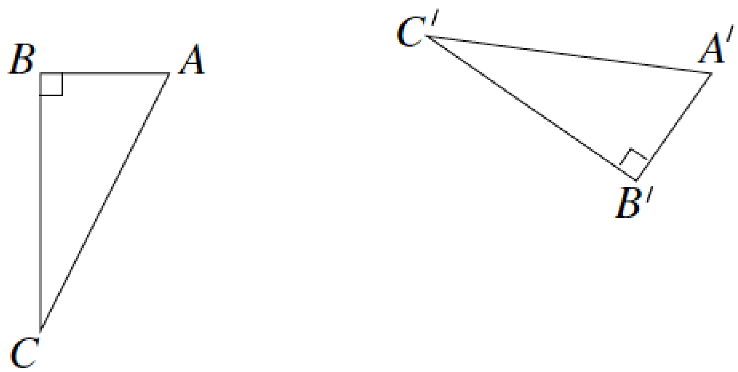
2. Looking only at segment AB, is it possible that a 180° rotation would map AB onto $A'B'$? Why or why not?

3. Looking only at segment AC, is it possible that a 180° rotation would map AC onto $A'C'$? Why or why not?

4. Connect point B to point B', point C to point C', and point A to point A'. What do you notice? What do you think that point is?

5. Would a rotation map triangle ABC onto triangle $'B'C'$? If so, define the rotation (i.e., degree and center). If not, explain why not.

6. The picture below shows right triangles ABC and $A'B'C'$, where the right angles are at B and B'. Given that $AB = A'B' = 1$, and $BC = B'C' = 2$, and that AB is not parallel to $A'B'$, is there a $180°$ rotation that would map ΔABC onto $\triangle A'B'C'$? Explain.

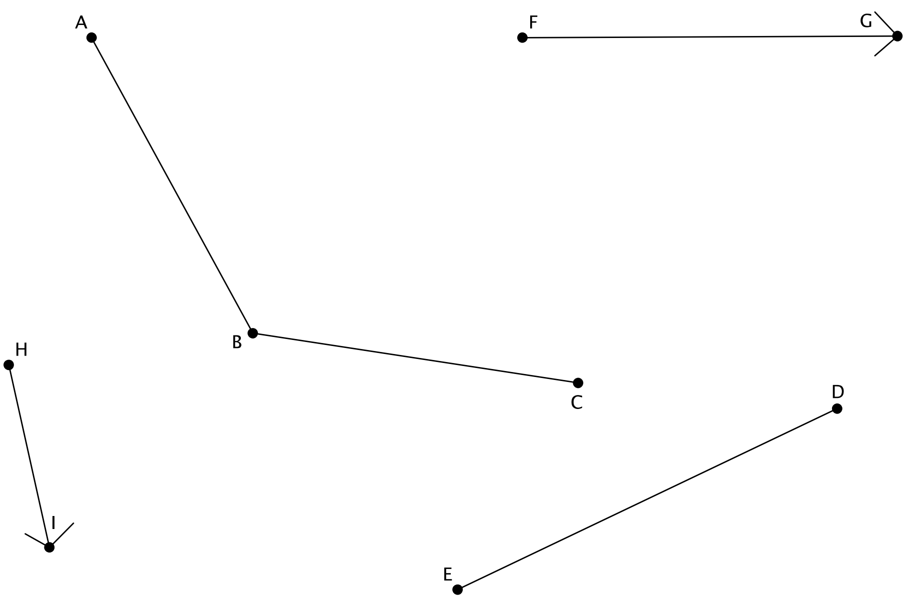


Lesson 7: Sequencing Translations

Classwork

Exploratory Challenge

1.



a. Translate $\angle ABC$ and segment ED along vector $\overrightarrow{FG}$. Label the translated images appropriately, i.e., $A'B'C'$ and $E'D'$.

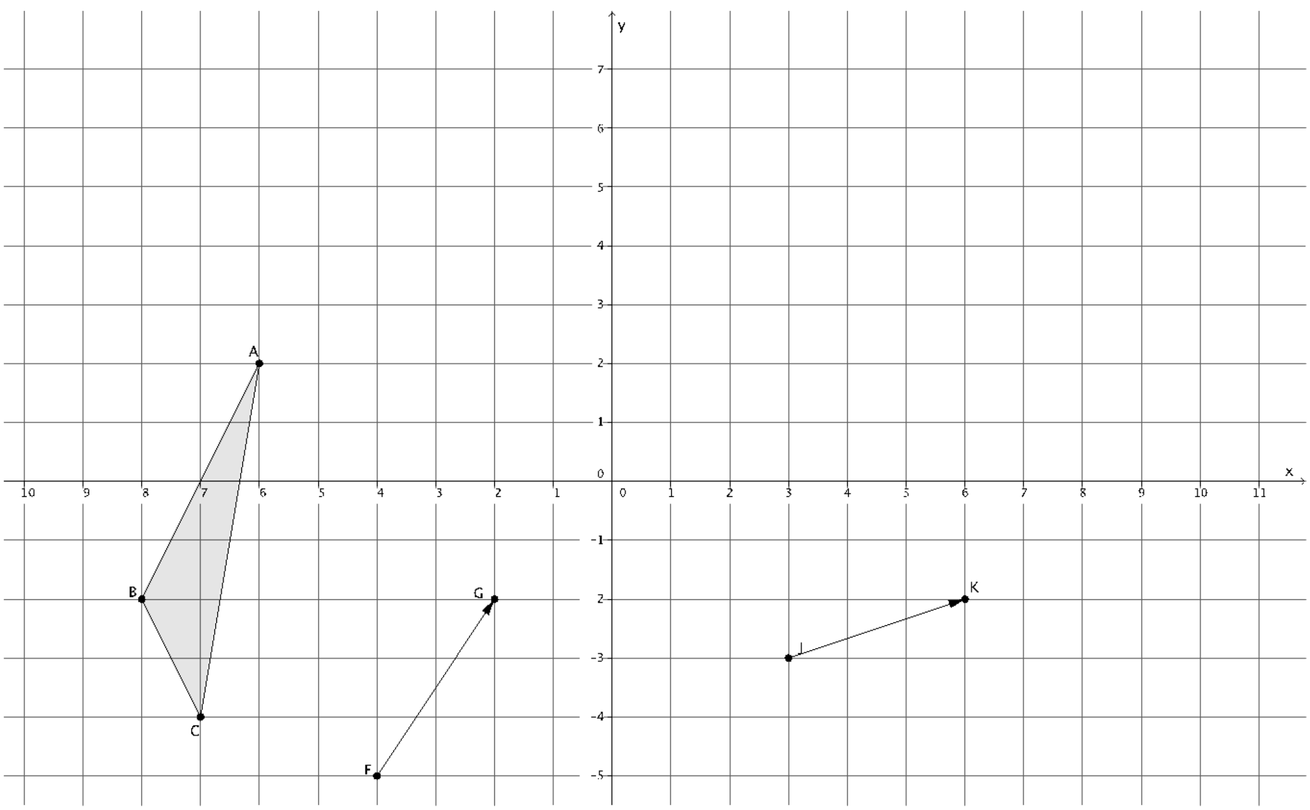
b. Translate $\angle A'B'C'$ and segment $E'D'$ along vector $\overrightarrow{HI}$. Label the translated images appropriately, i.e., $A''B''C''$ and $E''D''$.

c. How does the size of $\angle ABC$ compare to the size of $\angle A''B''C''$?

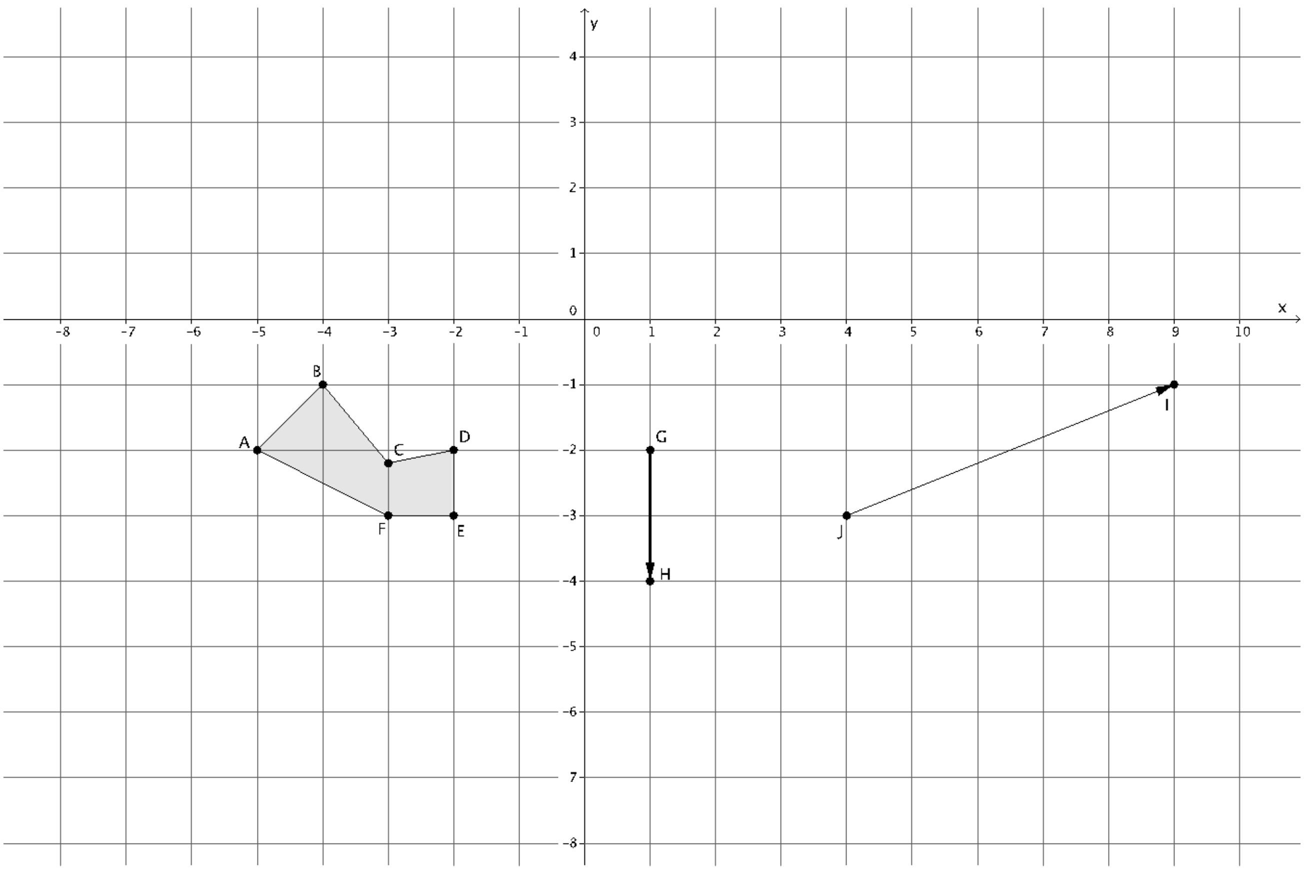
d. How does the length of segment ED compare to the length of the segment $E''D''$?

e. Why do you think what you observed in parts (d) and (e) were true?

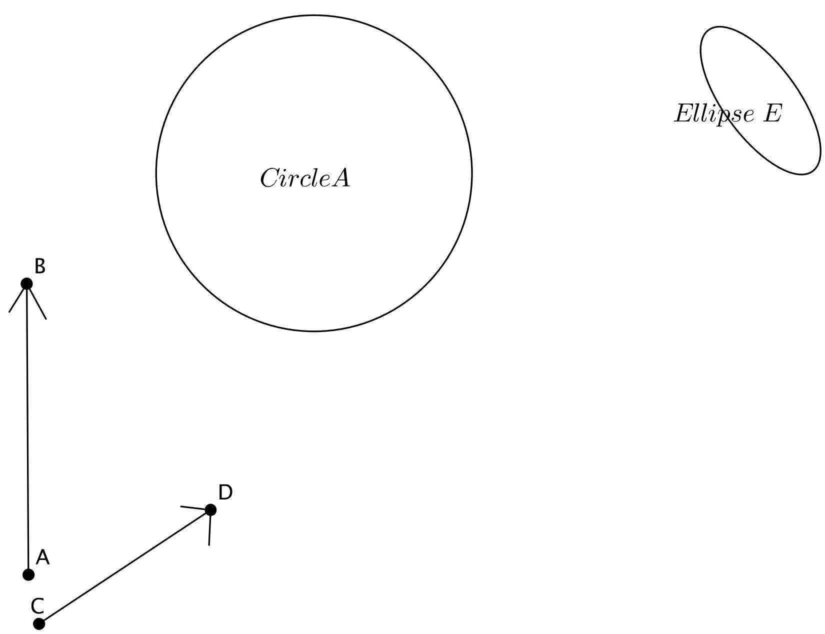
2. Translate $\triangle ABC$ along vector $\overrightarrow{FG}$ and then translate its image along vector $\overrightarrow{JK}$. Be sure to label the images appropriately.



3. Translate figure $ABCDEF$ along vector $\overrightarrow{GH}$. Then translate its image along vector $\overrightarrow{JI}$. Label each image appropriately.



4.

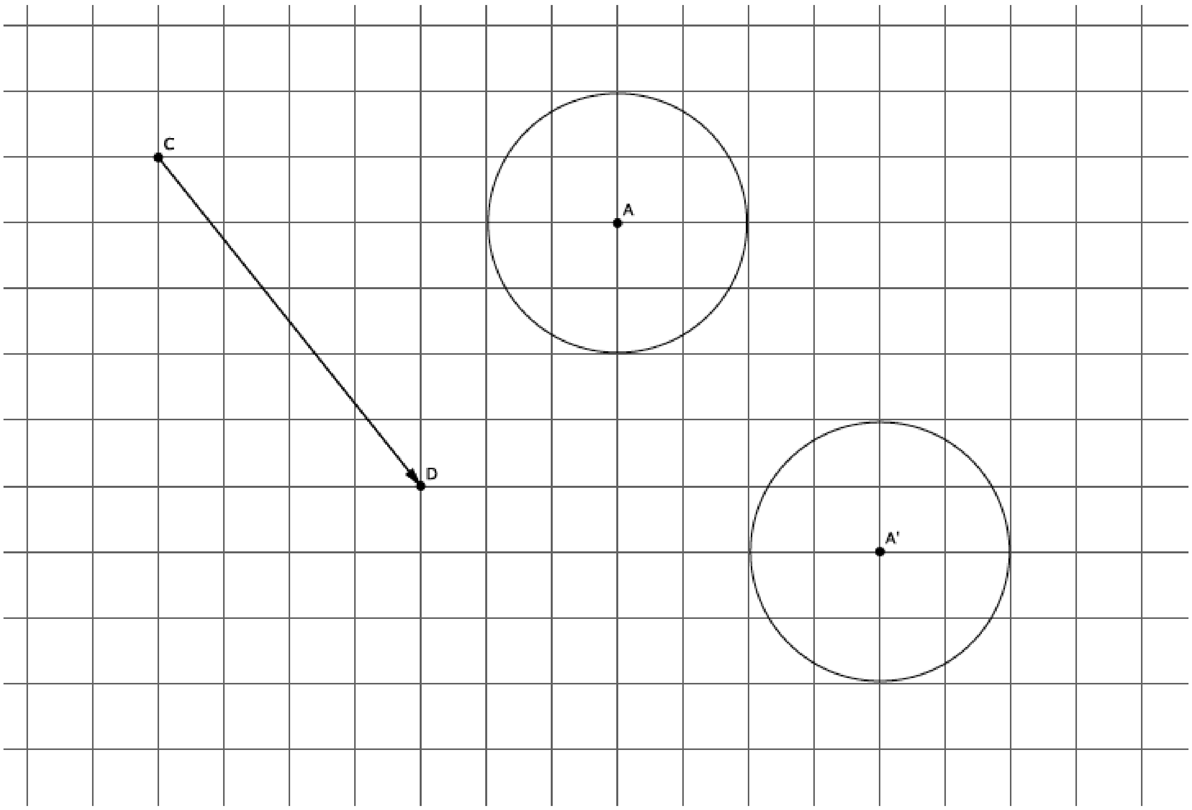


a. Translate Circle A and Ellipse E along vector $\overrightarrow{AB}$. Label the images appropriately.

b. Translate Circle A' and Ellipse E' along vector $\overrightarrow{CD}$. Label each image appropriately.

c. Did the size or shape of either figure change after performing the sequence of translations? Explain.

5. The picture below shows the translation of Circle A along vector $\overrightarrow{CD}$. Name the vector that will map the image of Circle A back to its original position.



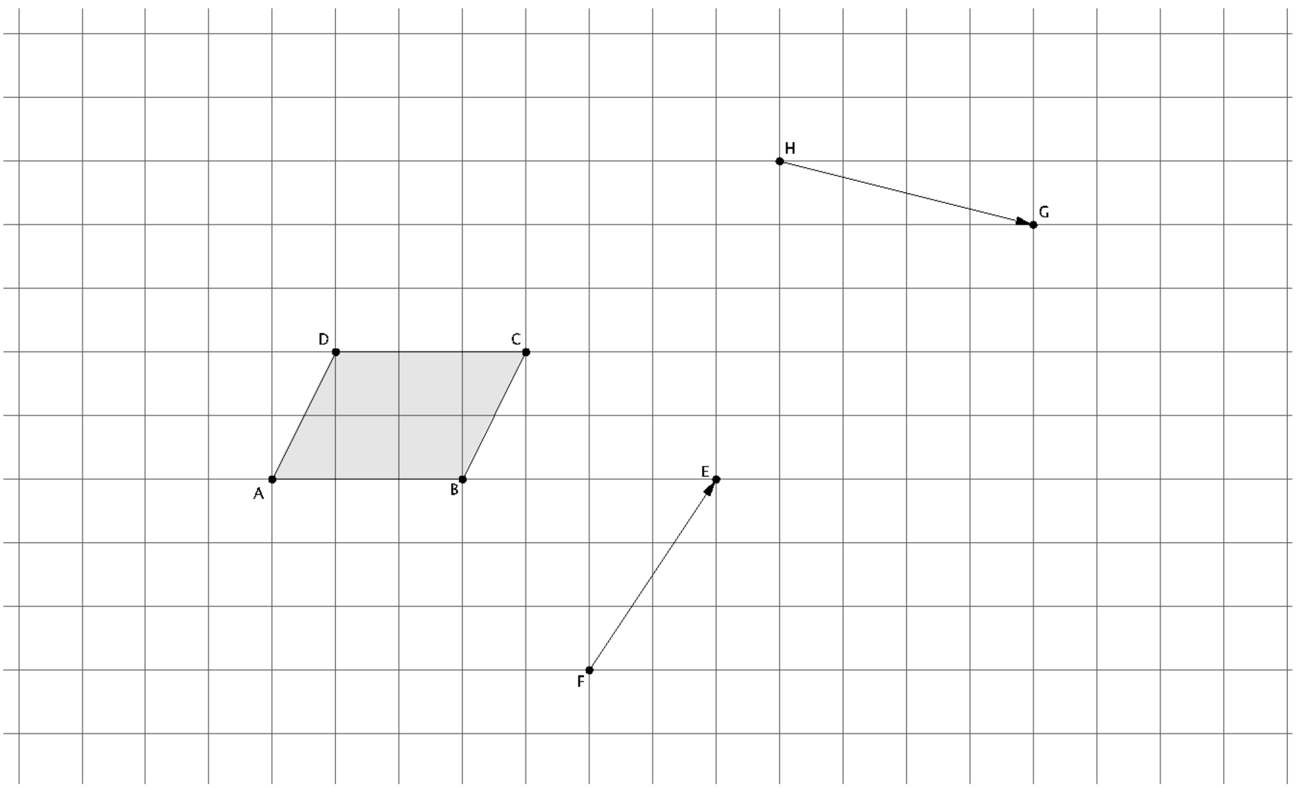
6. If a figure is translated along vector $\overrightarrow{QR}$, what translation takes the figure back to its original location?

Lesson Summary

- Translating a figure along one vector then translating its image along another vector is an example of a sequence of transformations.
- A sequence of translations enjoys the same properties as a single translation. Specifically, the figures' lengths and degrees of angles are preserved.
- If a figure undergoes two transformations, F and G, and is in the same place it was originally, then the figure has been mapped onto itself.

Problem Set

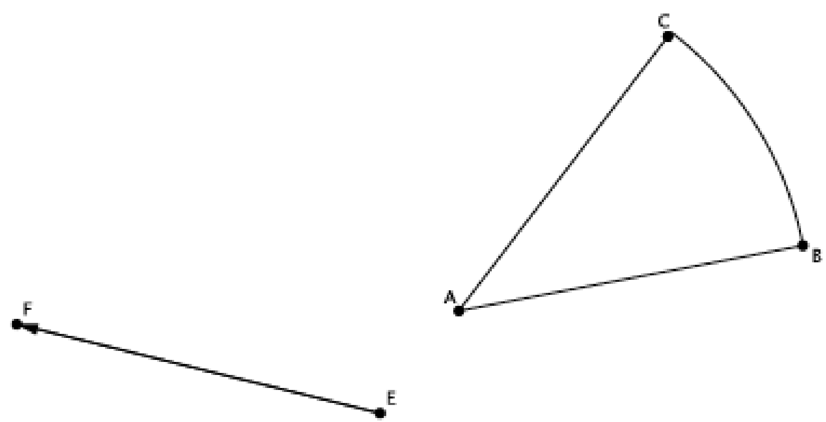
1. Sequence translations of parallelogram $ABCD$ (a quadrilateral in which both pairs of opposite sides are parallel) along vectors $\overrightarrow{HG}$ and $\overrightarrow{FE}$. Label the translated images.



2. What do you know about AD and BC compared with $A'D'$ and $B'C'$? Explain.

3. Are $A'B'$ and $A''B''$ equal in length? How do you know?

4. Translate the curved shape ABC along the given vector. Label the image.



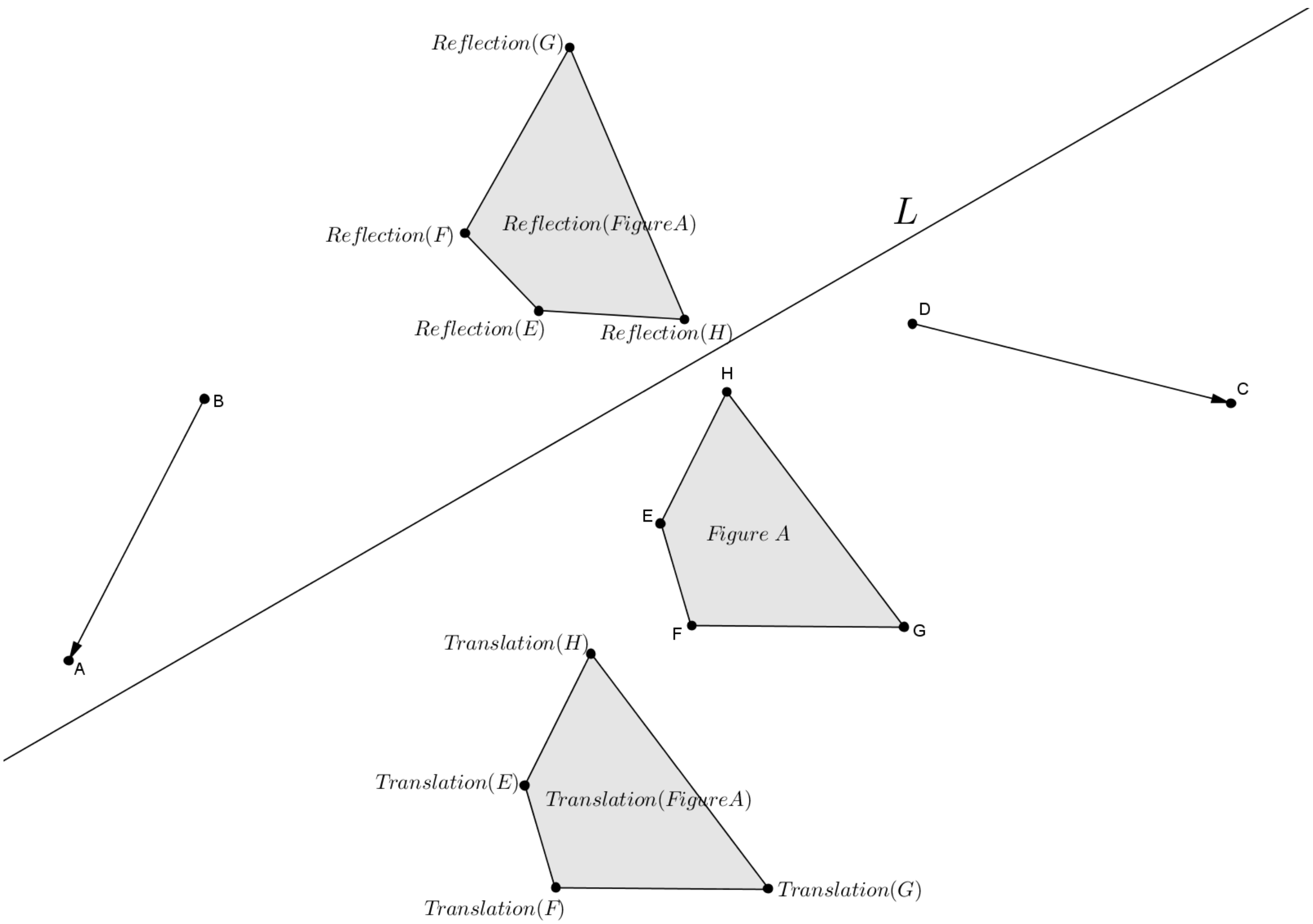
5. What vector would map the shape $A'B'C'$ back onto ABC?

Lesson 8: Sequencing Reflections and Translations

Classwork

Exercises 1–3

Use the figure below to answer Exercises 1–3.



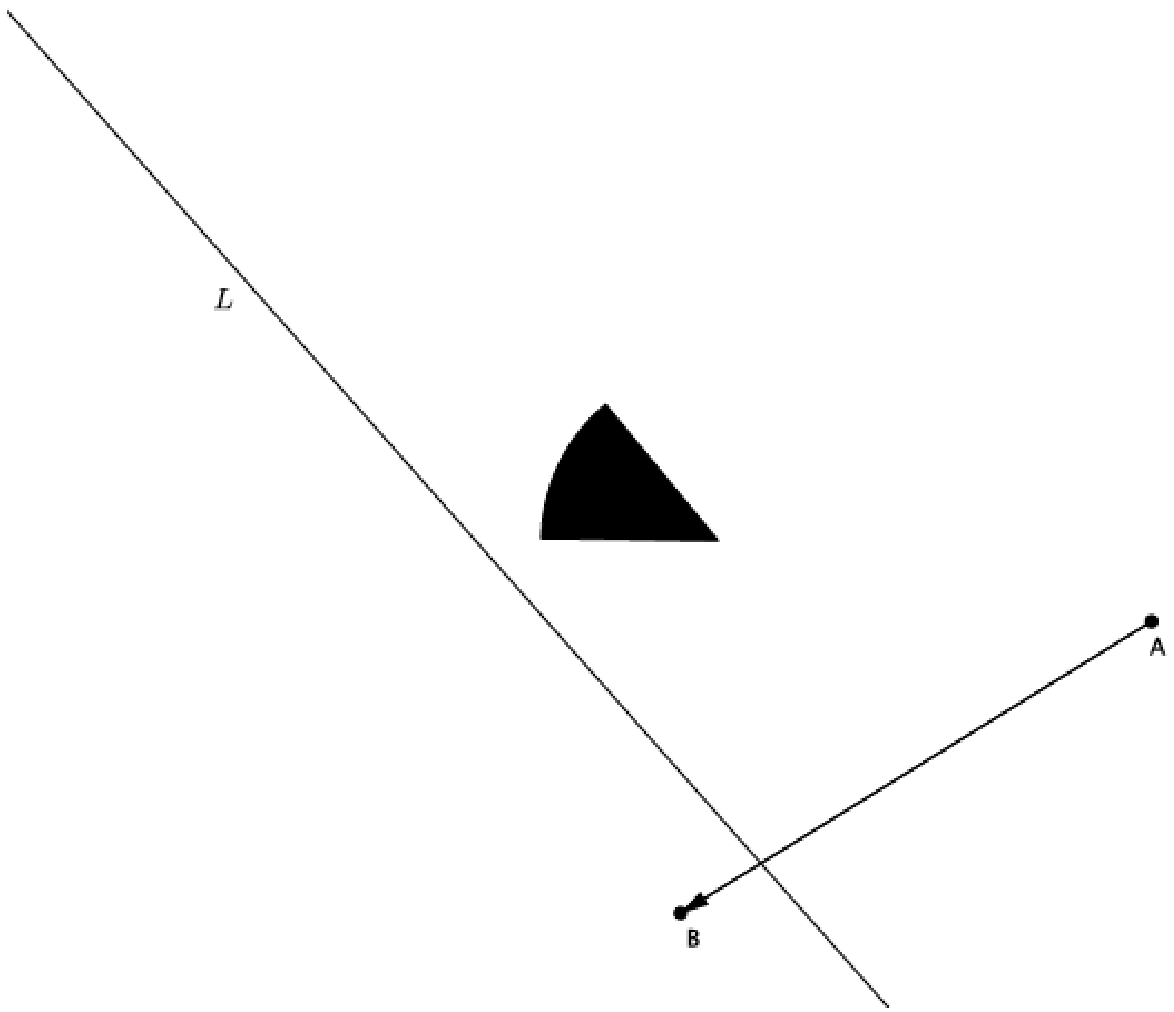
1. Figure A was translated along vector $\overrightarrow{BA}$ resulting in $Translation(Figure\ A)$. Describe a sequence of translations that would map Figure A back onto its original position.

2. Figure A was reflected across line L resulting in $Reflection(Figure\ A)$. Describe a sequence of reflections that would map Figure A back onto its original position.

3. Can $Translation_{\overrightarrow{BA}}$ of Figure A undo the transformation of $Translation_{\overrightarrow{DC}}$ of Figure A? Why or why not?

Exercises 4–7

Let S be the black figure.



4. Let there be the translation along vector $\overrightarrow{AB}$ and a reflection across line L.

   Use a transparency to perform the following sequence: Translate figure S; then, reflect figure S. Label the image S'.

5. Let there be the translation along vector $\overrightarrow{AB}$ and a reflection across line L.

   Use a transparency to perform the following sequence: Reflect figure S; then, translate figure S. Label the image S''.

6. Using your transparency, show that under a sequence of any two translations, $Translation$ and $Translation_0$ (along different vectors), that the sequence of the $Translation$ followed by the $Translation_0$ is equal to the sequence of the $Translation_0$ followed by the $Translation.$ That is, draw a figure, A, and two vectors. Show that the translation along the first vector, followed by a translation along the second vector, places the figure in the same location as when you perform the translations in the reverse order. (This fact will be proven in high school). Label the transformed image A'. Now, draw two new vectors and translate along them just as before. This time, label the transformed image A''. Compare your work with a partner. Was the statement "the sequence of the $Translation$ followed by the $Translation_0$ is equal to the sequence of the $Translation_0$ followed by the $Translation$" true in all cases? Do you think it will always be true?

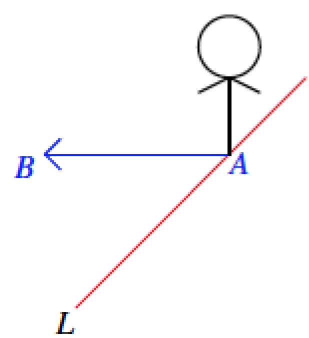
7. Does the same relationship you noticed in Exercise 6 hold true when you replace one of the translations with a reflection. That is, is the following statement true: A translation followed by a reflection is equal to a reflection followed by a translation?

Lesson Summary

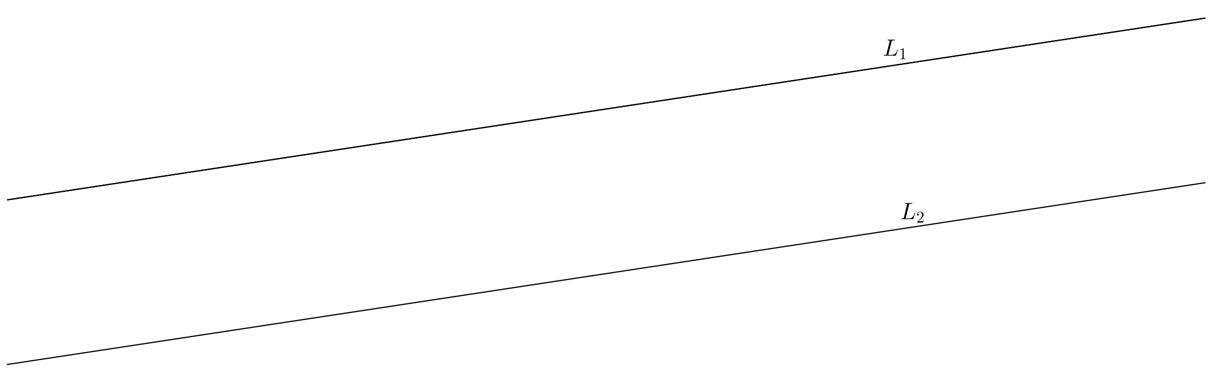
- A reflection across a line followed by a reflection across the same line places all figures in the plane back onto their original position.
- A reflection followed by a translation does not place a figure in the same location in the plane as a translation followed by a reflection. The order in which we perform a sequence of rigid motions matters.

Problem Set

1. Let there be a reflection across line L, and let there be a translation along vector $\overrightarrow{AB}$, as shown. If S denotes the black figure, compare the translation of S followed by the reflection of S with the reflection of S followed by the translation of S.



2. Let L_1 and L_2 be parallel lines, and let $Reflection_1$ and $Reflection_2$ be the reflections across L_1 and L_2, respectively (in that order). Show that a $Reflection_2$ followed by $Reflection_1$ is not equal to a $Reflection_1$ followed by $Reflection_2$. (Hint: Take a point on L_1 and see what each of the sequences does to it.)



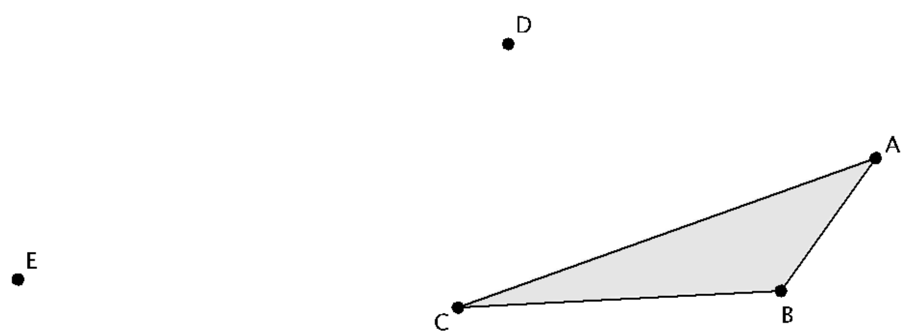
3. Let L_1 and L_2 be parallel lines, and let $Reflection_1$ and $Reflection_2$ be the reflections across L_1 and L_2, respectively (in that order). Can you guess what $Reflection_1$ followed by $Reflection_2$ is? Give as persuasive an argument as you can. (Hint: Examine the work you just finished for the last problem.)

Lesson 9: Sequencing Rotations

Classwork

Exploratory Challenge

1.



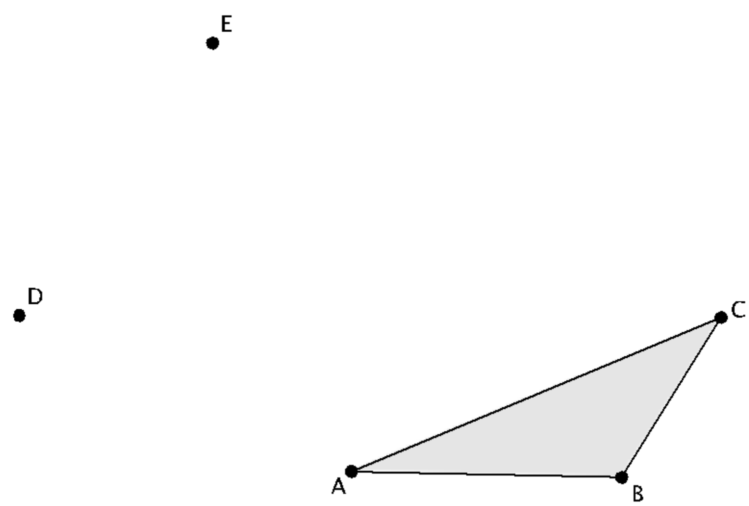
a. Rotate $\triangle ABC$ d degrees around center D. Label the rotated image as $\triangle A'B'C'$.

b. Rotate $\triangle A'B'C'$ d degrees around center E. Label the rotated image as $\triangle A''B''C''$.

c. Measure and label the angles and side lengths of $\triangle ABC$. How do they compare with the images $\triangle A'B'C'$ and $\triangle A''B''C''$?

d. How can you explain what you observed in part (c)? What statement can you make about properties of sequences of rotations as they relate to a single rotation?

2.



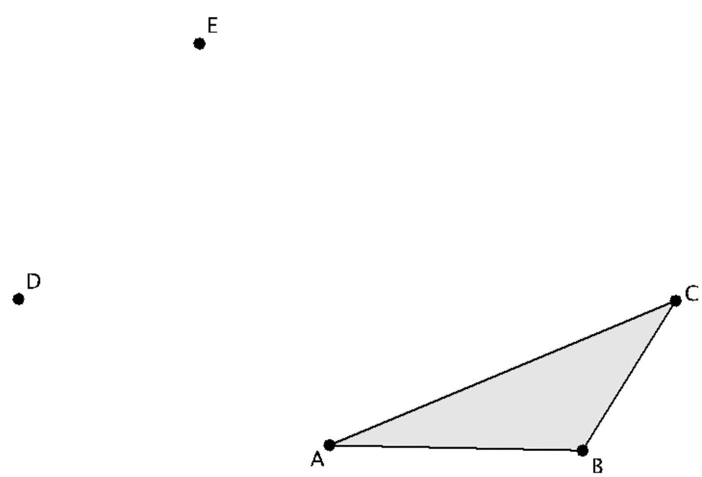
a. Rotate $\triangle ABC$ d degrees around center D, and then rotate again d degrees around center E. Label the image as $\triangle A'B'C'$ after you have completed both rotations.

b. Can a single rotation around center D map $\triangle A'B'C'$ onto $\triangle ABC$?

c. Can a single rotation around center E map $\triangle A'B'C'$ onto $\triangle ABC$?

d. Can you find a center that would map $\triangle A'B'C'$ onto $\triangle ABC$ in one rotation? If so, label the center F.

3.

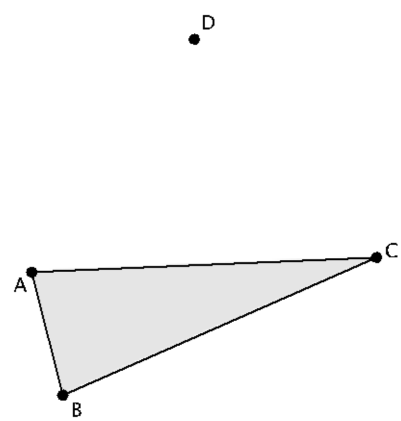


a. Rotate $\triangle ABC$ $90°$ (counterclockwise) around center D, and then rotate the image another $90°$ (counterclockwise) around center E. Label the image $\triangle A'B'C'$.

b. Rotate $\triangle ABC$ $90°$ (counterclockwise) around center E and then rotate the image another $90°$ (counterclockwise) around center D. Label the image $\triangle A''B''C''$.

c. What do you notice about the locations of $\triangle A'B'C'$ and $\triangle A''B''C''$? Does the order in which you rotate a figure around different centers have an impact on the final location of the figure's image?

4.

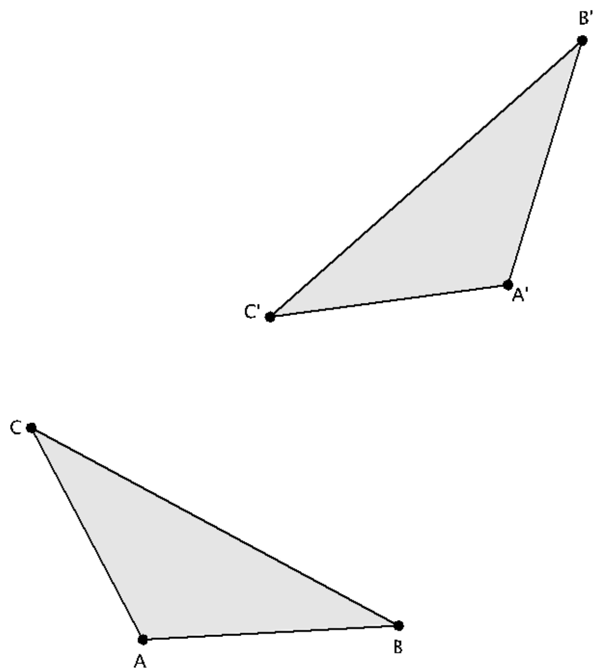


a. Rotate $\triangle ABC$ $90°$ (counterclockwise) around center D, and then rotate the image another $45°$ (counterclockwise) around center D. Label the $\triangle A'B'C'$.

b. Rotate $\triangle ABC$ $45°$ (counterclockwise) around center D, and then rotate the image another $90°$ (counterclockwise) around center D. Label the $\triangle A''B''C''$.

c. What do you notice about the locations of $\triangle A'B'C'$ and $\triangle A''B''C''$? Does the order in which you rotate a figure around the same center have an impact on the final location of the figure's image?

5. $\triangle ABC$ has been rotated around two different centers, and its image is $\triangle A'B'C'$. Describe a sequence of rigid motions that would map $\triangle ABC$ onto $\triangle A'B'C'$.

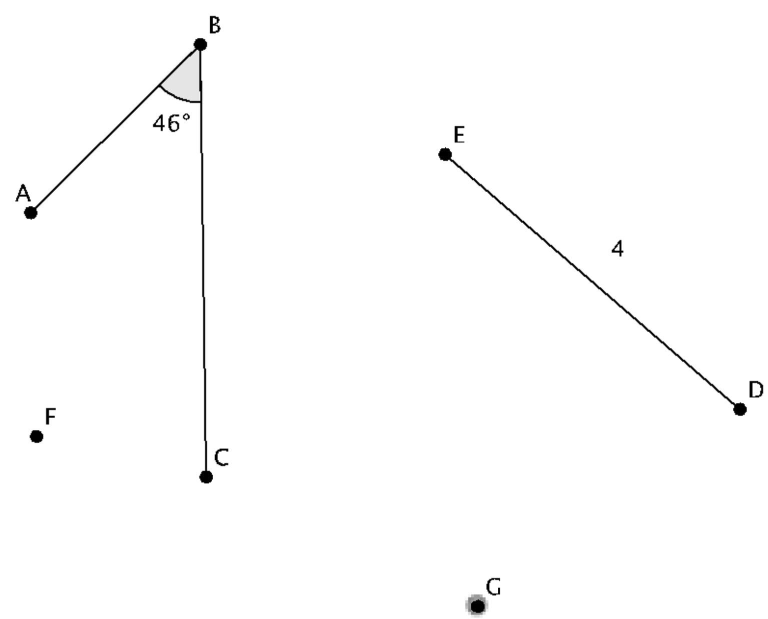


Lesson Summary

- Sequences of rotations have the same properties as a single rotation:
  - A sequence of rotations preserves degrees of measures of angles.
  - A sequence of rotations preserves lengths of segments.
- The order in which a sequence of rotations around different centers is performed matters with respect to the final location of the image of the figure that is rotated.
- The order in which a sequence of rotations around the same center is performed does not matter. The image of the figure will be in the same location.

Problem Set

1. Refer to the figure below.

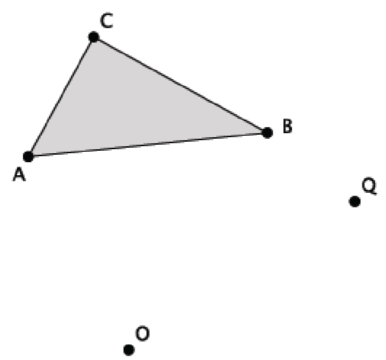


a. Rotate $\angle ABC$ and segment DE d degrees around center F, then d degrees around center G. Label the final location of the images as $\angle A'B'C'$ and $D'E'$.

b. What is the size of $\angle ABC$, and how does it compare to the size of $\angle A'B'C'$? Explain.

c. What is the length of segment DE, and how does it compare to the length of segment $D'E'$? Explain.

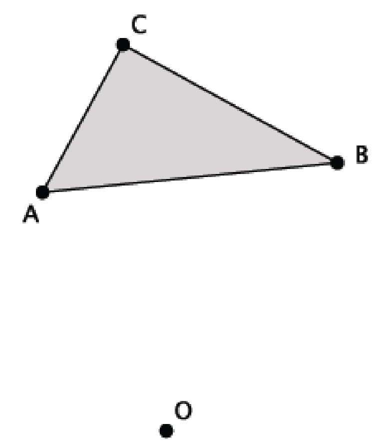
2. Refer to the figure given below.



   a. Let $Rotation_1$ be a counterclockwise rotation of $90°$ around the center O. Let $Rotation_2$ be a clockwise rotation of $(-45)°$ around the center Q. Determine the approximate location of $Rotation_1(\triangle ABC)$ followed by $Rotation_2$. Label the image of triangle ABC as $A'B'C'$.

   b. Describe the sequence of rigid motions that would map $\triangle ABC$ onto $\triangle A'B'C'$.

3. Refer to the figure given below.



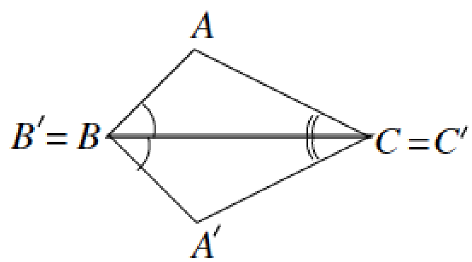
Let R be a rotation of $(-90)°$ around the center O. Let $Rotation_2$ be a rotation of $(-45)°$ around the same center O. Determine the approximate location of $Rotation_1(\triangle ABC)$ followed by $Rotation_2(\triangle ABC)$. Label the image of triangle ABC as $A'B'C'$.

Lesson 10: Sequences of Rigid Motions

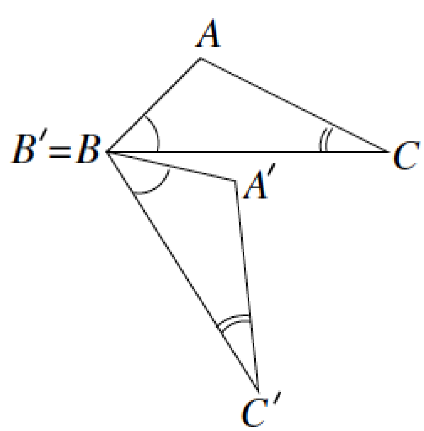
Classwork

Exercises

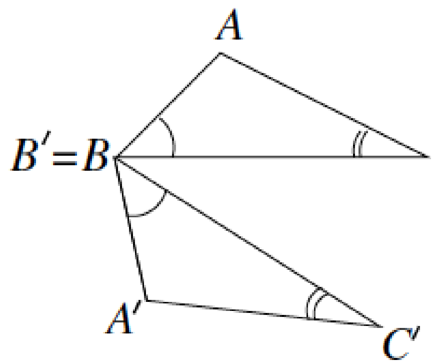
1. In the following picture, triangle ABC can be traced onto a transparency and mapped onto triangle $A'B'C'$. Which basic rigid motion, or sequence of, would map one triangle onto the other?



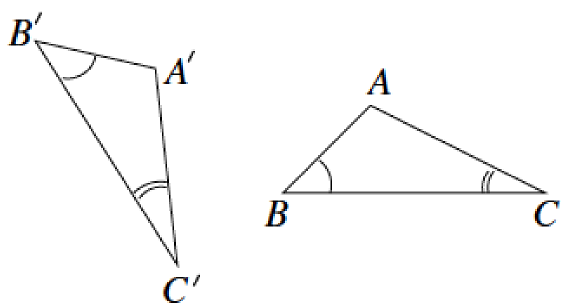
2. In the following picture, triangle ABC can be traced onto a transparency and mapped onto triangle $A'B'C'$. Which basic rigid motion, or sequence of, would map one triangle onto the other?



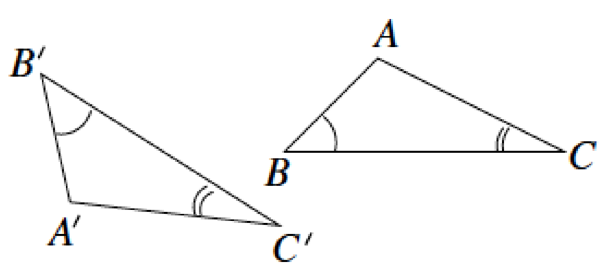
3. In the following picture, triangle ABC can be traced onto a transparency and mapped onto triangle $A'B'C'$.
   Which basic rigid motion, or sequence of, would map one triangle onto the other?



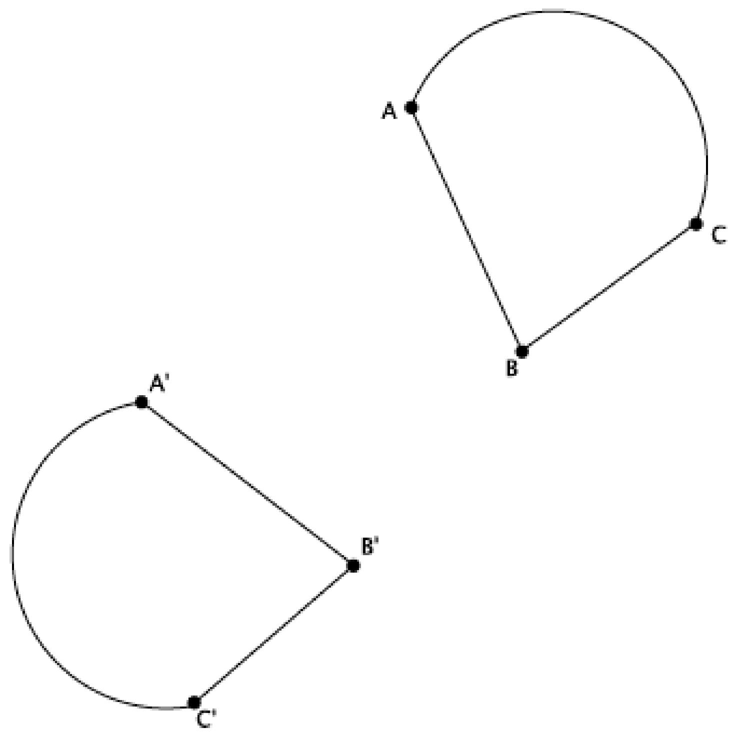
4. In the following picture, we have two pairs of triangles. In each pair, triangle ABC can be traced onto a transparency and mapped onto triangle $A'B'C'$.
   Which basic rigid motion, or sequence of, would map one triangle onto the other?
   Scenario 1:



Scenario 2:

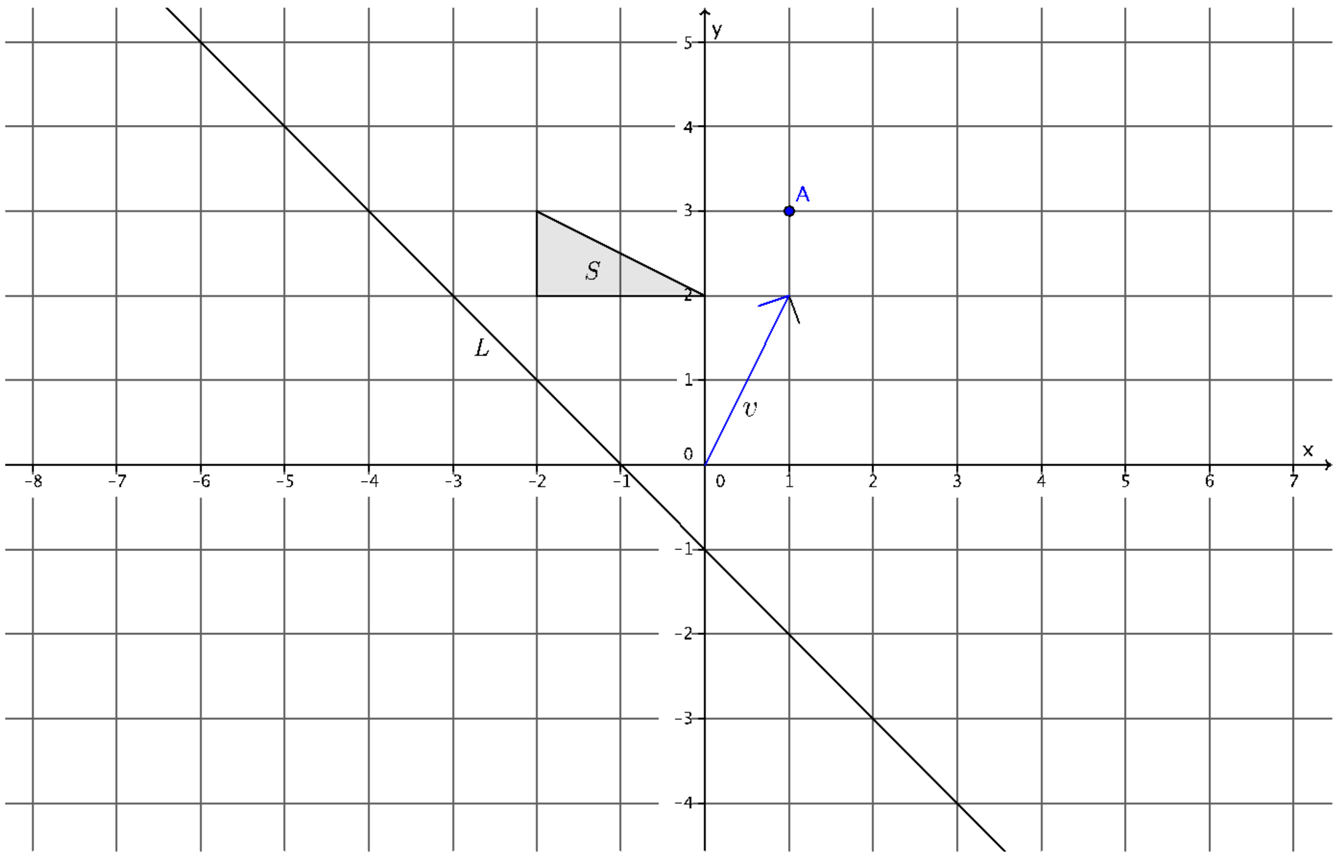


5. Let two figures ABC and $A'B'C'$ be given so that the length of curved segment AC equals the length of curved segment $A'C'$, $|\angle B| = |\angle B'| = 80°$, and $|AB| = |A'B'| = 5$. With clarity and precision, describe a sequence of rigid motions that would map figure ABC onto figure $A'B'C'$.



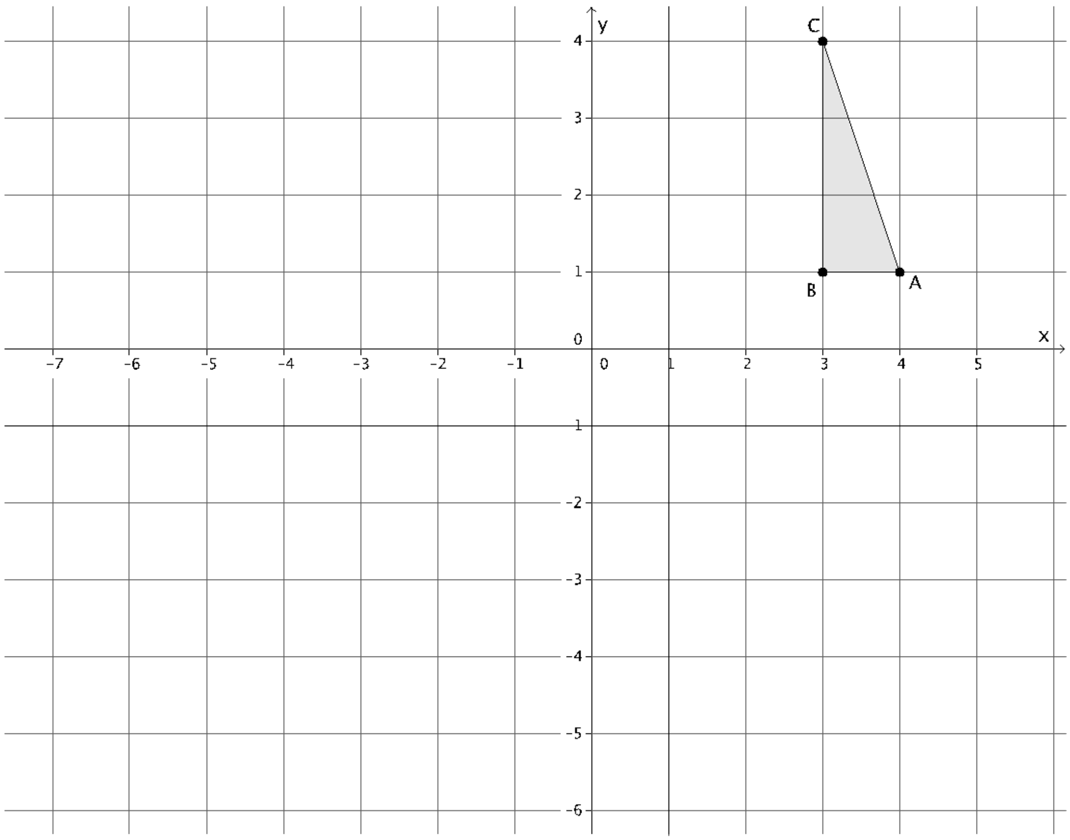
Problem Set

1. Let there be the translation along vector $\vec{v}$, let there be the rotation around point A, -90 degrees (clockwise), and let there be the reflection across line L. Let S be the figure as shown below. Show the location of S after performing the following sequence: a translation followed by a rotation followed by a reflection.



2. Would the location of the image of S in the previous problem be the same if the translation was performed last instead of first, i.e., does the sequence: translation followed by a rotation followed by a reflection equal a rotation followed by a reflection followed by a translation? Explain.

3. Use the same coordinate grid to complete parts (a)–(c).



a. Reflect triangle ABC across the vertical line, parallel to the y-axis, going through point $(1, 0)$. Label the transformed points A, B, C as A', B', C', respectively.

b. Reflect triangle $A'B'C'$ across the horizontal line, parallel to the x-axis going through point $(0, -1)$. Label the transformed points of A', B', C' as A'', B'', C'', respectively.

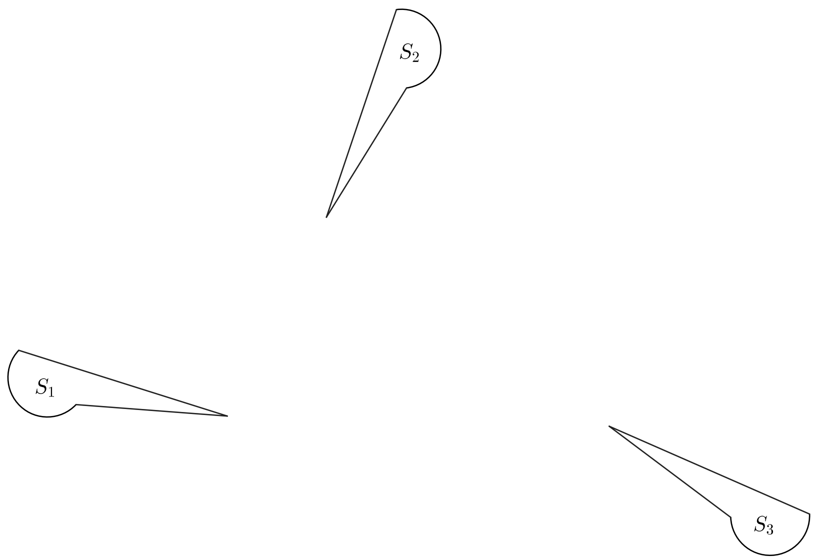
c. Is there a single rigid motion that would map triangle ABC to triangle $A''B''C''$?

Lesson 11: Definition of Congruence and Some Basic Properties

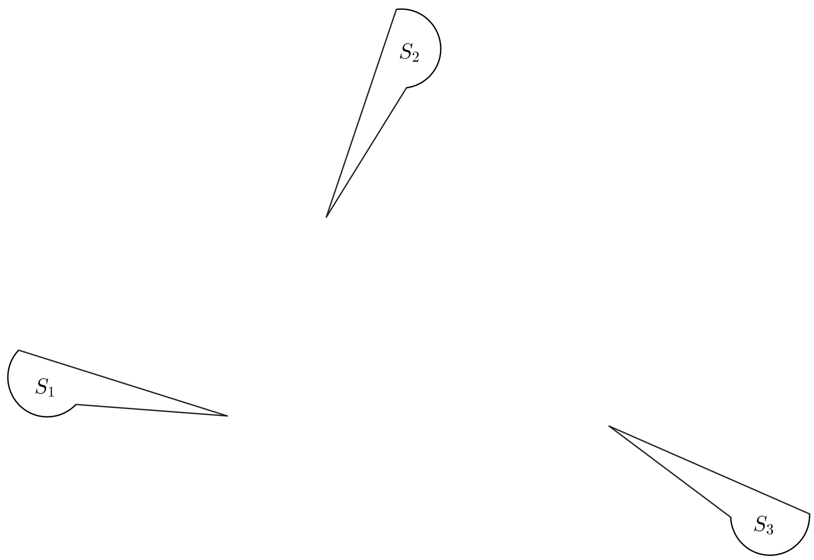
Classwork

Exercise 1

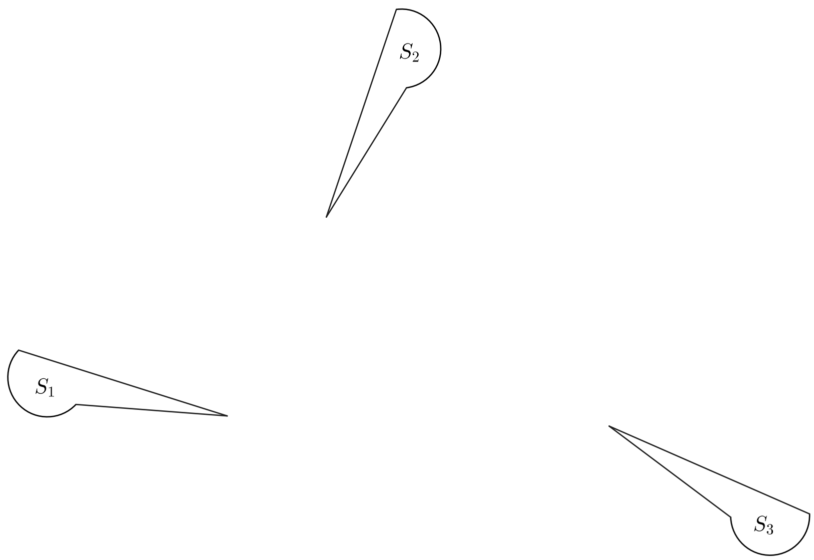
a. Describe the sequence of basic rigid motions that shows $S_1 \cong S_2$.



b. Describe the sequence of basic rigid motions that shows $S_2 \cong S_3$.

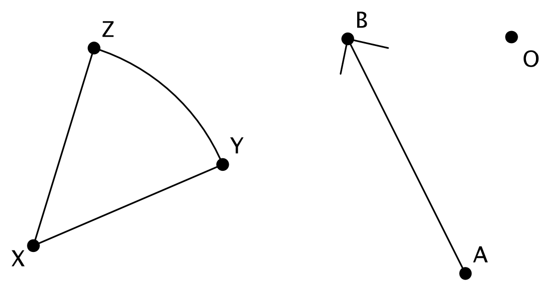


c. Describe a sequence of basic rigid motions that shows $S_1 \cong S_3$.



Exercise 2

Perform the sequence of a translation followed by a rotation of Figure XYZ, where T is a translation along a vector $\overrightarrow{AB}$, and R is a rotation of d degrees (you choose d) around a center O. Label the transformed figure $X'Y'Z'$. Will $XYZ \cong X'Y'Z'$?



Lesson Summary

Given that sequences enjoy the same basic properties of basic rigid motions, we can state three basic properties of congruences:

(Congruence 1) A congruence maps a line to a line, a ray to a ray, a segment to a segment, and an angle to an angle.

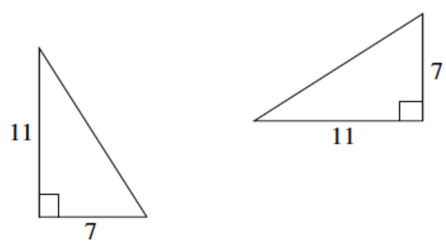
(Congruence 2) A congruence preserves lengths of segments.

(Congruence 3) A congruence preserves measures of angles.

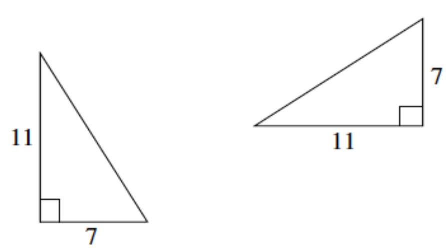
The notation used for congruence is ≅.

Problem Set

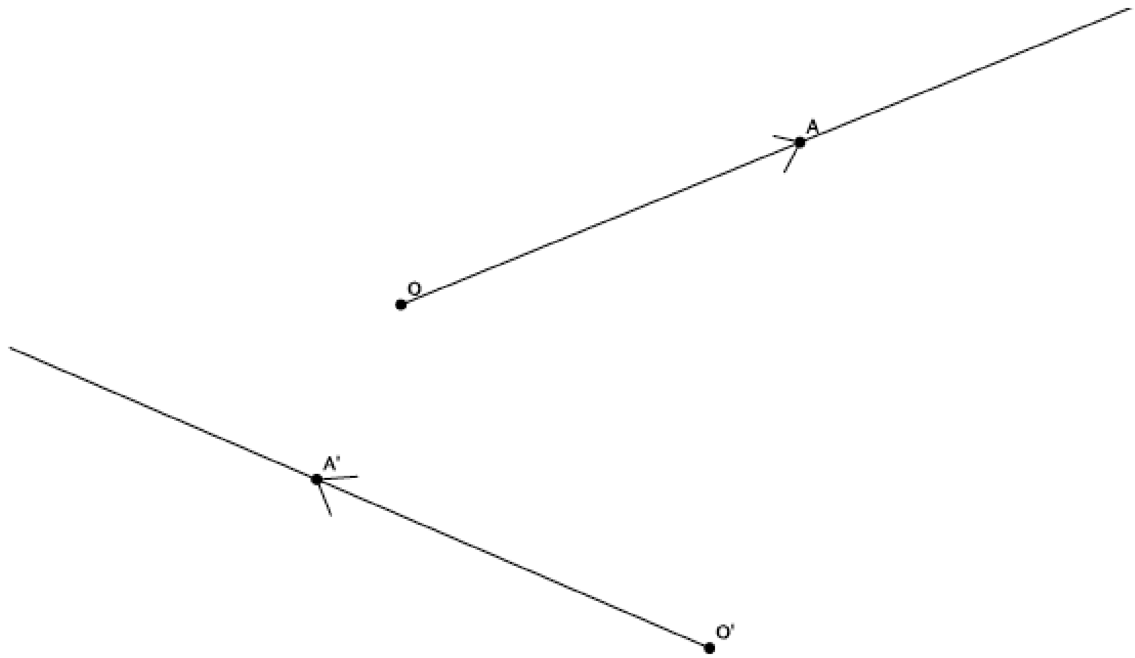
1. Given two right triangles with lengths shown below, is there one basic rigid motion that maps one to the other? Explain.



2. Are the two right triangles shown below congruent? If so, describe a congruence that would map one triangle onto the other.



3. Given two rays, $\overrightarrow{OA}$ and $\overrightarrow{O'A'}$:



a. Describe a congruence that maps $\overrightarrow{OA}$ to $\overrightarrow{O'A'}$.

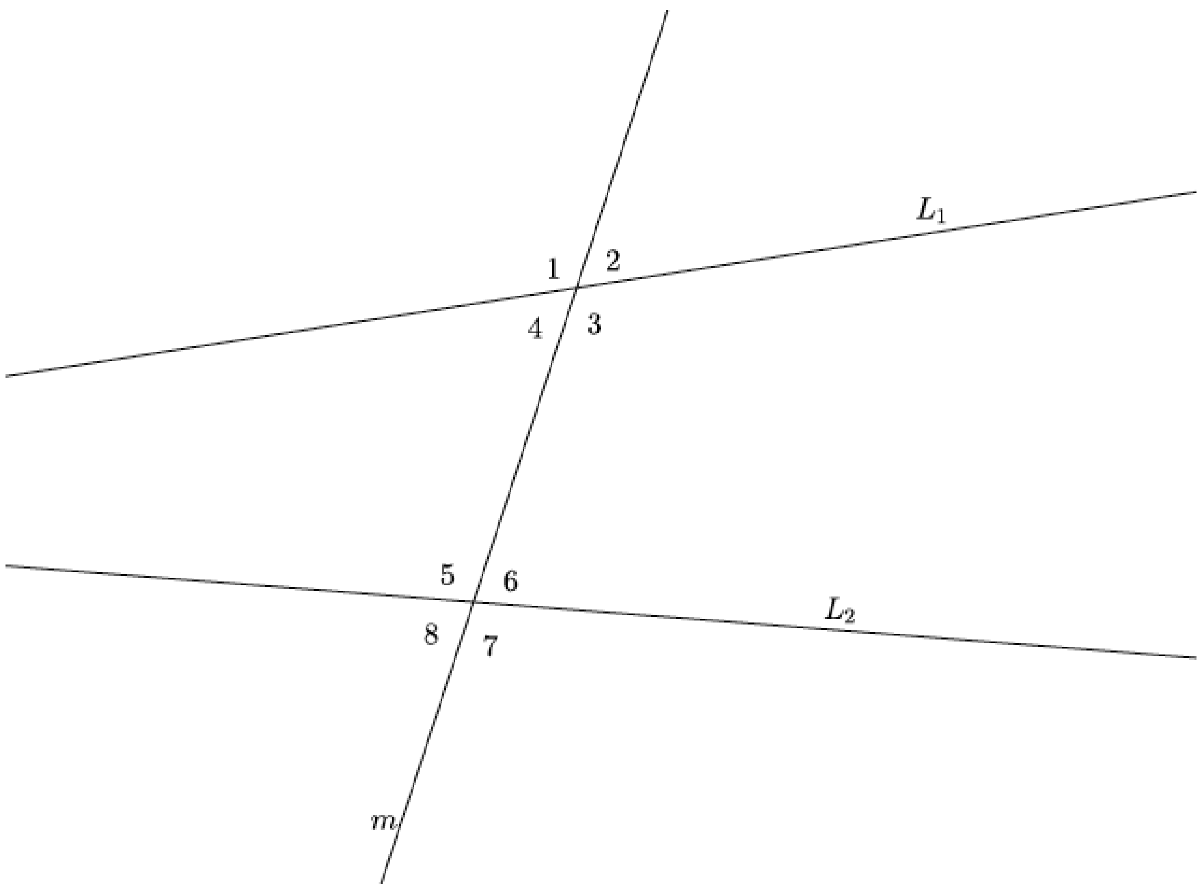
b. Describe a congruence that maps $\overrightarrow{O'A'}$ to $\overrightarrow{OA}$.

Lesson 12: Angles Associated with Parallel Lines

Classwork

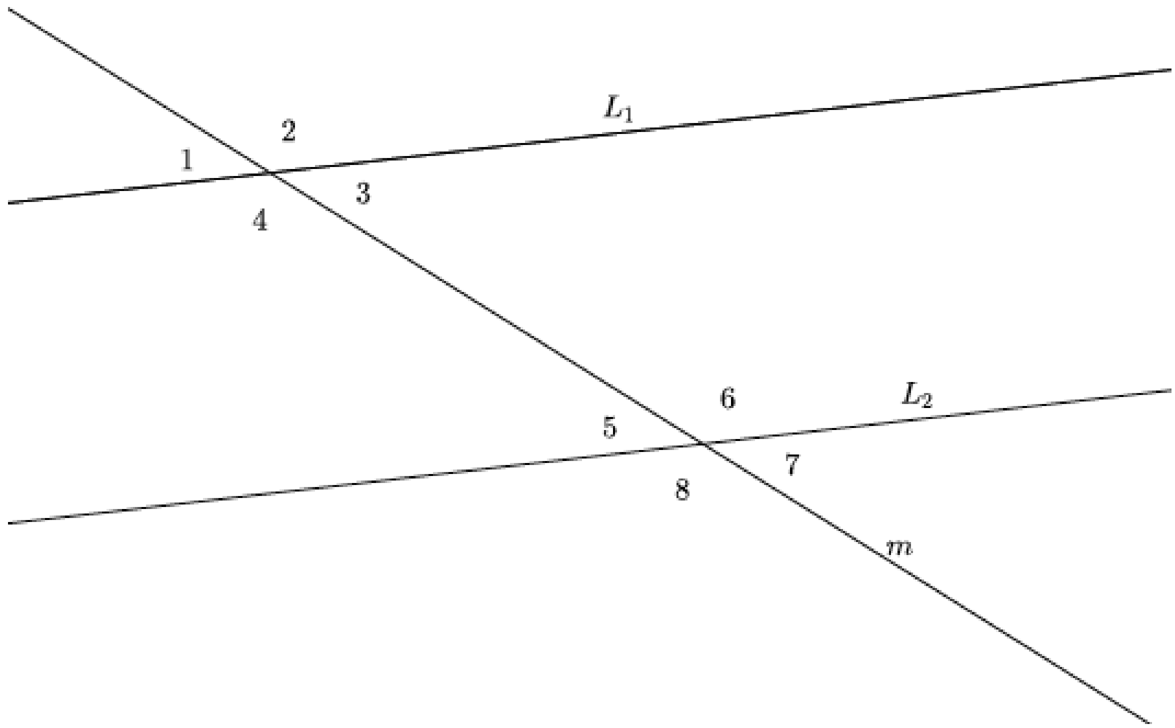
Exploratory Challenge 1

In the figure below, L_1 is not parallel to L_2, and m is a transversal. Use a protractor to measure angles 1–8. Which, if any, are equal? Explain why. (Use your transparency if needed.)



Exploratory Challenge 2

In the figure below, $L_1 \parallel L_2$, and m is a transversal. Use a protractor to measure angles 1–8. List the angles that are equal in measure.



a. What did you notice about the measures of $\angle 1$ and $\angle 5$? Why do you think this is so? (Use your transparency if needed.)

b. What did you notice about the measures of $\angle 3$ and $\angle 7$? Why do you think this is so? (Use your transparency if needed.) Are there any other pairs of angles with this same relationship? If so, list them.

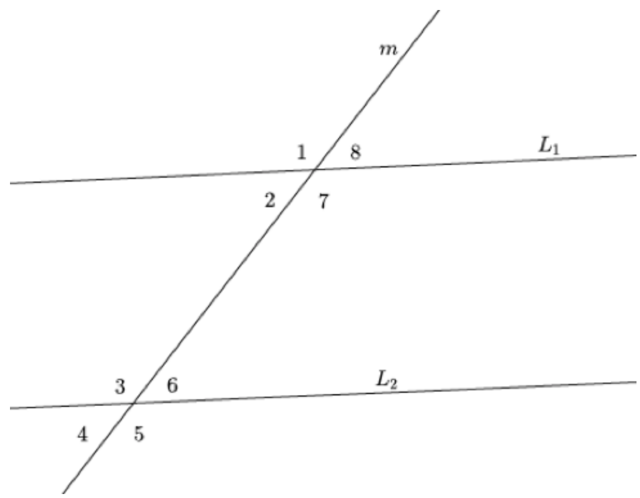
c. What did you notice about the measures of $\angle 4$ and $\angle 6$? Why do you think this is so? (Use your transparency if needed.) Is there another pair of angles with this same relationship?

Lesson Summary

Angles that are on the same side of the transversal in corresponding positions (above each of L_1 and L_2 or below each of L_1 and L_2) are called **corresponding angles**. For example, $\angle 2$ and $\angle 4$ are corresponding angles.

When angles are on opposite sides of the transversal and between (inside) the lines L_1 and L_2, they are called **alternate interior angle**s. For example, $\angle 3$ and $\angle 7$ are alternate interior angles.

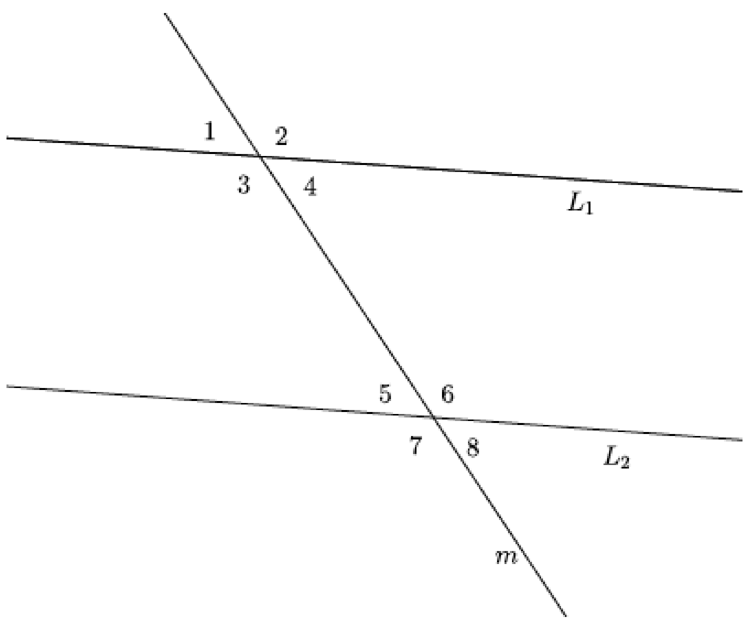
When angles are on opposite sides of the transversal and outside of the parallel lines (above L_1 and below L_2), they are called **alternate exterior** angles. For example, $\angle 1$ and $\angle 5$ are alternate exterior angles.



When parallel lines are cut by a transversal, any corresponding angles, any alternate interior angles, and any alternate exterior angles are equal in measure. If the lines are not parallel, then the angles are not equal in measure.

Problem Set

Use the diagram below to do Problems 1–6.



1. Identify all pairs of corresponding angles. Are the pairs of corresponding angles equal in measure? How do you know?

2. Identify all pairs of alternate interior angles. Are the pairs of alternate interior angles equal in measure? How do you know?

3. Use an informal argument to describe why $\angle 1$ and $\angle 8$ are equal in measure if $L_1 \parallel L_2$.

4. Assuming $L_1 \parallel L_2$ if the measure of $\angle 4$ is $73°$, what is the measure of $\angle 8$? How do you know?

5. Assuming $L_1 \parallel L_2$, if the measure of $\angle 3$ is $107°$ degrees, what is the measure of $\angle 6$? How do you know?

6. Assuming $L_1 \parallel L_2$, if the measure of $\angle 2$ is $107°$, what is the measure of $\angle 7$? How do you know?

7. Would your answers to Problems 4–6 be the same if you had not been informed that $L_1 \parallel L_2$? Why, or why not?

8. Use an informal argument to describe why $\angle 1$ and $\angle 5$ are equal in measure if $L_1 \parallel L_2$.

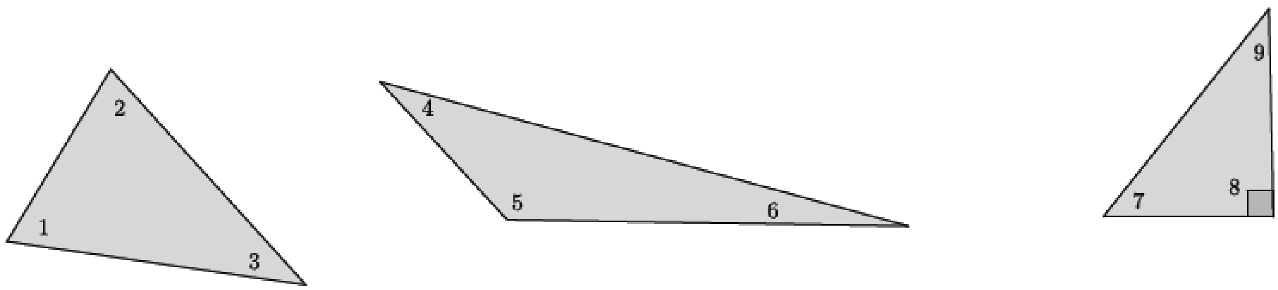
9. Use an informal argument to describe why $\angle 4$ and $\angle 5$ are equal in measure if $L_1 \parallel L_2$.

10. Assume that L_1 is not parallel to L_2. Explain why $\angle 3 \neq \angle 7$.

Lesson 13: Angle Sum of a Triangle

Classwork

Concept Development

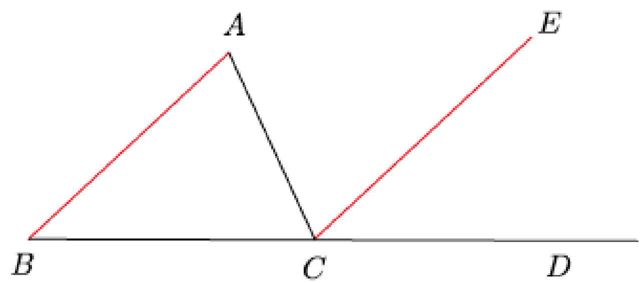


$$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + \angle 6 = \angle 7 + \angle 8 + \angle 9 = 180$$

Note that the sum of angles 7 and 9 must equal $90°$ because of the known right angle in the right triangle.

Exploratory Challenge 1

Let triangle ABC be given. On the ray from B to C, take a point D so that C is between B and D. Through point C, draw a line parallel to AB, as shown. Extend the parallel lines AB and CE. Line AC is the transversal that intersects the parallel lines.



a. Name the three interior angles of triangle ABC.

b. Name the straight angle.

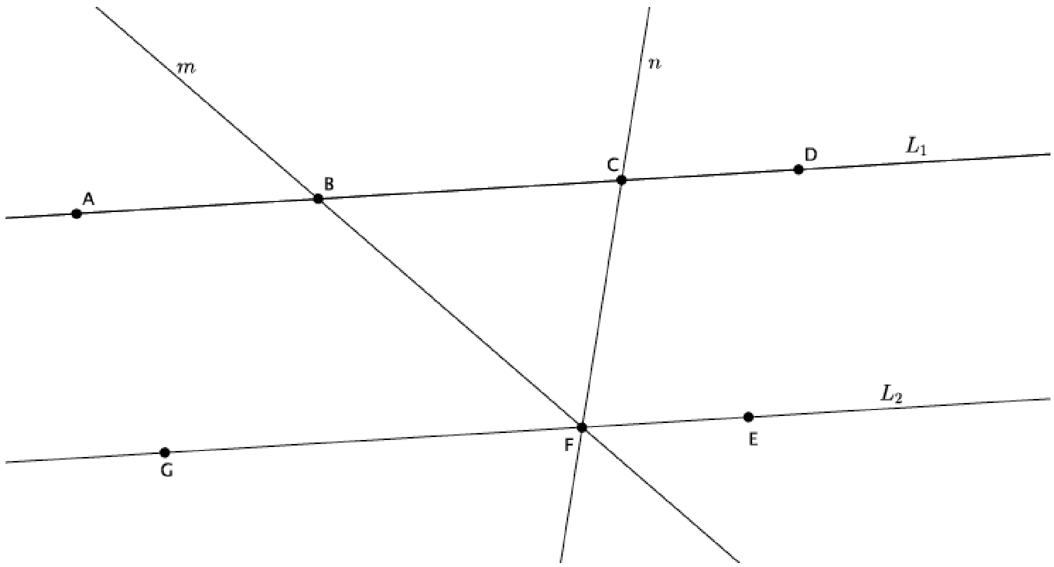
c. What kinds of angles are $\angle ABC$ and $\angle ECD$? What does that mean about their measures?

d. What kinds of angles are $\angle BAC$ and $\angle ECA$? What does that mean about their measures?

e. We know that $\angle BCD = \angle BCA + \angle ECA + \angle ECD = 180°$. Use substitution to show that the three interior angles of the triangle have a sum of $180°$.

Exploratory Challenge 2

The figure below shows parallel lines L_1 and L_2. Let m and n be transversals that intersect L_1 at points B and C, respectively, and L_2 at point F, as shown. Let A be a point on L_1 to the left of B, D be a point on L_1 to the right of C, G be a point on L_2 to the left of F, and E be a point on L_2 to the right of F.



a. Name the triangle in the figure.

b. Name a straight angle that will be useful in proving that the sum of the interior angles of the triangle is $180°$.

c. Write your proof below.

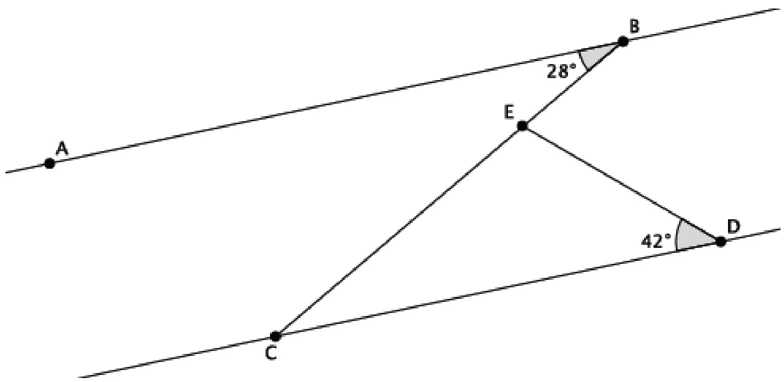
Lesson Summary

All triangles have a sum of interior angles equal to $180°$.

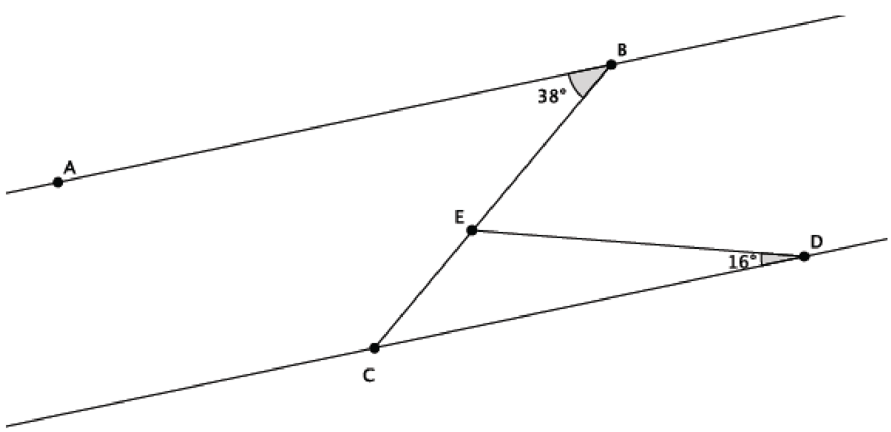
The proof that a triangle has a sum of interior angles equal to $180°$ is dependent upon the knowledge of straight angles and angle relationships of parallel lines cut by a transversal.

Problem Set

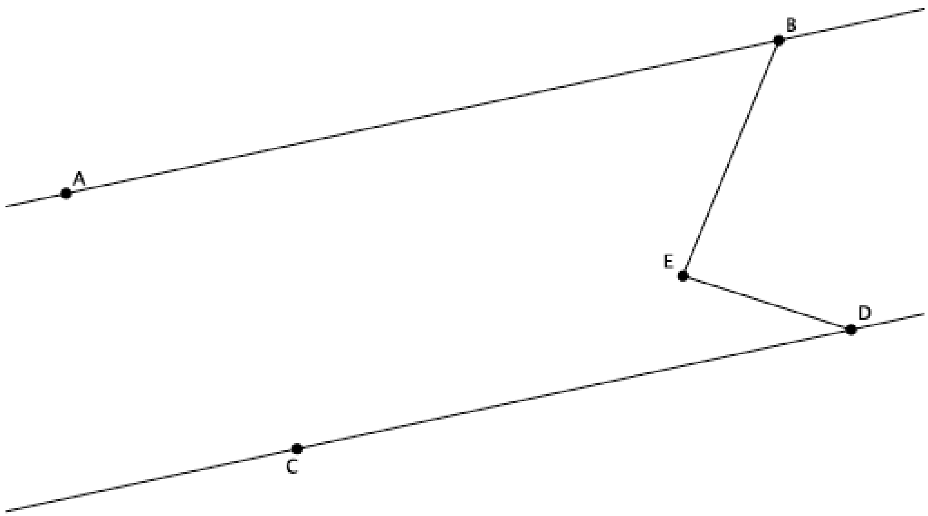
1. In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle ABC = 28°$, and the measure of angle $\angle EDC = 42°$. Find the measure of angle $\angle CED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle.



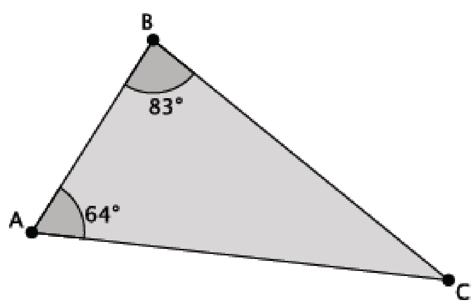
2. In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle ABE = 38°$, and the measure of angle $\angle EDC = 16°$. Find the measure of angle $\angle BED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Find the measure of angle $\angle CED$ first, and then use that measure to find the measure of angle $\angle BED$.)



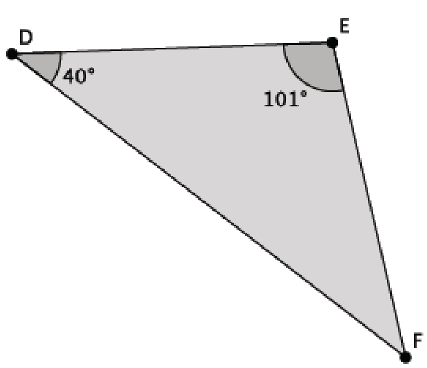
3. In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle ABE = 56°$, and the measure of angle $\angle EDC = 22°$. Find the measure of angle $\angle BED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Extend the segment BE so that it intersects line CD.)



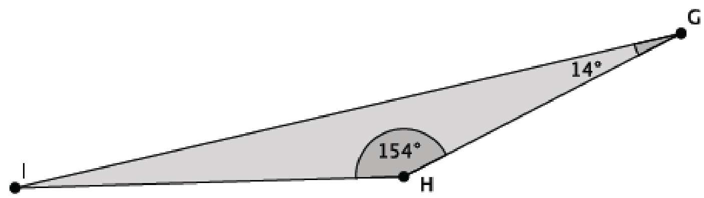
4. What is the measure of $\angle ACB$?



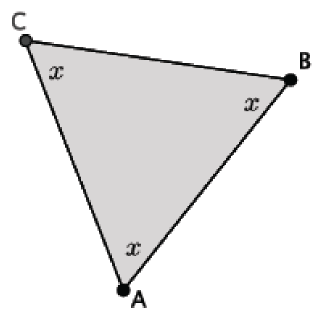
5. What is the measure of $\angle EFD$?



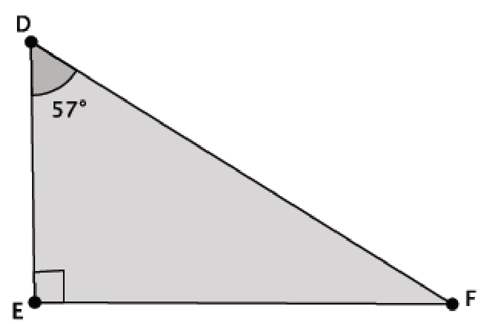
6. What is the measure of $\angle HIG$?



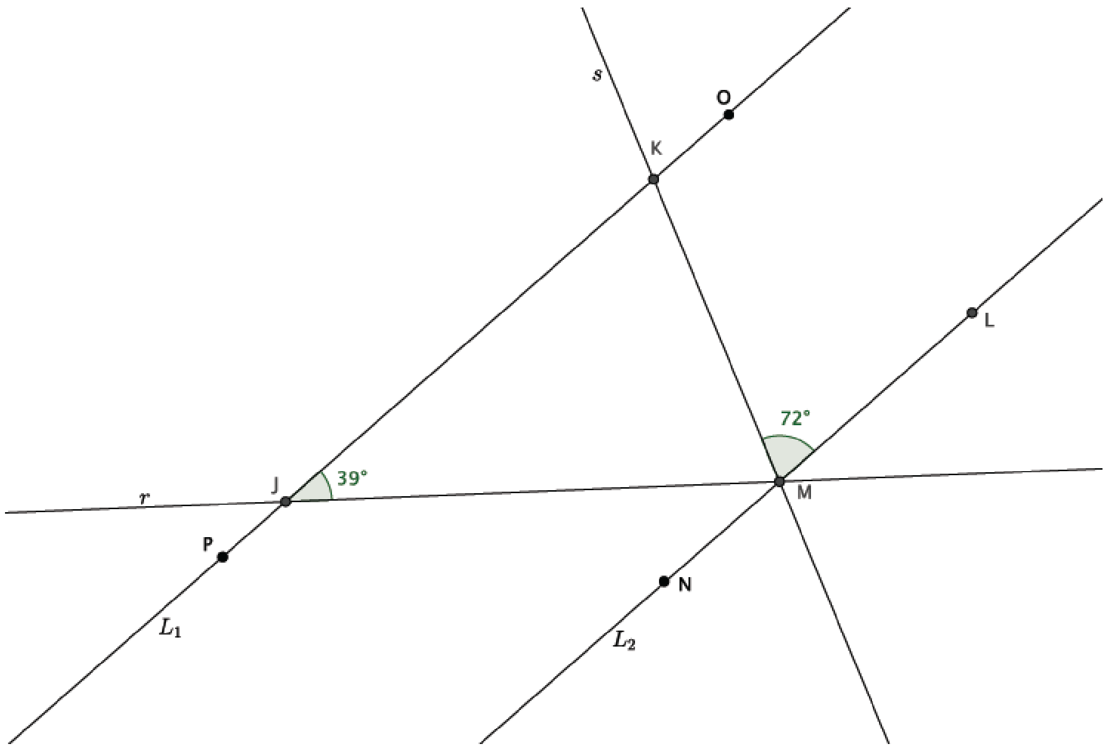
7. What is the measure of $\angle ABC$?



8. Triangle DEF is a right triangle. What is the measure of $\angle EFD$?



9. In the diagram below, lines L_1 and L_2 are parallel. Transversals r and s intersect both lines at the points shown below. Determine the measure of $\angle JMK$. Explain how you know you are correct.

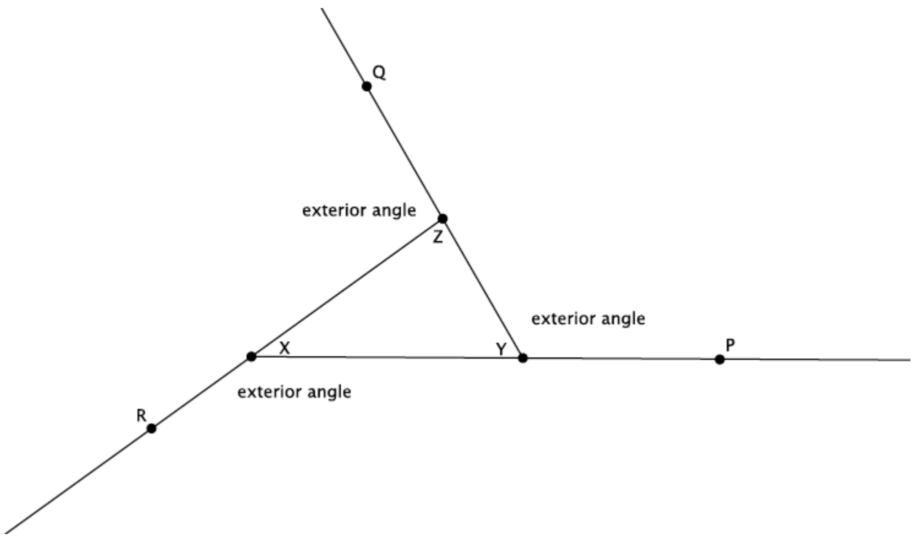


Lesson 14: More on the Angles of a Triangle

Classwork

Exercises 1–4

Use the diagram below to complete Exercises 1–4.



1. Name an exterior angle and the related remote interior angles.

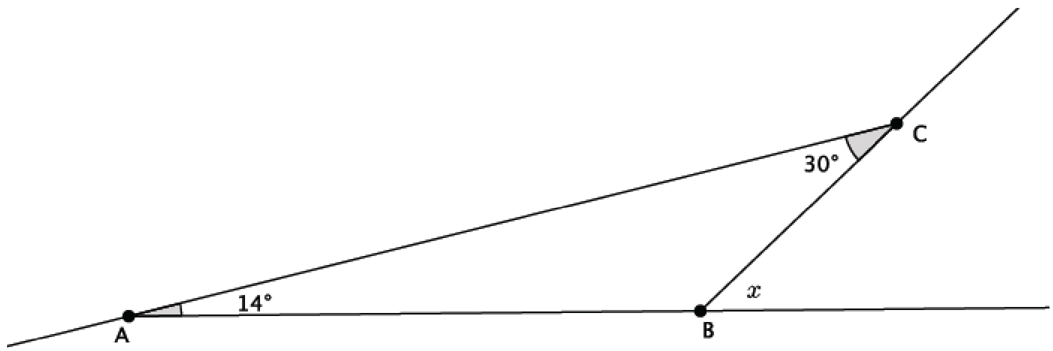
2. Name a second exterior angle and the related remote interior angles.

3. Name a third exterior angle and the related remote interior angles.

4. Show that the measure of an exterior angle is equal to the sum of the related remote interior angles.

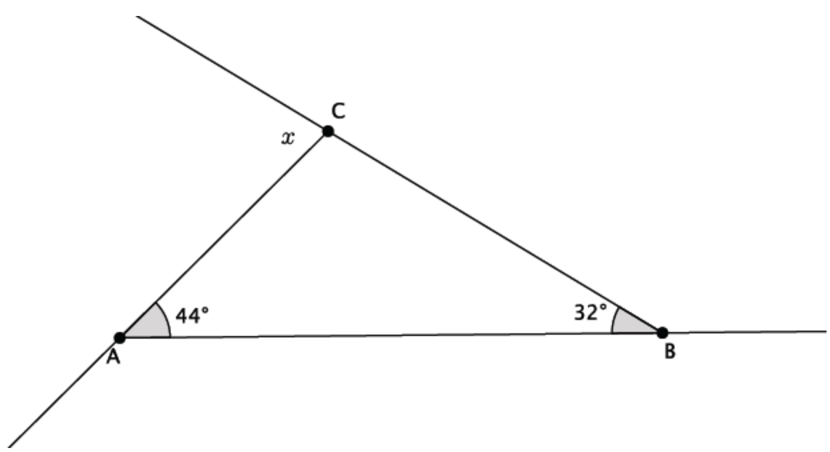
Example 1

Find the measure of angle x.



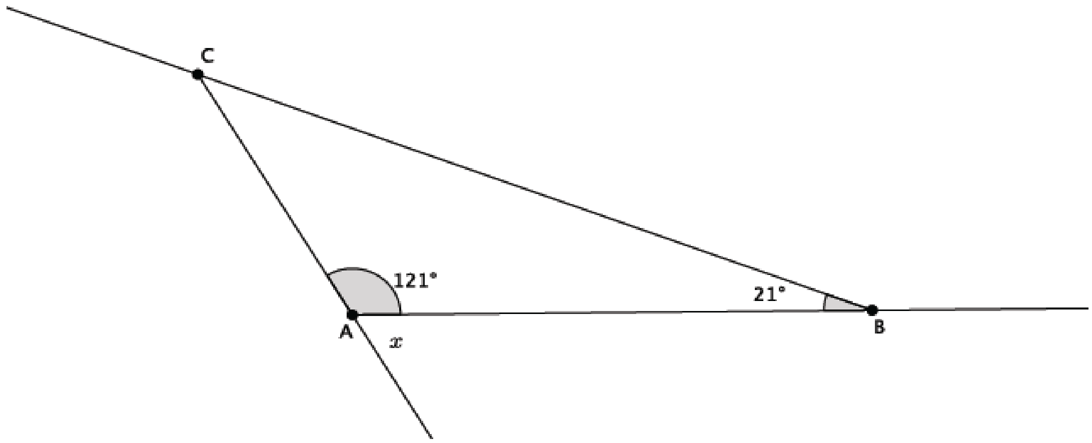
Example 2

Find the measure of angle x.



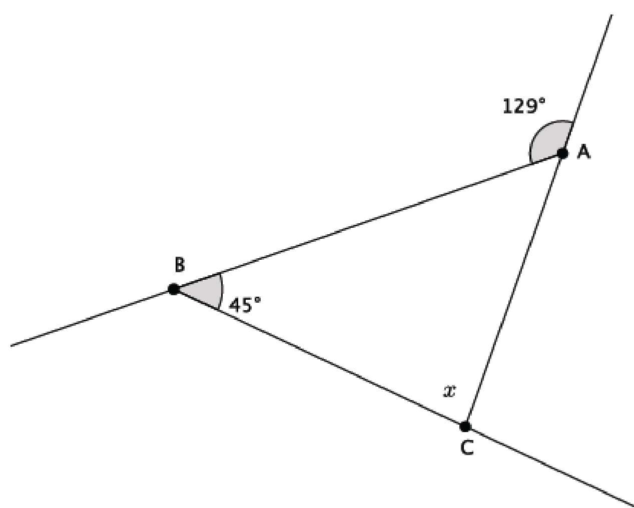
Example 3

Find the measure of angle x



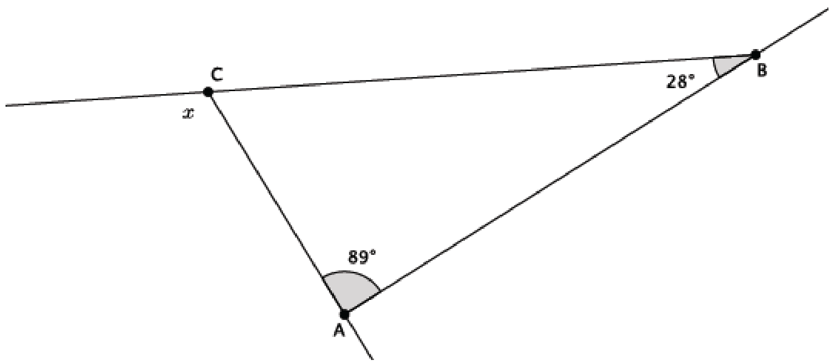
Example 4

Find the measure of angle x.

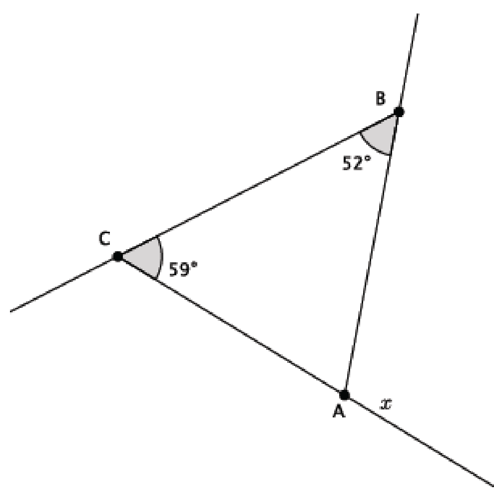


Exercises 5–10

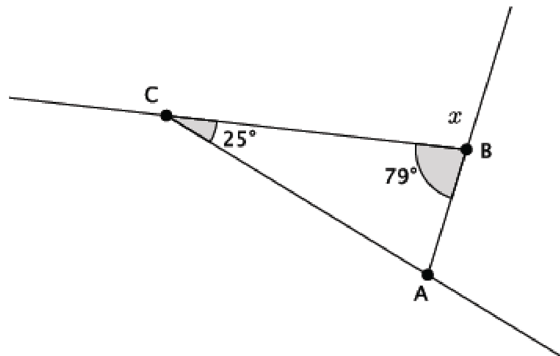
5. Find the measure of angle x. Present an informal argument showing that your answer is correct.



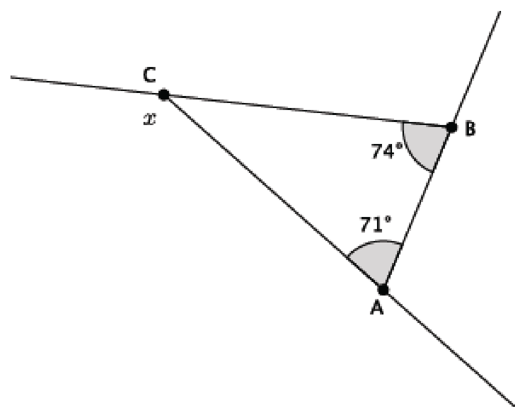
6. Find the measure of angle x. Present an informal argument showing that your answer is correct.



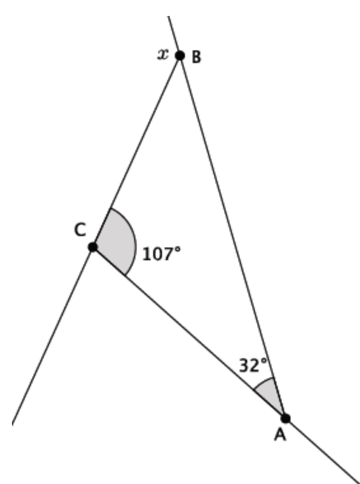
7. Find the measure of angle x. Present an informal argument showing that your answer is correct.



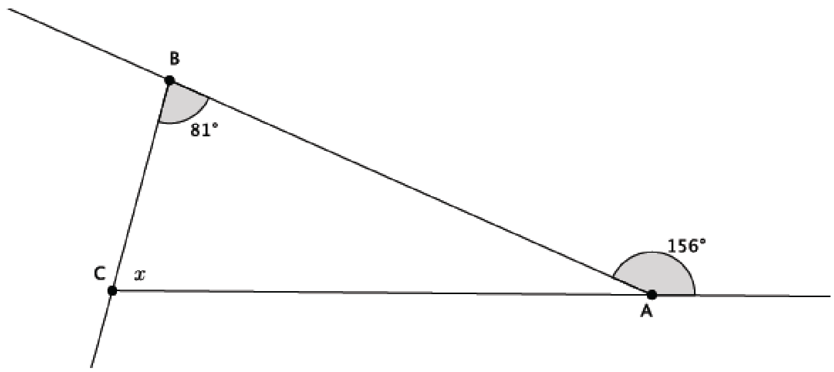
8. Find the measure of angle x. Present an informal argument showing that your answer is correct.



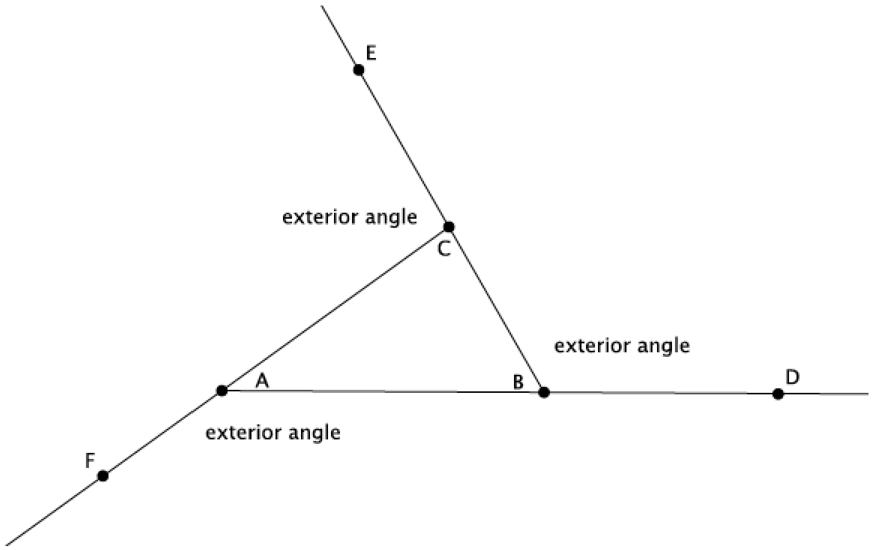
9. Find the measure of angle x. Present an informal argument showing that your answer is correct.



10. Find the measure of angle x. Present an informal argument showing that your answer is correct.



Lesson Summary

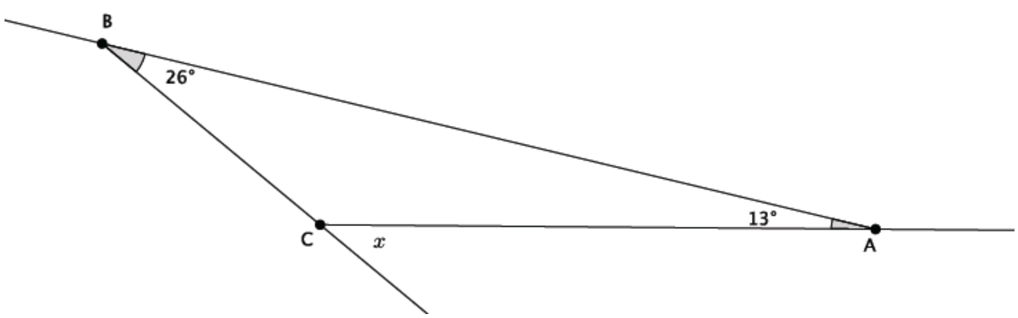


The sum of the remote interior angles of a triangle is equal to the measure of the related exterior angle. For example, $\angle CAB + \angle ABC = \angle ACE$.

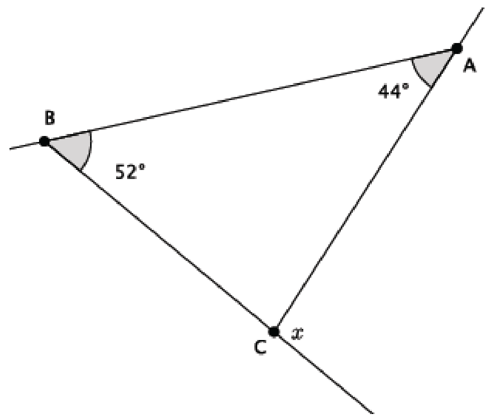
Problem Set

For each of the problems below, use the diagram to find the missing angle measure. Show your work.

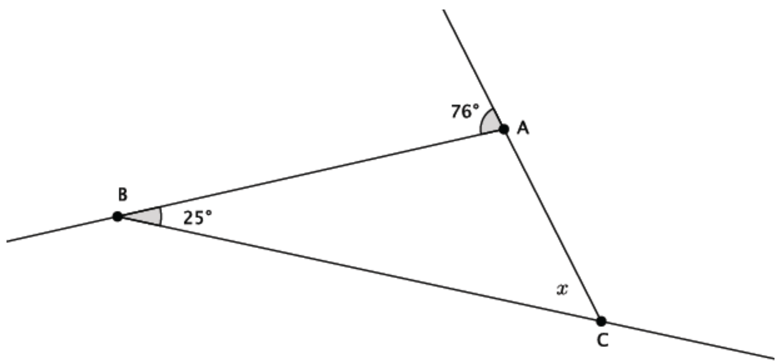
1. Find the measure of angle x. Present an informal argument showing that your answer is correct.



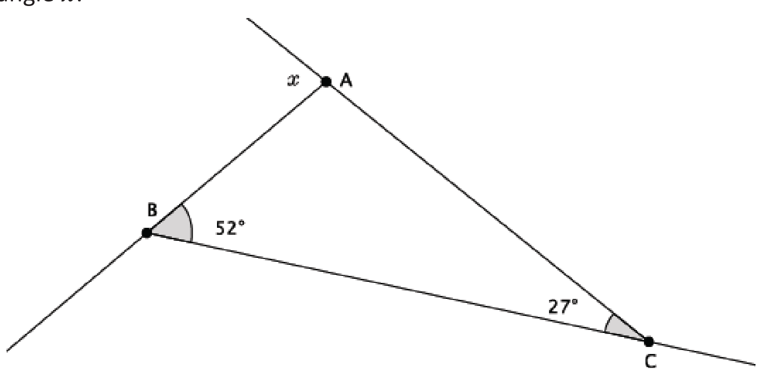
2. Find the measure of angle x.



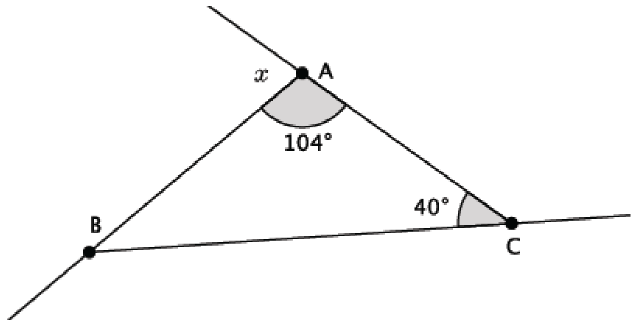
3. Find the measure of angle x. Present an informal argument showing that your answer is correct.



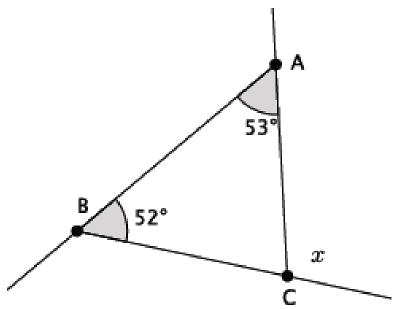
4. Find the measure of angle x.



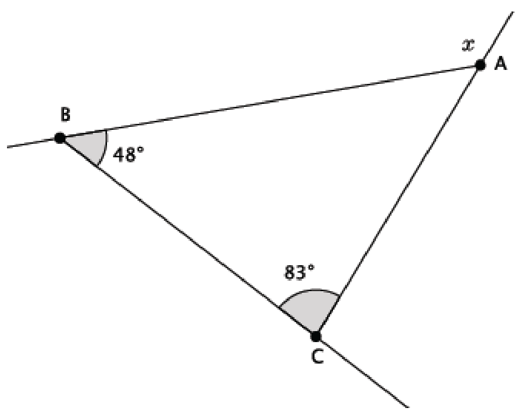
5. Find the measure of angle x.



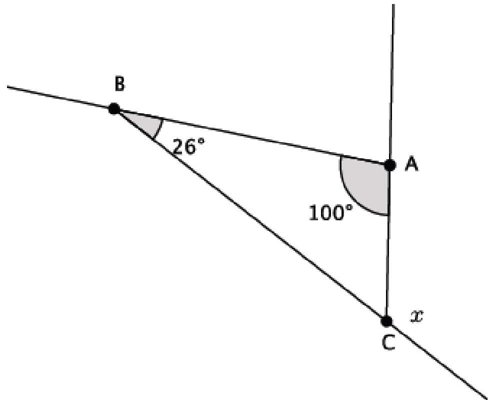
6. Find the measure of angle x.



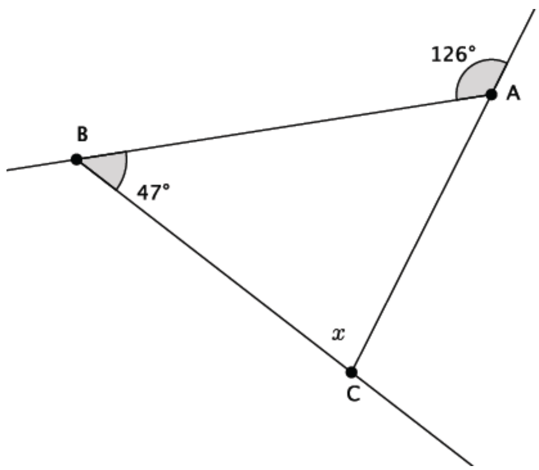
7. Find the measure of angle x.



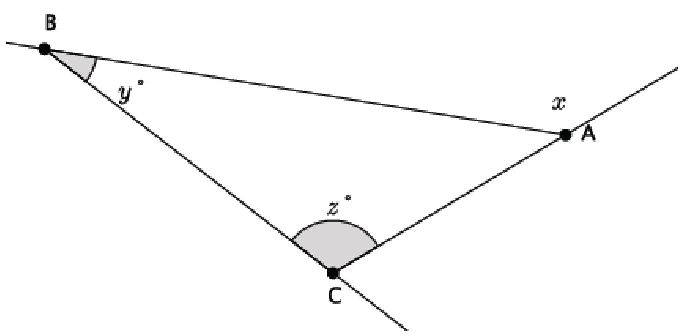
8. Find the measure of angle x.



9. Find the measure of angle x.



10. Write an equation that would allow you to find the measure of angle x. Present an informal argument showing that your answer is correct.



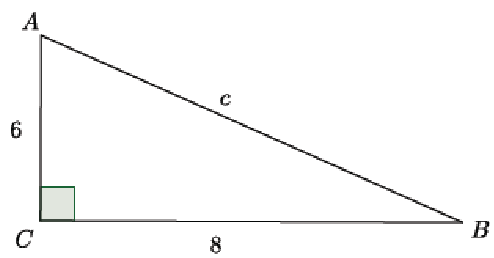
Lesson 15: Informal Proof of the Pythagorean Theorem

Classwork

Example 1

Now that we know what the Pythagorean theorem is, let's practice using it to find the length of a hypotenuse of a right triangle.

Determine the length of the hypotenuse of the right triangle.



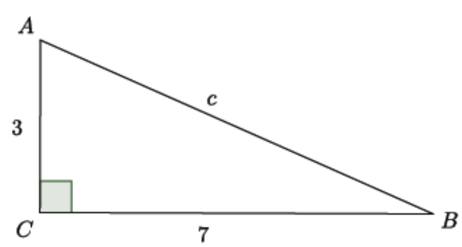
The Pythagorean theorem states that for right triangles $a^2 + b^2 = c^2$, where a and b are the legs and c is the hypotenuse. Then,

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 8^2 &= c^2 \\ 36 + 64 &= c^2 \\ 100 &= c^2. \end{aligned}$$

Since we know that $100 = 10^2$, we can say that the hypotenuse $c = 10$.

Example 2

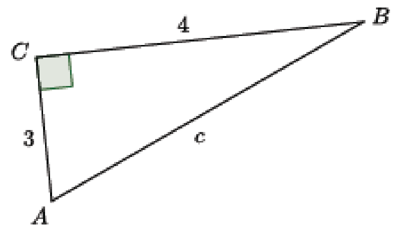
Determine the length of the hypotenuse of the right triangle.



Exercises 1–5

For each of the exercises, determine the length of the hypotenuse of the right triangle shown. Note: Figures not drawn to scale.

1.



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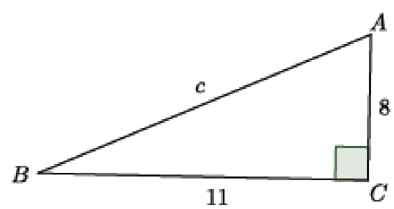

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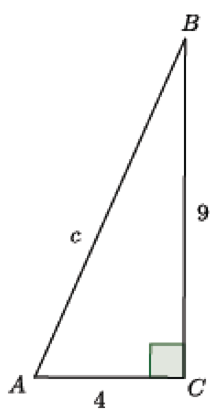

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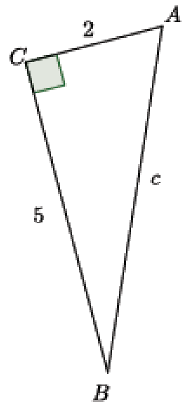

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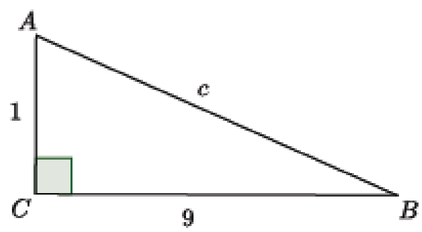
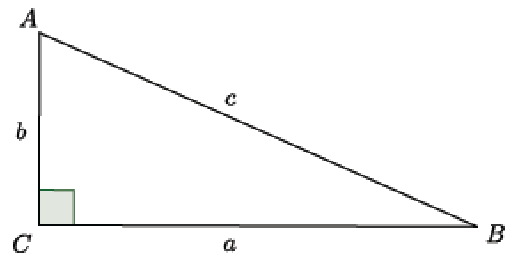


Lesson Summary

Given a right triangle ABC with C being the vertex of the right angle, then the sides AC and BC are called the *legs* of ΔABC, and AB is called the *hypotenuse* of ΔABC.

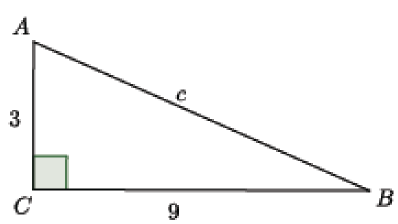


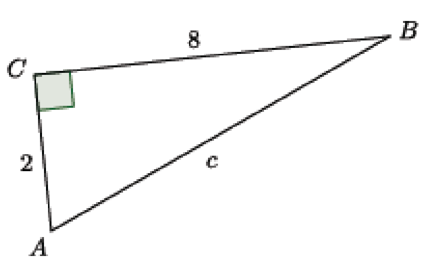
Take note of the fact that side a is opposite the angle A, side b is opposite the angle B, and side c is opposite the angle C.

The Pythagorean theorem states that for any right triangle, $a^2 + b^2 = c^2$.

Problem Set

For each of the problems below, determine the length of the hypotenuse of the right triangle shown. Note: Figures not drawn to scale.

1. 

2. 

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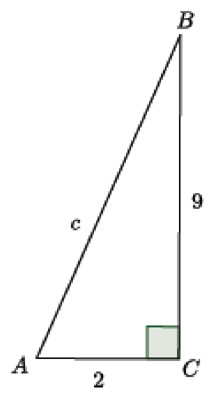

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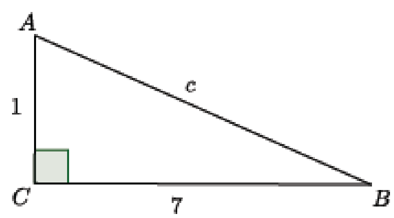

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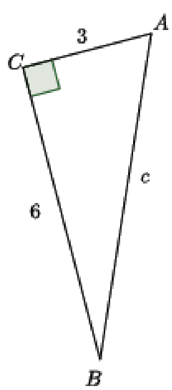

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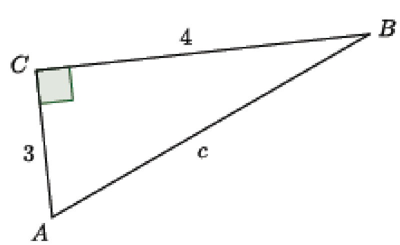

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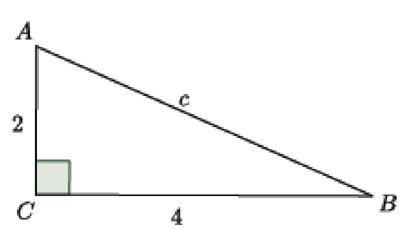

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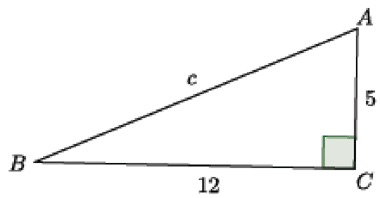

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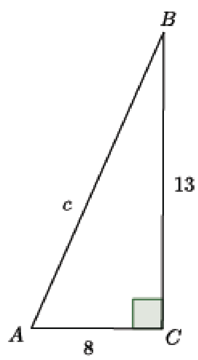

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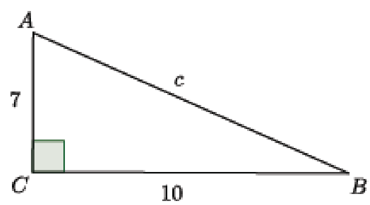

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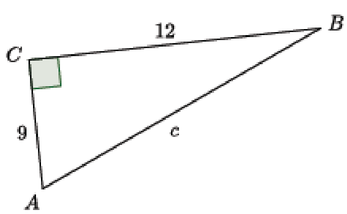
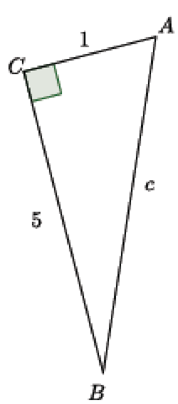


12.

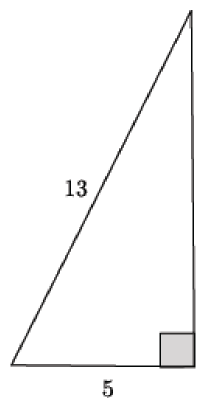


Lesson 16: Applications of the Pythagorean Theorem

Classwork

Example 1

Given a right triangle with a hypotenuse with length 13 units and a leg with length 5 units, as shown, determine the length of the other leg.

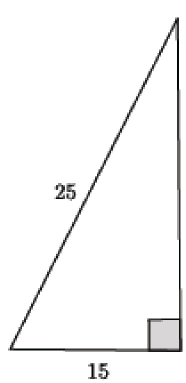


$$
\begin{aligned}
5^2 + b^2 &= 13^2 \\
5^2 - 5^2 + b^2 &= 13^2 - 5^2 \\
b^2 &= 13^2 - 5^2 \\
b^2 &= 169 - 25 \\
b^2 &= 144 \\
b &= 12
\end{aligned}
$$

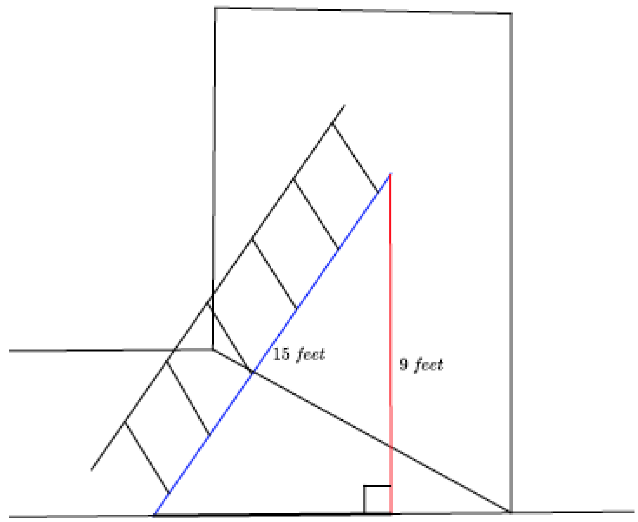
The length of the leg is 12 units.

Exercises 1–2

1. Use the Pythagorean theorem to find the missing length of the leg in the right triangle.

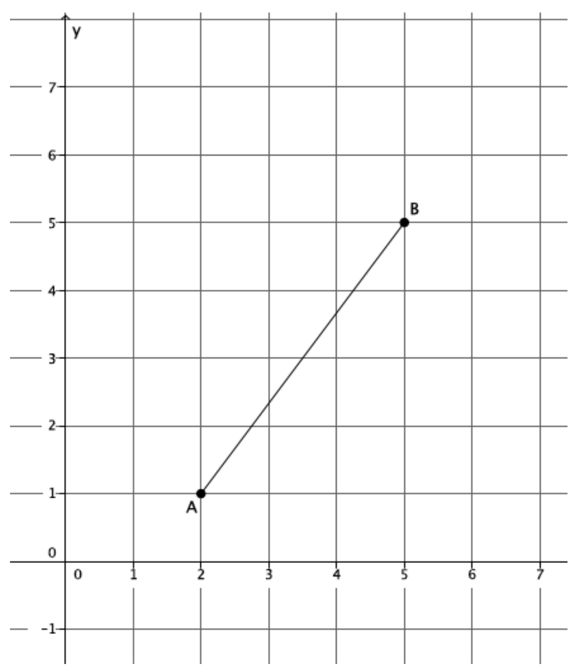


2. You have a 15-foot ladder and need to reach exactly 9 feet up the wall. How far away from the wall should you place the ladder so that you can reach your desired location?

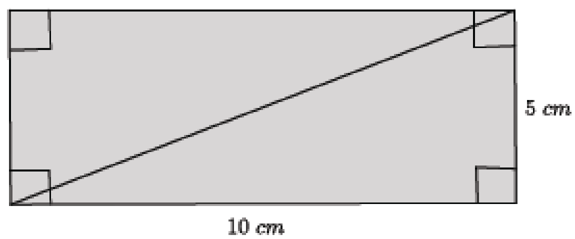


Exercises 3–6

3. Find the length of the segment AB, if possible.

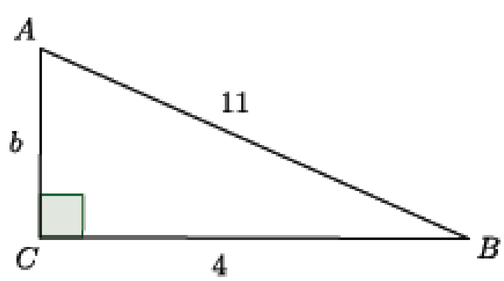


4. Given a rectangle with dimensions 5 cm and 10 cm, as shown, find the length of the diagonal, if possible.



5. A right triangle has a hypotenuse of length 13 in. and a leg with length 4 in. What is the length of the other leg?

6. Find the length of b in the right triangle below, if possible.



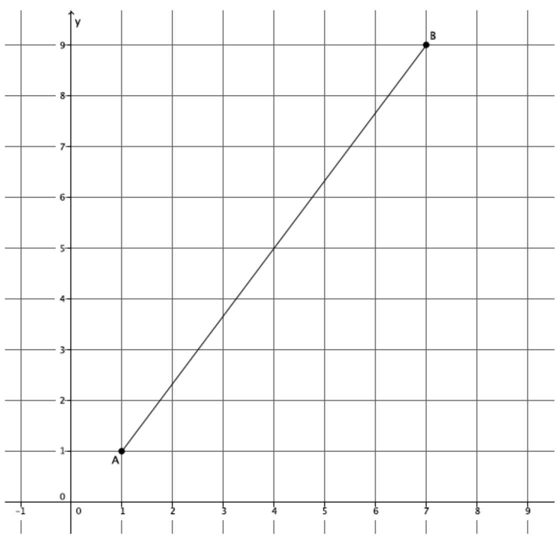
Lesson Summary

The Pythagorean theorem can be used to find the unknown length of a leg of a right triangle.

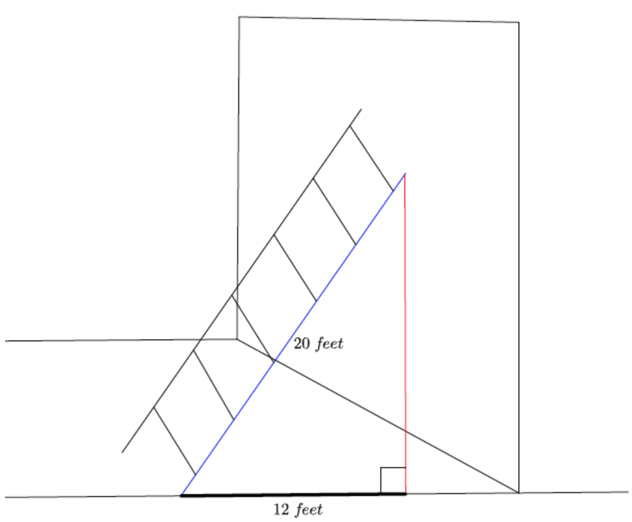
An application of the Pythagorean theorem allows you to calculate the length of a diagonal of a rectangle, the distance between two points on the coordinate plane, and the height that a ladder can reach as it leans against a wall.

Problem Set

1. Find the length of the segment AB shown below, if possible.



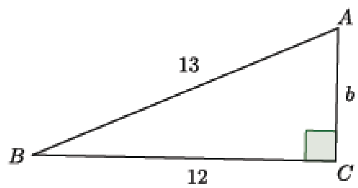
2. A 20-foot ladder is placed 12 feet from the wall, as shown. How high up the wall will the ladder reach?



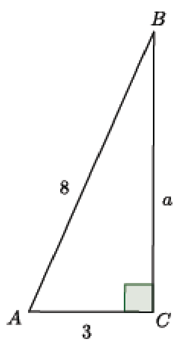
3. A rectangle has dimensions 6 in. by 12 in. What is the length of the diagonal of the rectangle?

Use the Pythagorean theorem to find the missing side lengths for the triangles shown in Problems 4–8.

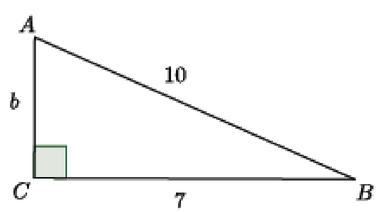
4. Determine the length of the missing side, if possible.



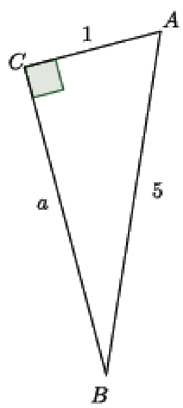
5. Determine the length of the missing side, if possible.



6. Determine the length of the missing side, if possible.



7. Determine the length of the missing side, if possible.



8. Determine the length of the missing side, if possible.