

Lesson 1: Percent

Classwork

Opening Exercise 1: Matching

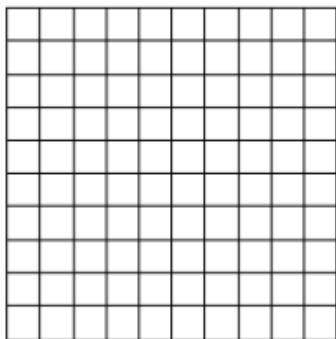
Match the percents with the correct sentence clues.

25%	I am half of a half. 5 cubic inches of water filled in a 20 cubic inch bottle.
50%	I am less than $\frac{1}{100}$. 25 out of 5,000 contestants won a prize.
30%	I am the chance of birthing a boy or a girl. Flip a coin, and it will land on heads or tails.
1%	I am less than a half but more than one-fourth. 15 out of 50 play drums in a band.
10%	I am equal to 1. 35 question out of 35 questions were answered correctly.
100%	I am more than 1. Instead of the \$1,200 expected to be raised, \$3,600 was collected for the school's fundraiser.
300%	I am a tenth of a tenth. One penny is this part of one dollar.
$\frac{1}{2}\%$	I am less than a fourth but more than a hundredth. \$11 out of \$110 earned is saved in the bank.

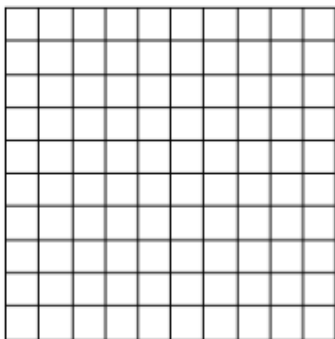
Opening Exercise 2

Color in the grids to represent the following fractions:

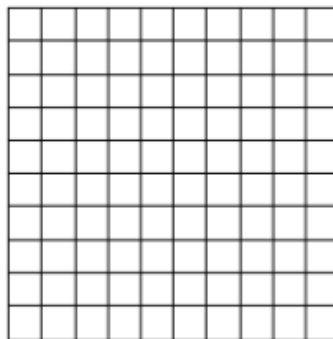
a. $\frac{30}{100}$



b. $\frac{3}{100}$



c. $\frac{1}{3}$



Example 1

Use the definition of the word *percent* to write each percent as a fraction and then as a decimal.

Percent	Fraction	Decimal
37.5%		
100%		
110%		
1%		
$\frac{1}{2}\%$		

Example 2

Fill in the chart by converting between fractions, decimals, and percents. Show your work in the space below.

Fraction	Decimal	Percent
		350%
	0.025	
$\frac{1}{8}$		

Exercise 1: Class Card Activity

Read your card to yourself (each student will have a different card), and work out the problem. When the exercise begins, listen carefully to the questions being read. When you have the card with the equivalent value, respond by reading your card aloud.

Examples:

0.22 should be read “twenty two-hundredths.”

$\frac{\frac{1}{5}}{1000}$ should be read “one-fifth thousandths” or “one-fifth over one thousand.”

$\frac{7}{300}$ should be read “seven-three hundredths” or “seven over three hundred.”

$\frac{200}{100}$ should be read “two hundred-hundredths” or “two hundred over one hundred.”

Lesson Summary

- *Percent* means per hundred. P percent is the same as $\frac{P}{100}$. Use % as a symbol for percent.
- Usually, there are three ways to write a number: a percent, a fraction, and a decimal. Fraction and decimal forms of the percent, P , are equivalent to $\frac{P}{100}$.

Problem Set

1. Create a model to represent the following percents.

- a. 90% b. 0.9% c. 900% d. $\frac{9}{10}\%$

2. Benjamin believes that $\frac{1}{2}\%$ is equivalent to 50%. Is he correct? Why or why not?

3. Order the following from least to greatest:

100%, $\frac{1}{100}$, 0.001%, $\frac{1}{10}$, 0.001, 1.1, 10, and $\frac{10,000}{100}$

4. Fill in the chart by converting between fractions, decimals, and percents. Show work in the space below.

Fraction	Decimal	Percent
		100%
	0.0825	
	6.25	
		$\frac{1}{8}\%$
$\frac{2}{300}$		
		33.3%
$\frac{3}{4}$ $\frac{100}{100}$		
		250%
	0.005	
$\frac{150}{100}$		
$5\frac{1}{2}$ $\frac{100}{100}$	0.055	$5\frac{1}{2}\%$

Lesson 2: Part of a Whole as a Percent

Classwork

Opening Exercise

- a. What is the whole unit in each scenario?

Scenario	Whole Unit
15 is what percent of 90?	
What number is 10% of 56?	
90% of a number is 180.	
A bag of candy contains 300 pieces and 25% of the pieces in the bag are red.	
Seventy percent (70%) of the students earned a B on the test.	
The 20 girls in the class represented 55% of the students in the class.	

- b. Read each problem and complete the table to record what you know.

Problem	Part	Percent	Whole
40% of the students on the field trip love the museum. If there are 20 students on the field trip, how many love the museum?			
40% of the students on the field trip love the museum. If 20 students love the museum, how many are on the field trip?			
20 students on the field trip love the museum. If there are 40 students on the field trip, what percent love the museum?			

Example 1: Visual Approaches to Finding a Part, Given a Percent of the Whole

In Ty's math class, 20% of students earned an A on a test. If there were 30 students in the class, how many got an A?

Exercise 1

In Ty's art class, 12% of the Flag Day art projects received a perfect score. There were 25 art projects turned in by Ty's class. How many of the art projects earned a perfect score? (Identify the whole.)

Example 2: A Numeric Approach to Finding a Part, Given a Percent of the Whole

In Ty's English class, 70% of the students completed an essay by the due date. There are 30 students in Ty's English class. How many completed the essay by the due date?

Example 3: An Algebraic Approach to Finding a Part, Given a Percent of the Whole

A bag of candy contains 300 pieces of which 28% are red. How many pieces are red?

Which quantity represents the whole?

Which of the terms in the percent equation is unknown? Define a letter (variable) to represent the unknown quantity.

Write an expression using the percent and the whole to represent the number of pieces of red candy.

Write and solve an equation to find the unknown quantity.

Exercise 2

A bag of candy contains 300 pieces of which 28% are red. How many pieces are not red?

- a. Write an equation to represent the number of pieces that are not red, n .

- b. Use your equation to find the number of pieces of candy that are not red.

- c. Jah-Lil told his math teacher that he could use the answer from part (b) and mental math to find the number of pieces of candy that are not red. Explain what Jah-Lil meant by that.

Example 4: Comparing Part of a Whole to the Whole with the Percent Formula

Zoey inflated 24 balloons for decorations at the middle school dance. If Zoey inflated 15% of the balloons that are inflated for the dance, how many balloons are there total? Solve the problem using the percent formula, and verify your answer using a visual model.

Example 5: Finding the Percent Given a Part of the Whole and the Whole

Haley is making admission tickets to the middle school dance. So far she has made 112 tickets, and her plan is to make 320 tickets. What percent of the admission tickets has Haley produced so far? Solve the problem using the percent formula, and verify your answer using a visual model.

Lesson Summary

- Visual models or numeric methods can be used to solve percent problems.
- An equation can be used to solve percent problems:

$$\text{Part} = \text{Percent} \times \text{Whole}.$$

Problem Set

1. Represent each situation using an equation. Check your answer with a visual model or numeric method.
 - a. What number is 40% of 90?
 - b. What number is 45% of 90?
 - c. 27 is 30% of what number?
 - d. 18 is 30% of what number?
 - e. 25.5 is what percent of 85?
 - f. 21 is what percent of 60?
2. 40% of the students on a field trip love the museum. If there are 20 students on the field trip, how many love the museum?
3. Maya spent 40% of her savings to pay for a bicycle that cost her \$85.
 - a. How much money was in her savings to begin with?
 - b. How much money does she have left in her savings after buying the bicycle?
4. Curtis threw 15 darts at a dartboard. 40% of his darts hit the bull's-eye. How many darts did not hit the bull's-eye?
5. A tool set is on sale for \$424.15. The original price of the tool set was \$499.00. What percent of the original price is the sale price?
6. Matthew scored a total of 168 points in basketball this season. He scored 147 of those points in the regular season, and the rest were scored in his only playoff game. What percent of his total points did he score in the playoff game?
7. Brad put 10 crickets in his pet lizard's cage. After one day, Brad's lizard had eaten 20% of the crickets he had put in the cage. By the end of the next day, the lizard had eaten 25% of the remaining crickets. How many crickets were left in the cage at the end of the second day?

8. A furnace used 40% of the fuel in its tank in the month of March and then used 25% of the remaining fuel in the month of April. At the beginning of March, there were 240 gallons of fuel in the tank. How much fuel (in gallons) was left at the end of April?
9. In Lewis County, there were 2,277 student athletes competing in spring sports in 2014. That was 110% of the number from 2013, which was 90% of the number from the year before. How many student athletes signed up for a spring sport in 2012?
10. Write a real-world word problem that could be modeled by the equation below. Identify the elements of the percent equation and where they appear in your word problem, and then solve the problem.

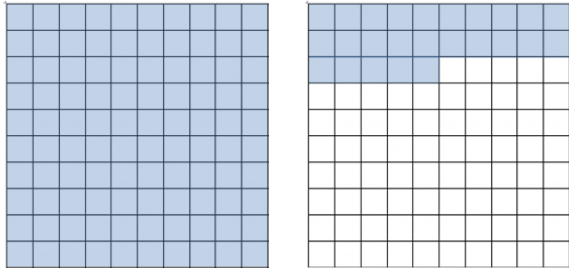
$$57.5 = p(250)$$

Lesson 3: Comparing Quantities with Percent

Classwork

Opening Exercise

If each 10×10 unit square represents one whole, then what percent is represented by the shaded region?



In the model above, 25% represents a quantity of 10 students. How many students does the shaded region represent?

Example 1

- The members of a club are making friendship bracelets to sell to raise money. Anna and Emily made 54 bracelets over the weekend. They need to produce 300 bracelets by the end of the week. What percent of the bracelets were they able to produce over the weekend?

- b. Anna produced 32 bracelets of the 54 bracelets produced by Emily and Anna over the weekend. Compare the number of bracelets that Emily produced as a percent of those that Anna produced.
- c. Write the number of bracelets that Anna produced as a percent of those that Emily produced.

Exercises 1–4

1. There are 750 students in the seventh-grade class and 625 students in the eighth-grade class at Kent Middle School.
- a. What percent is the seventh-grade class of the eighth-grade class at Kent Middle School?
- b. The principal will have to increase the number of eighth-grade teachers next year if the seventh-grade enrollment exceeds 110% of the current eighth-grade enrollment. Will she need to increase the number of teachers? Explain your reasoning.

Lesson Summary

- Visual models or arithmetic methods can be used to solve problems that compare quantities with percents.
- Equations can be used to solve percent problems using the basic equation
$$\text{Quantity} = \text{Percent} \times \text{Whole}.$$
- *Quantity* in the new percent formula is the equivalent of *part* in the original percent formula.

Problem Set

1. Solve each problem using an equation.
 - a. 49.5 is what percent of 33?
 - b. 72 is what percent of 180?
 - c. What percent of 80 is 90?
2. This year, Benny is 12 years old, and his mom is 48 years old.
 - a. What percent of his mom's age is Benny's age?
 - b. What percent of Benny's age is his mom's age?
 - c. In two years, what percent of his age will Benny's mom's age be at that time?
 - d. In 10 years, what percent will Benny's mom's age be of his age?
 - e. In how many years will Benny be 50% of his mom's age?
 - f. As Benny and his mom get older, Benny thinks that the percent of difference between their ages will decrease as well. Do you agree or disagree? Explain your reasoning.
3. This year, Benny is 12 years old. His brother Lenny's age is 175% of Benny's age. How old is Lenny?
4. When Benny's sister Penny is 24, Benny's age will be 125% of her age.
 - a. How old will Benny be then?
 - b. If Benny is 12 years old now, how old is Penny now? Explain your reasoning.
5. Benny's age is currently 200% of his sister Jenny's age. What percent of Benny's age will Jenny's age be in 4 years?

6. At an animal shelter, there are 15 dogs, 12 cats, 3 snakes, and 5 parakeets.
 - a. What percent of the number of cats is the number of dogs?
 - b. What percent of the number of cats is the number of snakes?
 - c. What percent less parakeets are there than dogs?
 - d. Which animal has 80% of the number of another animal?
 - e. Which animal makes up approximately 14% of the animals in the shelter?
7. Is 2 hours and 30 minutes more or less than 10% of a day? Explain your answer.
8. A club's membership increased from 25 to 30 members.
 - a. Express the new membership as a percent of the old membership.
 - b. Express the old membership as a percent of the new membership.
9. The number of boys in a school is 120% the number of girls at the school.
 - a. Find the number of boys if there are 320 girls.
 - b. Find the number of girls if there are 360 boys.
10. The price of a bicycle was increased from \$300 to \$450.
 - a. What percent of the original price is the increased price?
 - b. What percent of the increased price is the original price?
11. The population of Appleton is 175% of the population of Cherryton.
 - a. Find the population in Appleton if the population in Cherryton is 4,000 people.
 - b. Find the population in Cherryton if the population in Appleton is 10,500 people.

12. A statistics class collected data regarding the number of boys and the number of girls in each classroom at their school during homeroom. Some of their results are shown in the table below.

a. Complete the blank cells of the table using your knowledge about percent.

Number of Boys (x)	Number of Girls (y)	Number of Girls as a Percent of the Number of Boys
10	5	
	1	25%
18	12	
5	10	
4		50%
20		90%
	10	250%
	6	60%
11		200%
	5	$33\frac{1}{3}\%$
15		20%
	15	75%
6	18	
25	10	
10		110%
	2	10%
16		75%
	7	50%
3		200%
12	10	

- b. Using a coordinate plane and grid paper, locate and label the points representing the ordered pairs (x, y) .
- c. Locate all points on the graph that would represent classrooms in which the number of girls y is 100% of the number of boys x . Describe the pattern that these points make.

- d. Which points represent the classrooms in which the number of girls is greater than 100% of the number of boys? Which points represent the classrooms in which the number of girls is less than 100% of the number of boys? Describe the locations of the points in relation to the points in part (c).
- e. Find three ordered pairs from your table representing classrooms where the number of girls is the same percent of the number of boys. Do these points represent a proportional relationship? Explain your reasoning.
- f. Show the relationship(s) from part (e) on the graph, and label them with the corresponding equation(s).
- g. What is the constant of proportionality in your equation(s), and what does it tell us about the number of girls and the number of boys at each point on the graph that represents it? What does the constant of proportionality represent in the table in part (a)?

Lesson 4: Percent Increase and Decrease

Classwork

Opening Exercise

Cassandra likes jewelry. She has five rings in her jewelry box.

- a. In the box below, sketch Cassandra's five rings.



- b. Draw a double number line diagram relating the number of rings as a percent of the whole set of rings.
- c. What percent is represented by the whole collection of rings? What percent of the collection does each ring represent?

Example 1: Finding a Percent Increase

Cassandra's aunt said she will buy Cassandra another ring for her birthday. If Cassandra gets the ring for her birthday, what will be the percent increase in her ring collection?



Exercise 1

- a. Jon increased his trading card collection by 5 cards. He originally had 15 cards. What is the percent increase? Use the equation $\text{Quantity} = \text{Percent} \times \text{Whole}$ to arrive at your answer, and then justify your answer using a numeric or visual model.
- b. Suppose instead of increasing the collection by 5 cards, John increased his 15-card collection by just 1 card. Will the percent increase be the same as when Cassandra's ring collection increased by 1 ring (in Example 1)? Why or why not? Explain.
- c. Based on your answer to part (b), how is displaying change as a percent useful?

Discussion

"I will only pay 90% of my bill."	"10% of my bill will be subtracted from the original total."

Example 2: Percent Decrease

Ken said that he is going to reduce the number of calories that he eats during the day. Ken's trainer asked him to start off small and reduce the number of calories by no more than 7%. Ken estimated and consumed 2,200 calories per day instead of his normal 2,500 calories per day until his next visit with the trainer. Did Ken reduce his calorie intake by 7%? Justify your answer.

Exercise 2

Skylar is answering the following math problem:

The value of an investment decreased by 10%. The original amount of the investment was \$75.00. What is the current value of the investment?

- a. Skylar said 10% of \$75.00 is \$7.50, and since the investment decreased by that amount, you have to subtract \$7.50 from \$75.00 to arrive at the final answer of \$67.50. Create one algebraic equation that can be used to arrive at the final answer of \$67.50. Solve the equation to prove it results in an answer of \$67.50. Be prepared to explain your thought process to the class.

- b. Skylar wanted to show the proportional relationship between the dollar value of the original investment, x , and its value after a 10% decrease, y . He creates the table of values shown. Does it model the relationship? Explain. Then, provide a correct equation for the relationship Skylar wants to model.

x	y
75	7.5
100	10
200	20
300	30
400	40

Example 3: Finding a Percent Increase or Decrease

Justin earned 8 badges in Scouts as of the Scout Master's last report. Justin wants to complete 2 more badges so that he will have a total of 10 badges earned before the Scout Master's next report.

- a. If Justin completes the additional 2 badges, what will be the percent increase in badges?

- b. Express the 10 badges as a percent of the 8 badges.

- c. Does 100% plus your answer in part (a) equal your answer in part (b)? Why or why not?

Example 4: Finding the Original Amount Given a Percent Increase or Decrease

The population of cats in a rural neighborhood has declined in the past year by roughly 30%. Residents hypothesize that this is due to wild coyotes preying on the cats. The current cat population in the neighborhood is estimated to be 12. Approximately how many cats were there originally?

Example 5: Finding the Original Amount Given a Percent Increase or Decrease

Lu's math score on her achievement test in seventh grade was a 650. Her math teacher told her that her test level went up by 25% from her sixth grade test score level. What was Lu's test score level in sixth grade?

Closing

Phrase	Whole Unit (100%)
"Mary has 20% more money than John."	
"Anne has 15% less money than John."	
"What percent more (money) does Anne have than Bill?"	
"What percent less (money) does Bill have than Anne?"	

Lesson Summary

- Within each problem, there are keywords that determine if the problem represents a percent increase or a percent decrease.
- Equations can be used to solve percent problems using the basic equation
$$\text{Quantity} = \text{Percent} \times \text{Whole}.$$
- *Quantity* in the percent formula is the amount of change (increase or decrease) or the amount after the change.
- *Whole* in the percent formula represents the original amount.

Problem Set

1. A store advertises 15% off an item that regularly sells for \$300.
 - a. What is the sale price of the item?
 - b. How is a 15% discount similar to a 15% decrease? Explain.
 - c. If 8% sales tax is charged on the sale price, what is the total with tax?
 - d. How is 8% sales tax like an 8% increase? Explain.
2. An item that was selling for \$72.00 is reduced to \$60.00. Find the percent decrease in price. Round your answer to the nearest tenth.
3. A baseball team had 80 players show up for tryouts last year and this year had 96 players show up for tryouts. Find the percent increase in players from last year to this year.
4. At a student council meeting, there was a total of 60 students present. Of those students, 35 were female.
 - a. By what percent is the number of females greater than the number of males?
 - b. By what percent is the number of males less than the number of females?
 - c. Why is the percent increase and percent decrease in parts (a) and (b) different?
5. Once each day, Darlene writes in her personal diary and records whether the sun is shining or not. When she looked back through her diary, she found that over a period of 600 days, the sun was shining 60% of the time. She kept recording for another 200 days and then found that the total number of sunny days dropped to 50%. How many of the final 200 days were sunny days?
6. Henry is considering purchasing a mountain bike. He likes two bikes: One costs \$500, and the other costs \$600. He tells his dad that the bike that is more expensive is 20% more than the cost of the other bike. Is he correct? Justify your answer.

7. State two numbers such that the lesser number is 25% less than the greater number.
8. State two numbers such that the greater number is 75% more than the lesser number.
9. Explain the difference in your thought process for Problems 7 and 8. Can you use the same numbers for each problem? Why or why not?
10. In each of the following expressions, c represents the original cost of an item.
 - i. $0.90c$
 - ii. $0.10c$
 - iii. $c - 0.10c$
 - a. Circle the expression(s) that represents 10% of the original cost. If more than one answer is correct, explain why the expressions you chose are equivalent.
 - b. Put a box around the expression(s) that represents the final cost of the item after a 10% decrease. If more than one is correct, explain why the expressions you chose are equivalent.
 - c. Create a word problem involving a percent decrease so that the answer can be represented by expression (ii).
 - d. Create a word problem involving a percent decrease so that the answer can be represented by expression (i).
 - e. Tyler wants to know if it matters if he represents a situation involving a 25% decrease as $0.25x$ or $(1 - 0.25)x$. In the space below, write an explanation that would help Tyler understand how the context of a word problem often determines how to represent the situation.

Lesson 5: Finding One Hundred Percent Given Another Percent

Classwork

Opening Exercise

What are the whole number factors of 100? What are the multiples of those factors? How many multiples are there of each factor (up to 100)?

Factors of 100	Multiples of the Factors of 100	Number of Multiples
100	100	1
50	50, 100	2
1	1, 2, 3, 4, 5, 6, ... , 98, 99, 100	100

Example 1: Using a Modified Double Number Line with Percent

The 42 students who play wind instruments represent 75% of the students who are in band. How many students are in band?

Exercises 1–3

1. Bob's Tire Outlet sold a record number of tires last month. One salesman sold 165 tires, which was 60% of the tires sold in the month. What was the record number of tires sold?
2. Nick currently has 7,200 points in his fantasy baseball league, which is 20% more points than Adam. How many points does Adam have?
3. Kurt has driven 276 miles of his road trip but has 70% of the trip left to go. How many more miles does Kurt have to drive to get to his destination?

Example 2: Mental Math Using Factors of 100

Answer each part below using only mental math, and describe your method.

- If 39 is 1% of a number, what is that number? How did you find your answer?
- If 39 is 10% of a number, what is that number? How did you find your answer?
- If 39 is 5% of a number, what is that number? How did you find your answer?
- If 39 is 15% of a number, what is that number? How did you find your answer?
- If 39 is 25% of a number, what is that number? How did you find your answer?

Lesson Summary

To find 100% of the whole, you can use a variety of methods, including factors of 100 (1, 2, 4, 5, 10, 20, 25, 50, and 100) and double number lines. Both methods will require breaking 100% into equal-sized intervals. Use the greatest common factor of 100 and the percent corresponding to the part.

Problem Set

Use a double number line to answer Problems 1–5.

1. Tanner collected 360 cans and bottles while fundraising for his baseball team. This was 40% of what Reggie collected. How many cans and bottles did Reggie collect?
2. Emilio paid \$287.50 in taxes to the school district that he lives in this year. This year's taxes were a 15% increase from last year. What did Emilio pay in school taxes last year?
3. A snowmobile manufacturer claims that its newest model is 15% lighter than last year's model. If this year's model weighs 799 lb., how much did last year's model weigh?
4. Student enrollment at a local school is concerning the community because the number of students has dropped to 504, which is a 20% decrease from the previous year. What was the student enrollment the previous year?
5. The color of paint used to paint a race car includes a mixture of yellow and green paint. Scotty wants to lighten the color by increasing the amount of yellow paint 30%. If a new mixture contains 3.9 liters of yellow paint, how many liters of yellow paint did he use in the previous mixture?

Use factors of 100 and mental math to answer Problems 6–10. Describe the method you used.

6. Alexis and Tasha challenged each other to a typing test. Alexis typed 54 words in one minute, which was 120% of what Tasha typed. How many words did Tasha type in one minute?
7. Yoshi is 5% taller today than she was one year ago. Her current height is 168 cm. How tall was she one year ago?
8. Toya can run one lap of the track in 1 min. 3 sec., which is 90% of her younger sister Niki's time. What is Niki's time for one lap of the track?
9. An animal shelter houses only cats and dogs, and there are 25% more cats than dogs. If there are 40 cats, how many dogs are there, and how many animals are there total?

10. Angie scored 91 points on a test but only received a 65% grade on the test. How many points were possible on the test?

For Problems 11–17, find the answer using any appropriate method.

11. Robbie owns 15% more movies than Rebecca, and Rebecca owns 10% more movies than Joshua. If Rebecca owns 220 movies, how many movies do Robbie and Joshua each have?
12. 20% of the seventh-grade students have math class in the morning. $16\frac{2}{3}\%$ of those students also have science class in the morning. If 30 seventh-grade students have math class in the morning but not science class, find how many seventh-grade students there are.
13. The school bookstore ordered three-ring notebooks. They put 75% of the order in the warehouse and sold 80% of the rest in the first week of school. There are 25 notebooks left in the store to sell. How many three-ring notebooks did they originally order?
14. In the first game of the year, the modified basketball team made 62.5% of their foul shot free throws. Matthew made all 6 of his free throws, which made up for 25% of the team's free throws. How many free throws did the team miss altogether?
15. Aiden's mom calculated that in the previous month, their family had used 40% of their monthly income for gasoline, and 63% of that gasoline was consumed by the family's SUV. If the family's SUV used \$261.45 worth of gasoline last month, how much money was left after gasoline expenses?
16. Rectangle A is a scale drawing of Rectangle B and has 25% of its area. If Rectangle A has side lengths of 4 cm and 5 cm, what are the side lengths of Rectangle B?



17. Ted is a supervisor and spends 20% of his typical work day in meetings and 20% of that meeting time in his daily team meeting. If he starts each day at 7:30 a.m., and his daily team meeting is from 8:00 a.m. to 8:20 a.m., when does Ted's typical work day end?

Lesson 6: Fluency with Percents

Classwork

Opening Exercise

Solve the following problem using mental math only. Be prepared to discuss your method with your classmates.

Cory and Everett have collected model cars since the third grade. Cory has 80 model cars in his collection, which is 25% more than Everett has. How many model cars does Everett have?

Example 1: Mental Math and Percents

- a. 75% of the students in Jesse's class are 60 inches or taller. If there are 20 students in her class, how many students are 60 inches or taller?

- b. Bobbie wants to leave a tip for her waitress equal to 15% of her bill. Bobbie's bill for her lunch is \$18. How much money represents 15% of the bill?

- b. Express the number of fans for school B as a percent of the number of fans for school A.
- c. What percent more fans were there for school B than for school A?
4. Rectangle A has a length of 8 cm and a width of 16 cm. Rectangle B has the same area as the first, but its length is 62.5% of the length of the first rectangle. Express the width of Rectangle B as a percent of the width of Rectangle A. What percent more or less is the width of Rectangle B than the width of Rectangle A?

5. A plant in Mikayla's garden was 40 inches tall one day and was 4 feet tall one week later. By what percent did the plant's height increase over one week?
6. Loren must obtain a minimum number of signatures on a petition before it can be submitted. She was able to obtain 672 signatures, which is 40% more than she needs. How many signatures does she need?

Lesson Summary

- Identify the type of percent problem that is being asked as a comparison of quantities or a part of a whole.
- Identify what numbers represent the part, the whole, and the percent. And use the representation
$$\text{Quantity} = \text{Percent} \times \text{Whole}.$$
- A strategy to solving percents using mental math is to rewrite a percent using 1%, 5%, or 10%. These percents can be solved mentally. For example: $13\% = 10\% + 3(1\%)$. To find 13% of 70, find 10% of 70 as 7, 1% of 70 as 0.70, so 13% of 70 is $7 + 3(0.70) = 7 + 2.10 = 9.10$.

Problem Set

1. Micah has 294 songs stored in his phone, which is 70% of the songs that Jorge has stored in his phone. How many songs are stored on Jorge's phone?
2. Lisa sold 81 magazine subscriptions, which is 27% of her class's fundraising goal. How many magazine subscriptions does her class hope to sell?
3. Theresa and Isaiah are comparing the number of pages that they read for pleasure over the summer. Theresa read 2,210 pages, which was 85% of the number of pages that Isaiah read. How many pages did Isaiah read?
4. In a parking garage, the number of SUVs is 40% greater than the number of non-SUVs. Gina counted 98 SUVs in the parking garage. How many vehicles were parked in the garage?
5. The price of a tent was decreased by 15% and sold for \$76.49. What was the original price of the tent in dollars?
6. 40% of the students at Rockledge Middle School are musicians. 75% of those musicians have to read sheet music when they play their instruments. If 38 of the students can play their instruments without reading sheet music, how many students are there at Rockledge Middle School?
7. At Longbridge Middle School, 240 students said that they are an only child, which is 48% of the school's student enrollment. How many students attend Longbridge Middle School?
8. Grace and her father spent $4\frac{1}{2}$ hours over the weekend restoring their fishing boat. This time makes up 6% of the time needed to fully restore the boat. How much total time is needed to fully restore the boat?

9. Bethany's mother was upset with her because Bethany's text messages from the previous month were 218% of the amount allowed at no extra cost under her phone plan. Her mother had to pay for each text message over the allowance. Bethany had 5,450 text messages last month. How many text messages is she allowed under her phone plan at no extra cost?
10. Harry used 84% of the money in his savings account to buy a used dirt bike that cost him \$1,050. How much money is left in Harry's savings account?
11. 15% of the students in Mr. Riley's social studies classes watch the local news every night. Mr. Riley found that 136 of his students do not watch the local news. How many students are in Mr. Riley's social studies classes?
12. Grandma Bailey and her children represent about 9.1% of the Bailey family. If Grandma Bailey has 12 children, how many members are there in the Bailey family?
13. Shelley earned 20% more money in tips waitressing this week than last week. This week she earned \$72.00 in tips waitressing. How much money did she earn last week in tips?
14. Lucy's savings account has 35% more money than her sister Edy's. Together, the girls have saved a total of \$206.80. How much money has each girl saved?
15. Bella spent 15% of her paycheck at the mall, and 40% of that was spent at the movie theater. Bella spent a total of \$13.74 at the movie theater for her movie ticket, popcorn, and a soft drink. How much money was in Bella's paycheck?
16. On a road trip, Sara's brother drove 47.5% of the trip, and Sara drove 80% of the remainder. If Sara drove for 4 hours and 12 minutes, how long was the road trip?

Lesson 7: Markup and Markdown Problems

Classwork

Example 1: A Video Game Markup

Games Galore Super Store buys the latest video game at a wholesale price of \$30.00. The markup rate at Game's Galore Super Store is 40%. You use your allowance to purchase the game at the store. How much will you pay, not including tax?

- Write an equation to find the price of the game at Games Galore Super Store. Explain your equation.
- Solve the equation from part (a).
- What was the total markup of the video game? Explain.
- You and a friend are discussing markup rate. He says that an easier way to find the total markup is by multiplying the wholesale price of \$30.00 by 40%. Do you agree with him? Why or why not?

Example 2: Black Friday

A \$300 mountain bike is discounted by 30%, and then discounted an additional 10% for shoppers who arrive before 5:00 a.m.

- Find the sales price of the bicycle.
- In all, by how much has the bicycle been discounted in dollars? Explain.
- After both discounts were taken, what was the total percent discount?
- Instead of purchasing the bike for \$300, how much would you save if you bought it before 5:00 a.m.?

Exercises 1–3

1. Sasha went shopping and decided to purchase a set of bracelets for 25% off of the regular price. If Sasha buys the bracelets today, she will receive an additional 5%. Find the sales price of the set of bracelets with both discounts. How much money will Sasha save if she buys the bracelets today?



2. A golf store purchases a set of clubs at a wholesale price of \$250. Mr. Edmond learned that the clubs were marked up 200%. Is it possible to have a percent increase greater than 100%? What is the retail price of the clubs?
3. Is a percent increase of a set of golf clubs from \$250 to \$750 the same as a markup rate of 200%? Explain.

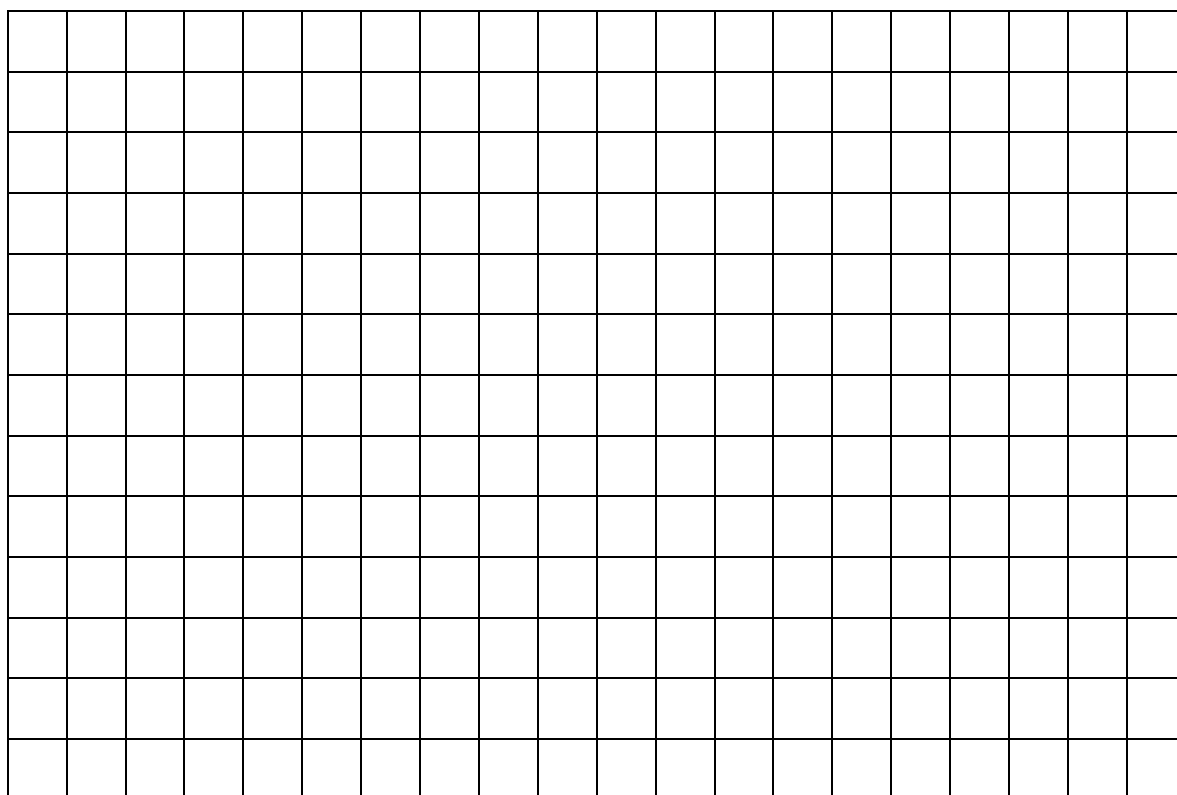
Example 3: Working Backward

A car that normally sells for \$20,000 is on sale for \$16,000. The sales tax is 7.5%.

- What percent of the original price of the car is the final price?
- Find the discount rate.
- By law, sales tax has to be applied to the discount price. However, would it be better for the consumer if the 7.5% sales tax was calculated before the 20% discount was applied? Why or why not?
- Write an equation applying the commutative property to support your answer to part (c).

Exercise 4

- a. Write an equation to determine the selling price in dollars, p , on an item that is originally priced s dollars after a markup of 25%.
- b. Create and label a table showing five possible pairs of solutions to the equation.
- c. Create and label a graph of the equation.



- d. Interpret the points $(0,0)$ and $(1,r)$.

Exercise 5

Use the following table to calculate the markup or markdown rate. Show your work. Is the relationship between the original price and selling price proportional or not? Explain.

Original Price, m (in dollars)	Selling Price, p (in dollars)
\$1,750	\$1,400
\$1,500	\$1,200
\$1,250	\$1,000
\$1,000	\$800
\$750	\$600

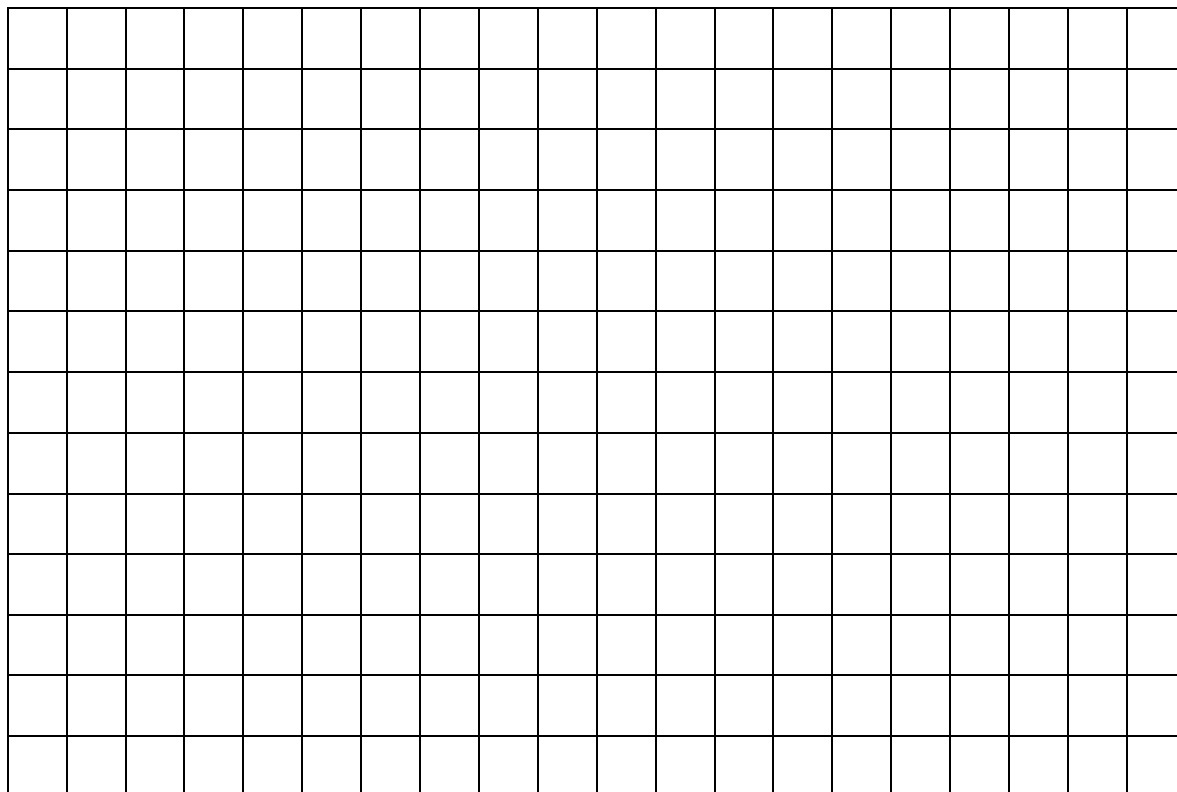
Lesson Summary

- To find the markup or markdown of an item, multiply the whole by $(1 \pm m)$, where m is the markup/markdown rate.
- To apply multiple discount rates to the price of an item, you must find the first discount price and then use this answer to get the second discount price.

Problem Set

1. You have a coupon for an additional 25% off the price of any sale item at a store. The store has put a robotics kit on sale for 15% off the original price of \$40. What is the price of the robotics kit after both discounts?
2. A sign says that the price marked on all music equipment is 30% off the original price. You buy an electric guitar for the sale price of \$315.
 - a. What is the original price?
 - b. How much money did you save off the original price of the guitar?
 - c. What percent of the original price is the sale price?
3. The cost of a New York Yankee baseball cap is \$24.00. The local sporting goods store sells it for \$30.00. Find the markup rate.

4. Write an equation to determine the selling price in dollars, p , on an item that is originally priced s dollars after a markdown of 15%.
- Create and label a table showing five possible pairs of solutions to the equation.
 - Create and label a graph of the equation.



- Interpret the points $(0,0)$ and $(1,r)$.
5. At the amusement park, Laura paid \$6.00 for a small cotton candy. Her older brother works at the park, and he told her they mark up the cotton candy by 300%. Laura does not think that is mathematically possible. Is it possible, and if so, what is the price of the cotton candy before the markup?
6. A store advertises that customers can take 25% off the original price and then take an extra 10% off. Is this the same as a 35% off discount? Explain.
7. An item that costs \$50.00 is marked 20% off. Sales tax for the item is 8%. What is the final price, including tax?
- Solve the problem with the discount applied before the sales tax.
 - Solve the problem with the discount applied after the sales tax.
 - Compare your answers in parts (a) and (b). Explain.

8. The sale price for a bicycle is \$315. The original price was first discounted by 50% and then discounted an additional 10%. Find the original price of the bicycle.
9. A ski shop has a markup rate of 50%. Find the selling price of skis that cost the storeowner \$300.
10. A tennis supply store pays a wholesaler \$90 for a tennis racquet and sells it for \$144. What is the markup rate?
11. A shoe store is selling a pair of shoes for \$60 that has been discounted by 25%. What was the original selling price?
12. A shoe store has a markup rate of 75% and is selling a pair of shoes for \$133. Find the price the store paid for the shoes.
13. Write $5\frac{1}{4}\%$ as a simple fraction.
14. Write $\frac{3}{8}$ as a percent.
15. If 20% of the 70 faculty members at John F. Kennedy Middle School are male, what is the number of male faculty members?
16. If a bag contains 400 coins, and $33\frac{1}{2}\%$ are nickels, how many nickels are there? What percent of the coins are not nickels?
17. The temperature outside is 60 degrees Fahrenheit. What would be the temperature if it is increased by 20%?

Lesson 8: Percent Error Problems

Classwork

Example 1: How Far Off

Find the absolute error for the following problems. Explain what the absolute error means in context.

- a. Taylor's Measurement 1

- b. Connor's Measurement 1

- c. Jordan's Measurement 2

Example 2: How Right Is Wrong?

- a. Find the percent error for Taylor's Measurement 1. What does this mean?

- b. From Example 1, part (b), find the percent error for Connor's Measurement 1. What does this mean?
- c. From Example 1, part (c), find the percent error for Jordan's Measurement 2. What does it mean?
- d. What is the purpose of finding percent error?

Exercises 1–3

Calculate the percent error for Problems 1–3. Leave your final answer in fraction form, if necessary.

1. A realtor expected 18 people to show up for an open house, but 25 attended.

Lesson Summary

- The absolute error is defined as $|a - x|$, where x is the exact value of a quantity and a is an approximate value.
- The percent error is defined as $\frac{|a - x|}{|x|} \times 100\%$.
- The absolute error will tell how big the error is but the percent error compares the error to the actual value. A good measurement has a small percent error.

Problem Set

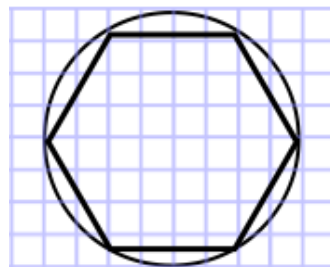
1. The odometer in Mr. Washington's car does not work correctly. The odometer recorded 13.2 miles for his last trip to the hardware store, but he knows the distance traveled is 15 miles. What is the percent error? Use a calculator and the percent error formula to help find the answer. Show your steps.
2. The actual length of a soccer field is 500 feet. A measuring instrument shows the length to be 493 feet. The actual width of the field is 250 feet, but the recorded width is 246.5 feet. Answer the following questions based on this information. Round all decimals to the nearest tenth.
 - a. Find the percent error for the length of the soccer field.
 - b. Find the percent error of the area of the soccer field.
 - c. Explain why the values from parts (a) and (b) are different.

250 feet



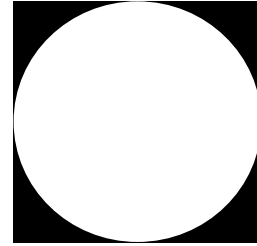
500 feet

3. Kayla's class went on a field trip to an aquarium. One tank had 30 clown fish. She miscounted the total number of clown fish in the tank and recorded it as 24 fish. What is Kayla's percent error?
4. Sid used geometry software to draw a circle of radius 4 units on a grid. He estimated the area of the circle by counting the squares that were mostly inside the circle and got an answer of 52 square units.



- a. Is his estimate too large or too small?
- b. Find the percent error in Sid's estimation to the nearest hundredth using the π key on your calculator.

5. The exact value for the density of aluminum is 2.699 g/cm^3 . Working in the science lab at school, Joseph finds the density of a piece of aluminum to be 2.75 g/cm^3 . What is Joseph's percent error? (Round to the nearest hundredths.)
6. The world's largest marathon, The New York City Marathon, is held on the first Sunday in November each year. It is estimated that anywhere between 2 million and 2.5 million spectators will line the streets to cheer on the marathon runners. At most, what is the percent error?
7. A circle is inscribed inside a square, which has a side length of 12.6 cm. Jared estimates the area of the circle to be about 80% of the area of the square and comes up with an estimate of 127 cm^2 .
- Find the absolute error from Jared's estimate to two decimal places using the π key on your calculator.
 - Find the percent error of Jared's estimate to two decimal places using the π key on your calculator.
 - Do you think Jared's estimate was reasonable?
 - Would this method of computing the area of a circle always be too large?
8. In a school library, 52% of the books are paperback. If there are 2,658 books in the library, how many of them are not paperback to the nearest whole number?
9. Shaniqua has 25% less money than her older sister Jennifer. If Shaniqua has \$180, how much money does Jennifer have?
10. An item that was selling for \$1,102 is reduced to \$806. To the nearest whole, what is the percent decrease?
11. If 60 calories from fat is 75% of the total number of calories in a bag of chips, find the total number of calories in the bag of chips.



12.6 cm

Lesson 9: Problem Solving When the Percent Changes

Classwork

Example 1

The amount of money Tom has is 75% of Sally's amount of money. After Sally spent \$120 and Tom saved all his money, Tom's amount of money is 50% more than Sally's. How much money did each have at the beginning? Use a visual model and a percent line to solve the problem.

Example 2

Erin and Sasha went to a candy shop. Sasha bought 50% more candies than Erin. After Erin bought 8 more candies, Sasha had 20% more. How many candies did Erin and Sasha have at first?

- Model the situation using a visual model.

- b. How many candies did Erin have at first? Explain.

Example 3

Kimberly and Mike have an equal amount of money. After Kimberly spent \$50 and Mike spent \$25, Mike's money is 50% more than Kimberly's. How much did Kimberly and Mike have at first?

- a. Use an equation to solve the problem.
- b. Use a visual model to solve the problem.
- c. Which method do you prefer and why?

Exercise

Todd has 250% more video games than Jaylon. Todd has 56 video games in his collection. He gives Jaylon 8 of his games. How many video games did Todd and Jaylon have in the beginning? How many do they have now?

Lesson Summary

- To solve a changing percent problem, identify the first whole and then the second whole. To relate the part, whole, and percent, use the formula

$$\text{Quantity} = \text{Percent} \times \text{Whole}.$$

- Models, such as double number lines, can help visually show the change in quantities and percents.

Problem Set

1. Solve each problem using an equation.
 - a. What is 150% of 625?
 - b. 90 is 40% of what number?
 - c. What percent of 520 is 40? Round to the nearest hundredth of a percent.
2. The actual length of a machine is 12.25 cm. The measured length is 12.2 cm. Round the answer to part (b) to the nearest hundredth of a percent.
 - a. Find the absolute error.
 - b. Find the percent error.
3. A rowing club has 600 members. 60% of them are women. After 200 new members joined the club, the percentage of women was reduced to 50%. How many of the new members are women?
4. 40% of the marbles in a bag are yellow. The rest are orange and green. The ratio of the number of orange to the number of green is 4:5. If there are 30 green marbles, how many yellow marbles are there? Use a visual model to show your answer.
5. Susan has 50% more books than Michael. Michael has 40 books. If Michael buys 8 more books, will Susan have more or less books than Michael? What percent more or less will Susan's books be? Use any method to solve the problem.
6. Harry's amount of money is 75% of Kayla's amount of money. After Harry earned \$30 and Kayla earned 25% more of her money, Harry's amount of money is 80% of Kayla's money. How much money did each have at the beginning? Use a visual model to solve the problem.

Lesson 10: Simple Interest

Classwork

To find the simple interest, use:

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$

$$I = P \times r \times t$$

$$I = Prt$$

- r is the percent of the principal that is paid over a period of time (usually per year).
- t is the time.
- r and t must be compatible. For example, if r is an annual interest rate, then t must be written in years.

Example 1: Can Money Grow? A Look at Simple Interest

Larry invests \$100 in a savings plan. The plan pays $4\frac{1}{2}\%$ interest each year on his \$100 account balance.

a. How much money will Larry earn in interest after 3 years? After 5 years?

b. How can you find the balance of Larry's account at the end of 5 years?

Exercise 1

Find the balance of a savings account at the end of 10 years if the interest earned each year is 7.5%. The principal is \$500.

Example 2: Time Other Than One Year

A \$1,000 savings bond earns simple interest at the rate of 3% each year. The interest is paid at the end of every month. How much interest will the bond have earned after 3 months?

Example 3: Solving for P , r , or t

Mrs. Williams wants to know how long it will take an investment of \$450 to earn \$200 in interest if the yearly interest rate is 6.5%, paid at the end of each year.

Exercise 2

Write an equation to find the amount of simple interest, A , earned on a \$600 investment after $1\frac{1}{2}$ years, if the semi-annual (6-month) interest rate is 2%.

Exercise 3

A \$1,500 loan has an annual interest rate of $4\frac{1}{4}\%$ on the amount borrowed. How much time has elapsed if the interest is now \$127.50?

Lesson Summary

- Interest earned over time can be represented by a proportional relationship between time, in years, and interest.
- The simple interest formula is

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$

$$I = P \times r \times t$$

$$I = Prt$$

r is the percent of the principal that is paid over a period of time (usually per year)

t is the time

- The rate, r , and time, t , must be compatible. If r is the annual interest rate, then t must be written in years.

Problem Set

- Enrique takes out a student loan to pay for his college tuition this year. Find the interest on the loan if he borrowed \$2,500 at an annual interest rate of 6% for 15 years.
- Your family plans to start a small business in your neighborhood. Your father borrows \$10,000 from the bank at an annual interest rate of 8% rate for 36 months. What is the amount of interest he will pay on this loan?
- Mr. Rodriguez invests \$2,000 in a savings plan. The savings account pays an annual interest rate of 5.75% on the amount he put in at the end of each year.
 - How much will Mr. Rodriguez earn if he leaves his money in the savings plan for 10 years?
 - How much money will be in his savings plan at the end of 10 years?
 - Create (and label) a graph in the coordinate plane to show the relationship between time and the amount of interest earned for 10 years. Is the relationship proportional? Why or why not? If so, what is the constant of proportionality?
 - Explain what the points (0, 0) and (1, 115) mean on the graph.
 - Using the graph, find the balance of the savings plan at the end of seven years.
 - After how many years will Mr. Rodriguez have increased his original investment by more than 50%? Show your work to support your answer.

Challenge Problem

4. George went on a game show and won \$60,000. He wanted to invest it and found two funds that he liked. Fund 250 earns 15% interest annually, and Fund 100 earns 8% interest annually. George does not want to earn more than \$7,500 in interest income this year. He made the table below to show how he could invest the money.

	I	P	r	t
Fund 100		x	0.08	1
Fund 250		$60,000 - x$	0.15	1
Total	7,500	60,000		

- Explain what value x is in this situation.
- Explain what the expression $60,000 - x$ represents in this situation.
- Using the simple interest formula, complete the table for the amount of interest earned.
- Write an equation to show the total amount of interest earned from both funds.
- Use algebraic properties to solve for x and the principal, in dollars, George could invest in Fund 100. Show your work.
- Use your answer from part (e) to determine how much George could invest in Fund 250.
- Using your answers to parts (e) and (f), how much interest would George earn from each fund?

Lesson 11: Tax, Commissions, Fees, and Other Real-World

Percent Problems

Classwork

Opening Exercise

How are each of the following percent applications different, and how are they the same? Solve each problem, and then compare your solution process for each problem.

- a. Silvio earns 10% for each car sale he makes while working at a used car dealership. If he sells a used car for \$2,000, what is his commission?

- b. Tu's family stayed at a hotel for 10 nights on their vacation. The hotel charged a 10% room tax, per night. How much did they pay in room taxes if the room cost \$200 per night?

- c. Eric bought a new computer and printer online. He had to pay 10% in shipping fees. The items totaled \$2,000. How much did the shipping cost?

- d. Selena had her wedding rehearsal dinner at a restaurant. The restaurant's policy is that gratuity is included in the bill for large parties. Her father said the food and service were exceptional, so he wanted to leave an extra 10% tip on the total amount of the bill. If the dinner bill totaled \$2,000, how much money did her father leave as the extra tip?

Exercises

Show all work; a calculator may be used for calculations.

The school board has approved the addition of a new sports team at your school.

1. The district ordered 30 team uniforms and received a bill for \$2,992.50. The total included a 5% discount.
 - a. The school needs to place another order for two more uniforms. The company said the discount will not apply because the discount only applies to orders of \$1,000 or more. How much will the two uniforms cost?
 - b. The school district does not have to pay the 8% sales tax on the \$2,992.50 purchase. Estimate the amount of sales tax the district saved on the \$2,992.50 purchase. Explain how you arrived at your estimate.
 - c. A student who loses a uniform must pay a fee equal to 75% of the school's cost of the uniform. For a uniform that cost the school \$105, will the student owe more or less than \$75 for the lost uniform? Explain how to use mental math to determine the answer.
 - d. Write an equation to represent the proportional relationship between the school's cost of a uniform and the amount a student must pay for a lost uniform. Use u to represent the uniform cost and s to represent the amount a student must pay for a lost uniform. What is the constant of proportionality?

2. A taxpayer claims the new sports team caused his school taxes to increase by 2%.
- a. Write an equation to show the relationship between the school taxes before and after a 2% increase. Use b to represent the dollar amount of school tax before the 2% increase and t to represent the dollar amount of school tax after the 2% increase.

- b. Use your equation to complete the table below, listing at least 5 pairs of values.

b	t
1,000	
2,000	
	3,060
	6,120

- c. On graph paper, graph the relationship modeled by the equation in part (a). Be sure to label the axes and scale.
- d. Is the relationship proportional? Explain how you know.
- e. What is the constant of proportionality? What does it mean in the context of the situation?
- f. If a taxpayers' school taxes rose from \$4,000 to \$4,020, was there a 2% increase? Justify your answer using your graph, table, or equation.

3. The sports booster club sold candles as a fundraiser to support the new team. The club earns a commission on its candle sales (which means it receives a certain percentage of the total dollar amount sold). If the club gets to keep 30% of the money from the candle sales, what would the club's total sales have to be in order to make at least \$500?
4. Christian's mom works at the concession stand during sporting events. She told him they buy candy bars for \$0.75 each and mark them up 40% to sell at the concession stand. What is the amount of the markup? How much does the concession stand charge for each candy bar?

With your group, brainstorm solutions to the problems below. Prepare a poster that shows your solutions and math work. A calculator may be used for calculations.

5. For the next school year, the new soccer team will need to come up with \$600.
 - a. Suppose the team earns \$500 from the fundraiser at the start of the current school year, and the money is placed for one calendar year in a savings account earning 0.5% simple interest annually. How much money will the team still need to raise to meet next year's expenses?
 - b. Jeff is a member of the new sports team. His dad owns a bakery. To help raise money for the team, Jeff's dad agrees to provide the team with cookies to sell at the concession stand for next year's opening game. The team must pay back the bakery \$0.25 for each cookie it sells. The concession stand usually sells about 60 to 80 baked goods per game. Using your answer from part (a), determine a percent markup for the cookies the team plans to sell at next year's opening game. Justify your answer.

- c. Suppose the team ends up selling 78 cookies at next year's opening game. Find the percent error in the number of cookies that you estimated would be sold in your solution to part (b).

Percent Error = $\frac{|a-x|}{|x|} \cdot 100\%$, where x is the exact value and a is the approximate value.

Lesson Summary

- There are many real-world problems that involve percents. For example, gratuity (tip), commission, fees, and taxes are applications found daily in the real world. They each increase the total, so all questions like these reflect a percent increase. Likewise, markdowns and discounts decrease the total, so they reflect a percent decrease.
- Regardless of the application, the percent relationship can be represented as

$$\text{Quantity(Part)} = \text{Percent(\%)} \times \text{Whole}$$

Problem Set

1. A school district's property tax rate rises from 2.5% to 2.7% to cover a \$300,000 budget deficit (shortage of money). What is the value of the property in the school district to the nearest dollar? (Note: Property is assessed at 100% of its value.)
2. Jake's older brother Sam has a choice of two summer jobs. He can either work at an electronics store or at the school's bus garage. The electronics store would pay him to work 15 hours per week. He would make \$8 per hour plus a 2% commission on his electronics sales. At the school's bus garage, Sam could earn \$300 per week working 15 hours cleaning buses. Sam wants to take the job that pays him the most. How much in electronics would Sam have to sell for the job at the electronics store to be the better choice for his summer job?
3. Sarah lost her science book. Her school charges a lost book fee equal to 75% of the cost of the book. Sarah received a notice stating she owed the school \$60 for the lost book.
 - a. Write an equation to represent the proportional relationship between the school's cost for the book and the amount a student must pay for a lost book. Let B represent the school's cost of the book in dollars and N represent the student's cost in dollars.
 - b. What is the constant or proportionality? What does it mean in the context of this situation?
 - c. How much did the school pay for the book?

4. In the month of May, a certain middle school has an average daily absentee rate of 8% each school day. The absentee rate is the percent of students who are absent from school each day.
- Write an equation that shows the proportional relationship between the number of students enrolled in the middle school and the average number of students absent each day during the month of May. Let s represent the number of students enrolled in school, and let a represent the average number of students absent each day in May.
 - Use your equation to complete the table. List 5 possible values for s and a .

s	a

- Identify the constant of proportionality, and explain what it means in the context of this situation.
 - Based on the absentee rate, determine the number of students absent on average from school during the month of May if there are 350 students enrolled in the middle school.
5. The equation shown in the box below could relate to many different percent problems. Put an X next to each problem that could be represented by this equation. For any problem that does not match this equation, explain why it does not. $\text{Quantity} = 1.05 \cdot \text{Whole}$

_____ Find the amount of an investment after 1 year with 0.5% interest paid annually.

_____ Write an equation to show the amount paid for an item including tax, if the tax rate is 5%.

_____ A proportional relationship has a constant of proportionality equal to 105%.

Whole	0	100	200	300	400	500
Quantity	0	105	210	315	420	525

_____ Mr. Hendrickson sells cars and earns a 5% commission on every car he sells. Write an equation to show the relationship between the price of a car Mr. Hendrickson sold and the amount of commission he earns.

Lesson 12: The Scale Factor as a Percent for a Scale Drawing

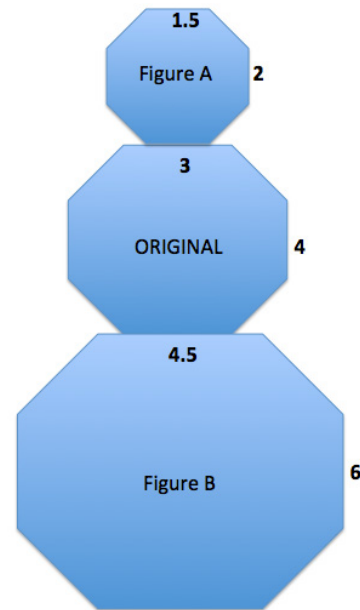
Classwork

Compare the corresponding lengths of Figure A to the original octagon in the middle. This is an example of a particular type of *scale drawing* called a

_____. Explain why it is called that.

Compare the corresponding lengths of Figure B to the original octagon in the middle. This is an example of a particular type of *scale drawing* called an

_____. Explain why it is called that.



The *scale factor* is the quotient of any length in the scale drawing to its corresponding length in the actual drawing.

Use what you recall from Module 1 to determine the scale factors between the original figure and Figure A and the original figure and Figure B.

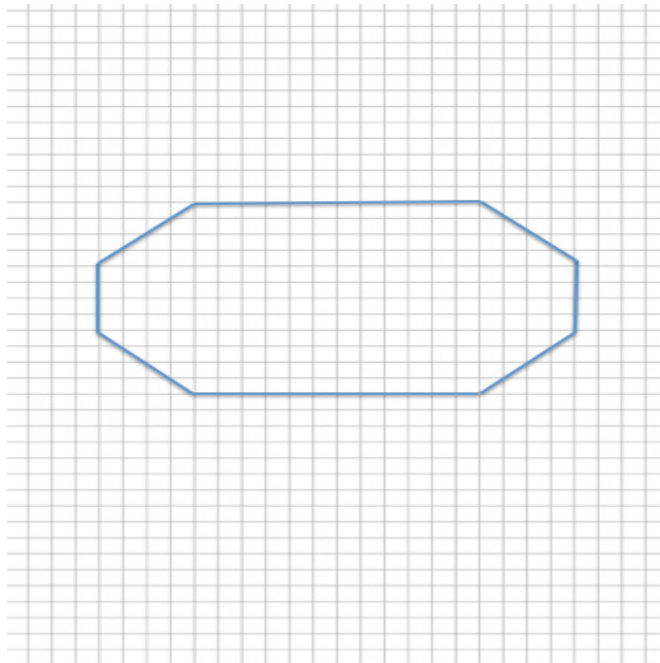
Using the diagram, complete the chart to determine the horizontal and vertical scale factors. Write answers as a percent and as a concluding statement using the previously learned reduction and enlargement vocabulary.

	Horizontal Measurement in Scale Drawing	Vertical Measurement in Scale Drawing	Concluding Statement
Figure A			
Figure B			

Example 1

Create a snowman on the accompanying grid. Use the octagon given as the middle of the snowman with the following conditions:

- a. Calculate the width, neck, and height for the figure at the right.



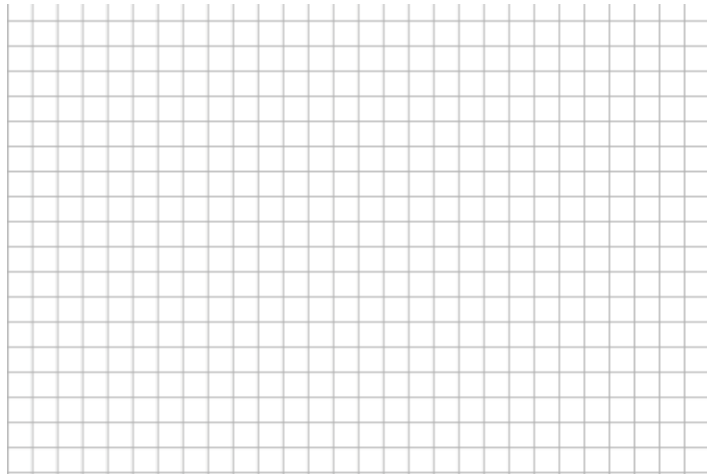
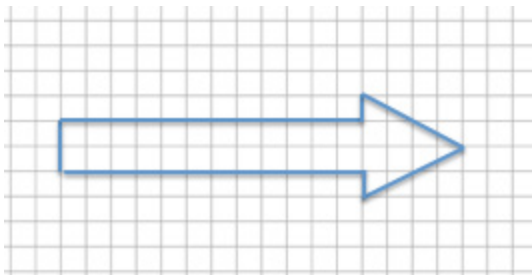
- b. To create the head of the snowman, make a scale drawing of the middle of the snowman with a scale factor of 75%. Calculate the new lengths for the width, neck, and height.

- c. To create the bottom of the snowman, make a scale drawing of the middle of the snowman with a scale factor of 125%. Calculate the new lengths for the width, neck, and height.

- d. Is the head a reduction or enlargement of the middle?
- e. Is the bottom a reduction or enlargement of the middle?
- f. What is the significance of the scale factor as it relates to 100%? What happens when such scale factors are applied?
- g. Use the dimensions you calculated in parts (b) and (c) to draw the complete snowman.

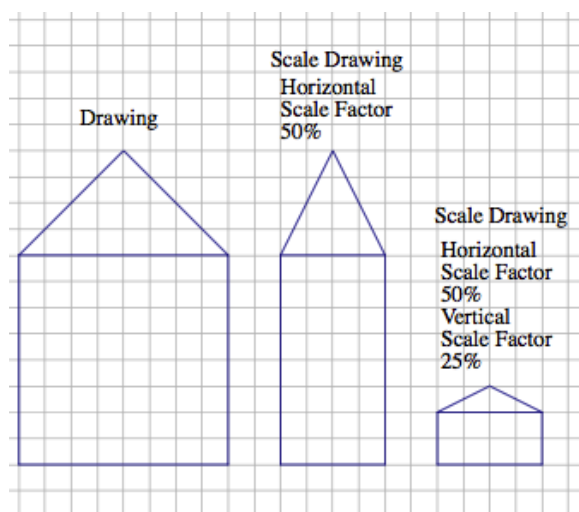
Example 2

Create a scale drawing of the arrow below using a scale factor of 150%.

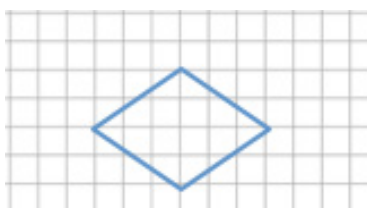


Example 3: Scale Drawing Where the Horizontal and Vertical Scale Factors Are Different

Sometimes it is helpful to make a scale drawing where the horizontal and vertical scale factors are different, such as when creating diagrams in the field of engineering. Having differing scale factors may distort some drawings. For example, when you are working with a very large horizontal scale, you sometimes must exaggerate the vertical scale in order to make it readable. This can be accomplished by creating a drawing with two scales. Unlike the scale drawings with just one scale factor, these types of scale drawings may look distorted. Next to the drawing below is a scale drawing with a horizontal scale factor of 50% and vertical scale factor of 25% (given in two steps). Explain how each drawing is created.

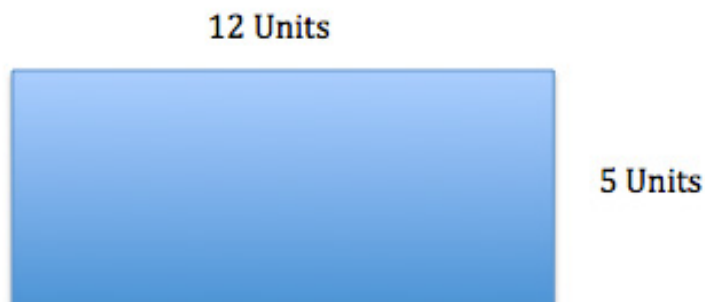
**Exercise 1**

Create a scale drawing of the following drawing using a horizontal scale factor of $183\frac{1}{3}\%$ and a vertical scale factor of 25%.



Exercise 2

Chris is building a rectangular pen for his dog. The dimensions are 12 units long and 5 units wide.



Chris is building a second pen that is 60% the length of the original and 125% the width of the original. Write equations to determine the length and width of the second pen.

Lesson Summary

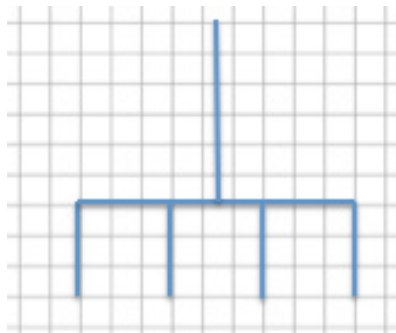
The scale factor is the number that determines whether the new drawing is an enlargement or a reduction of the original. If the scale factor is greater than 100%, then the resulting drawing will be an enlargement of the original drawing. If the scale factor is less than 100%, then the resulting drawing will be a reduction of the original drawing.

When a scale factor is mentioned, assume that it refers to both vertical and horizontal factors. It will be noted if the horizontal and vertical factors are intended to be different.

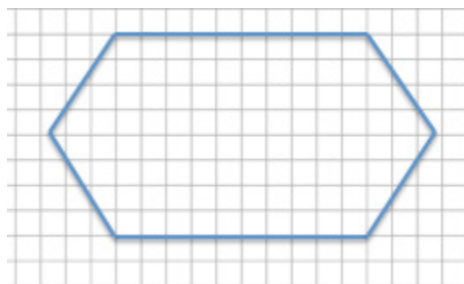
To create a scale drawing with both the same vertical and horizontal factors, determine the horizontal and vertical distances of the original drawing. Using the given scale factor, determine the new corresponding lengths in the scale drawing by writing a numerical equation that requires the scale factor to be multiplied by the original length. Draw new segments based on the calculations from the original segments. If the scale factors are different, determine the new corresponding lengths the same way but use the unique given scale factor for each of the horizontal length and vertical length.

Problem Set

1. Use the diagram below to create a scale drawing using a scale factor of $133\frac{1}{3}\%$. Write numerical equations to find the horizontal and vertical distances in the scale drawing.



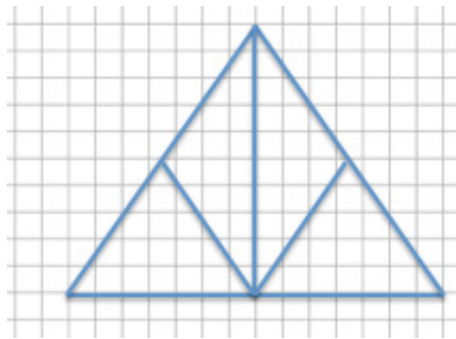
2. Create a scale drawing of the original drawing given below using a horizontal scale factor of 80% and a vertical scale factor of 175%. Write numerical equations to find the horizontal and vertical distances.



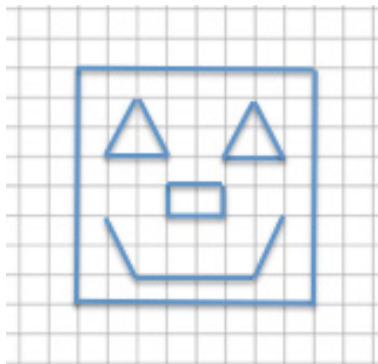
3. The accompanying diagram shows that the length of a pencil from its eraser to its tip is 7 units and that the eraser is 1.5 units wide. The picture was placed on a photocopy machine and reduced to $66\frac{2}{3}\%$. Find the new size of the pencil and sketch a drawing. Write numerical equations to find the new dimensions.



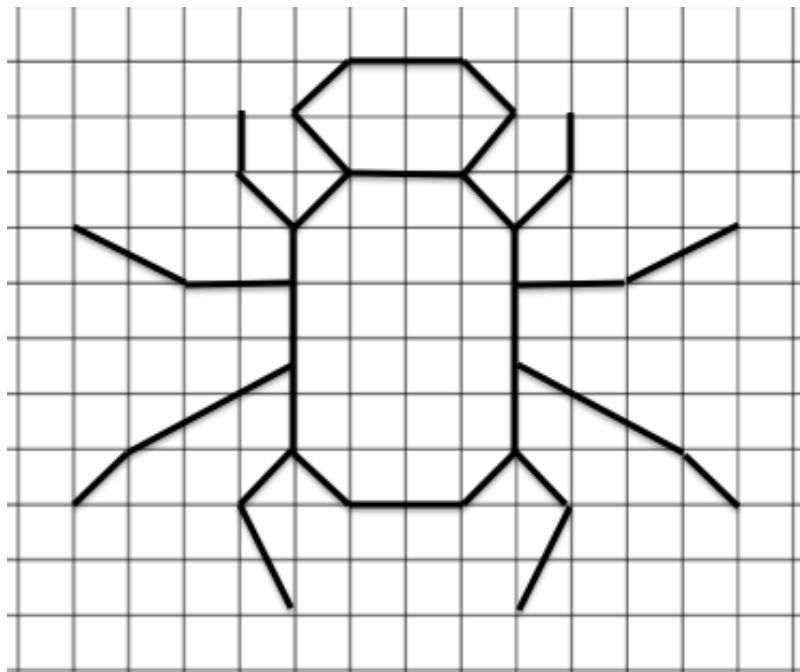
4. Use the diagram to answer each question.
- What are the corresponding horizontal and vertical distances in a scale drawing if the scale factor is 25%? Use numerical equations to find your answers.



- What are the corresponding horizontal and vertical distances in a scale drawing if the scale factor is 160%? Use a numerical equation to find your answers.
5. Create a scale drawing of the original drawing below using a horizontal scale factor of 200% and a vertical scale factor of 250%.



6. Using the diagram below, on grid paper sketch the same drawing using a horizontal scale factor of 50% and a vertical scale factor of 150%.



Lesson 13: Changing Scales

Classwork

Opening Exercise

Scale factor: $\frac{\text{length in SCALE drawing}}{\text{Corresponding length in ORIGINAL drawing}}$

Describe, using percentages, the difference between a reduction and an enlargement.

Use the two drawings below to complete the chart. Calculate the first row (Drawing 1 to Drawing 2) only.



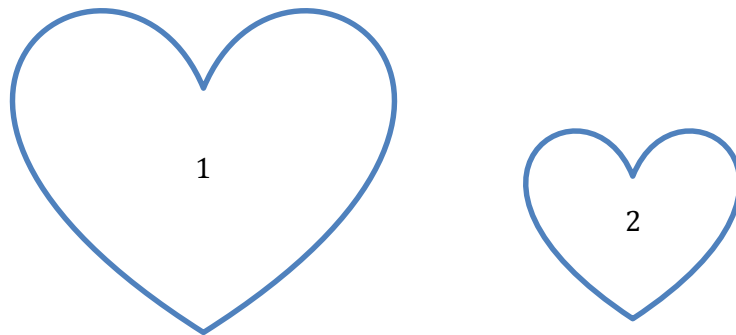
	Quotient of Corresponding Horizontal Distances	Quotient of Corresponding Vertical Distances	Scale Factor as a Percent	Reduction or Enlargement?
Drawing 1 to Drawing 2				
Drawing 2 to Drawing 1				

Compare Drawing 2 to Drawing 1. Using the completed work in the first row, make a conjecture (statement) about what the second row of the chart will be. Justify your conjecture without computing the second row.

Compute the second row of the chart. Was your conjecture proven true? Explain how you know.

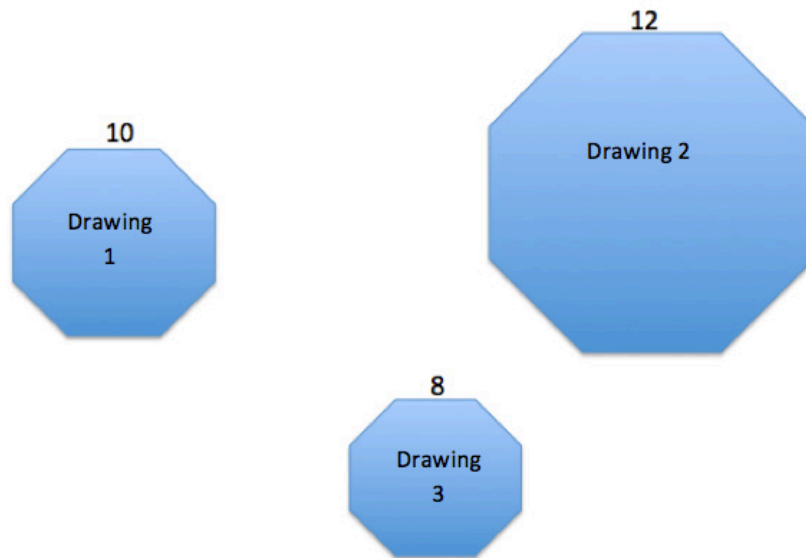
Example 1

The scale factor from Drawing 1 to Drawing 2 is 60%. Find the scale factor from Drawing 2 to Drawing 1. Explain your reasoning.



Example 2

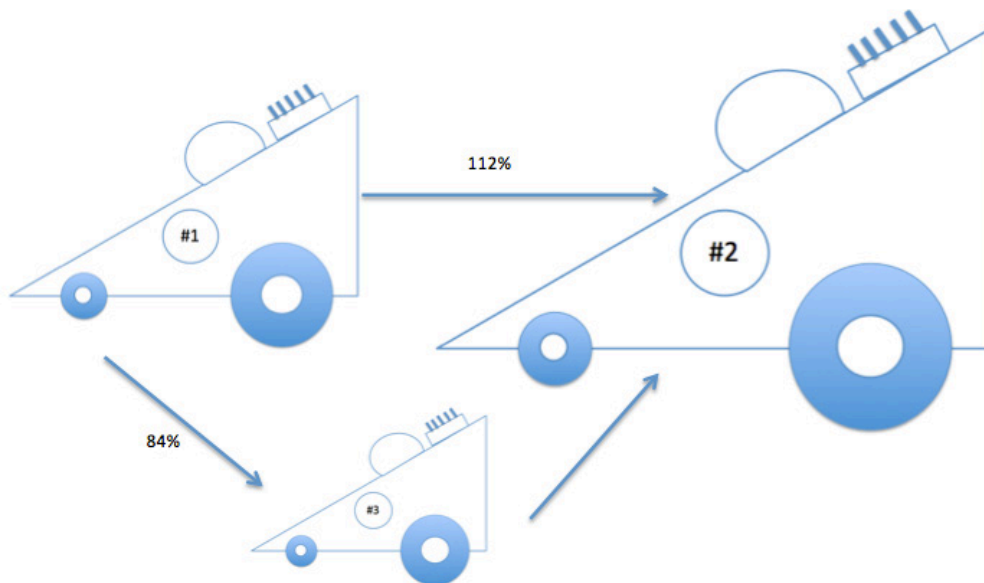
A regular octagon is an eight-sided polygon with side lengths that are all equal. All three octagons are scale drawings of each other. Use the chart and the side lengths to compute each scale factor as a percent. How can we check our answers?



Actual Drawing to Scale Drawing	Scale Factor	Equation to Illustrate Relationship
Drawing 1 to Drawing 2		
Drawing 1 to Drawing 3		
Drawing 2 to Drawing 1		
Drawing 2 to Drawing 3		
Drawing 3 to Drawing 1		
Drawing 3 to Drawing 2		

Example 3

The scale factor from Drawing 1 to Drawing 2 is 112%, and the scale factor from Drawing 1 to Drawing 3 is 84%. Drawing 2 is also a scale drawing of Drawing 3. Is Drawing 2 a reduction or an enlargement of Drawing 3? Justify your answer using the scale factor. The drawing is not necessarily drawn to scale.



Explain how you could use the scale factors from Drawing 1 to Drawing 2 (112%) and from Drawing 2 to Drawing 3 (75%) to show that the scale factor from Drawing 1 to Drawing 3 is 84%.

Lesson Summary

To compute the scale factor from one drawing to another, use the representation:

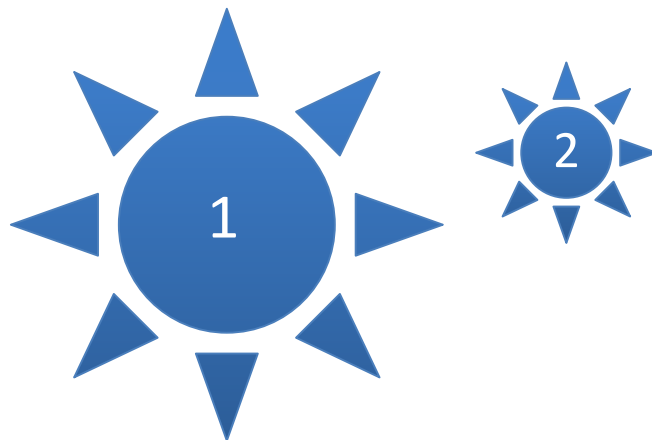
$$\text{Quantity} = \text{Percent} \times \text{Whole}$$

where the whole is the length in the actual or original drawing and the quantity is the length in the scale drawing.

If the lengths of the sides are not provided but two scale factors are provided, use the same relationship but use the scale factors as the whole and quantity instead of the given measurements.

Problem Set

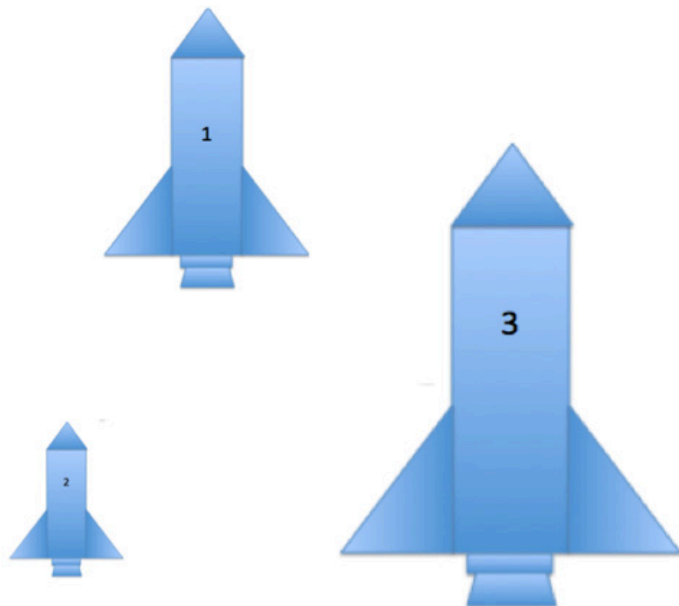
1. The scale factor from Drawing 1 to Drawing 2 is $41\frac{2}{3}\%$. Justify why Drawing 1 is a scale drawing of Drawing 2 and why it is an enlargement of Drawing 2. Include the scale factor in your justification.



2. The scale factor from Drawing 1 to Drawing 2 is 40%, and the scale factor from Drawing 2 to Drawing 3 is 37.5%. What is the scale factor from Drawing 1 to Drawing 3? Explain your reasoning, and check your answer using an example.



3. Traci took a photograph and printed it to be a size of 4 units by 4 units as indicated in the diagram. She wanted to enlarge the original photograph to a size of 5 units by 5 units and 10 units by 10 units.
- Sketch the different sizes of photographs.
 - What was the scale factor from the original photo to the photo that is 5 units by 5 units?
 - What was the scale factor from the original photo to the photo that is 10 units by 10 units?
 - What was the scale factor from the 5 × 5 photo to the 10 × 10 photo?
 - Write an equation to verify how the scale factor from the original photo to the enlarged 10 × 10 photo can be calculated using the scale factors from the original to the 5 × 5, and then from the 5 × 5 to the 10 × 10.
4. The scale factor from Drawing 1 to Drawing 2 is 30%, and the scale factor from Drawing 1 to Drawing 3 is 175%. What are the scale factors of each given relationship? Then, answer the question that follows.
- Drawing 2 to Drawing 3
 - Drawing 3 to Drawing 1
 - Drawing 3 to Drawing 2
 - How can you check your answers?

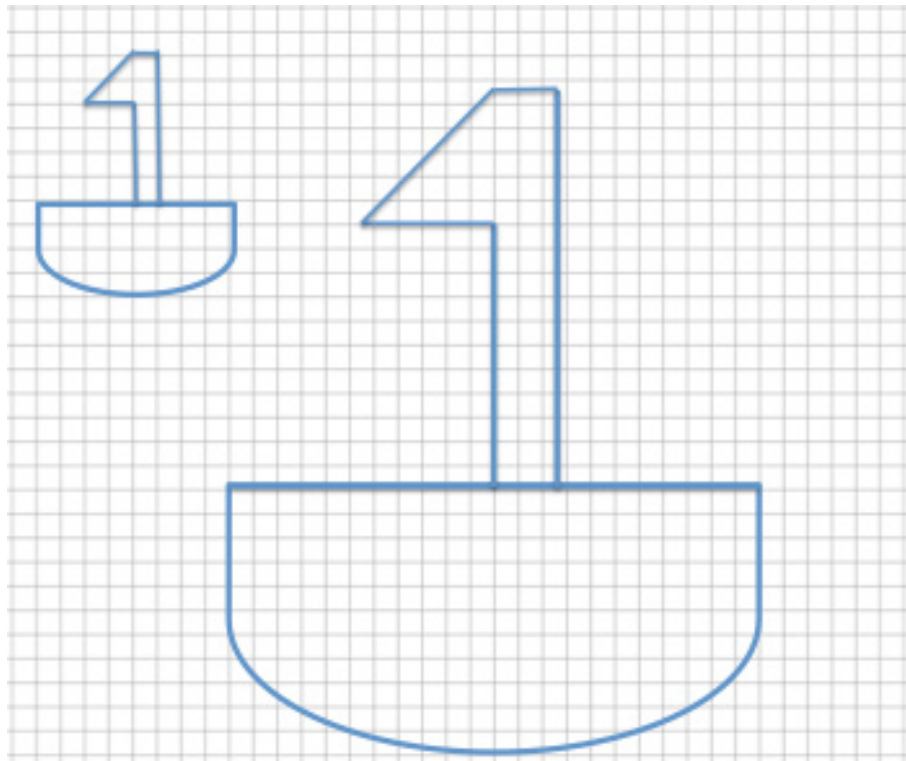


Lesson 14: Computing Actual Lengths from a Scale Drawing

Classwork

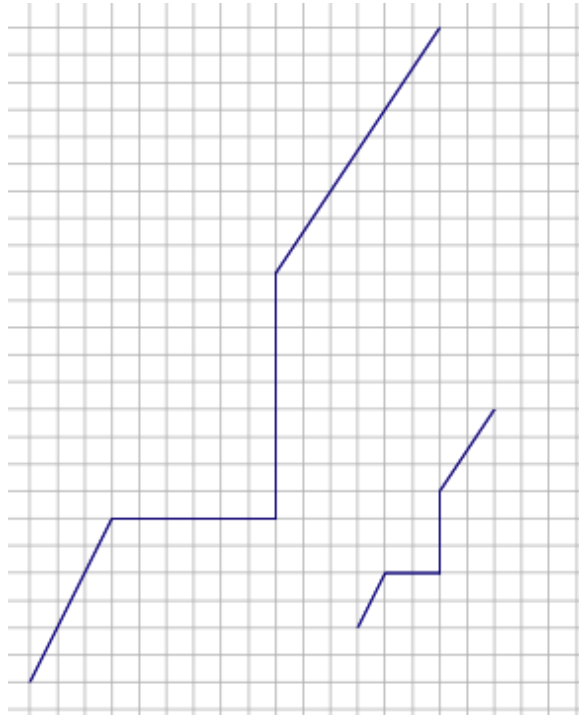
Example 1

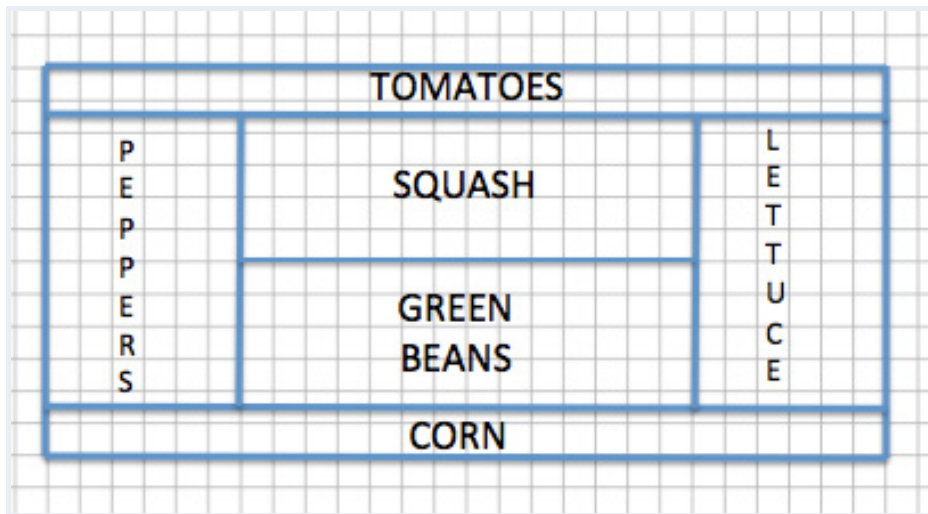
The distance around the entire small boat is 28.4 units. The larger figure is a scale drawing of the smaller drawing of the boat. State the scale factor as a percent, and then use the scale factor to find the distance around the scale drawing.



Exercise 1

The length of the longer path is 32.4 units. The shorter path is a scale drawing of the longer path. Find the length of the shorter path, and explain how you arrived at your answer.



Example 2: Time to Garden

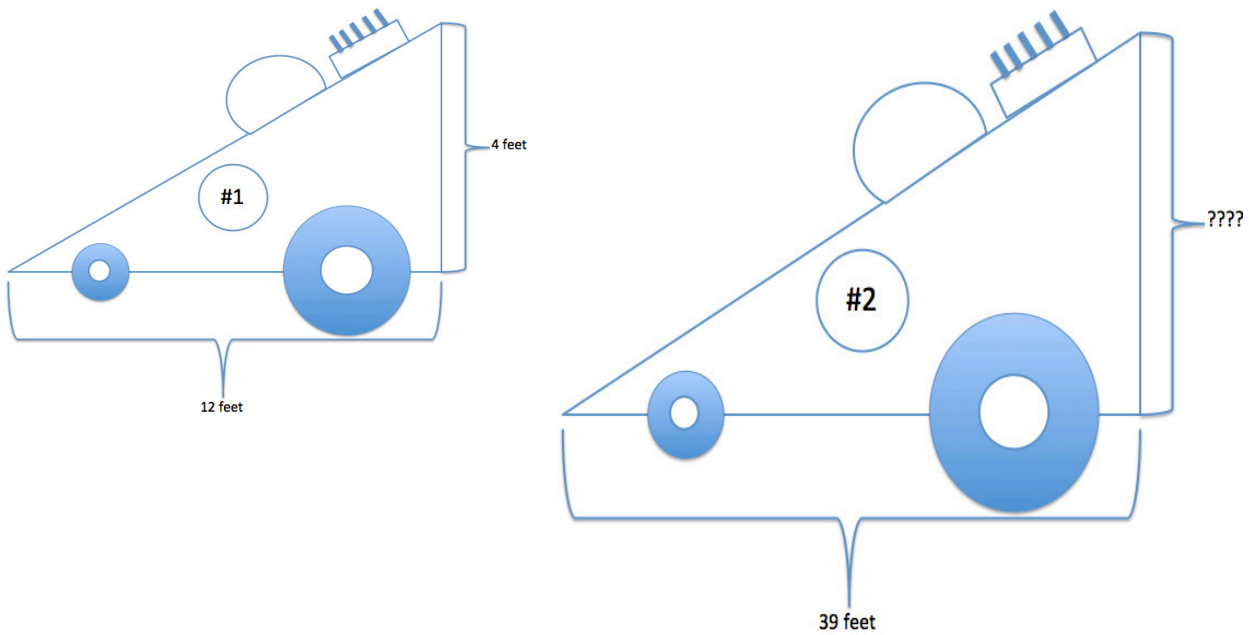
Sherry designed her garden as shown in the diagram above. The distance between any two consecutive vertical grid lines is 1 foot, and the distance between any two consecutive horizontal grid lines is also 1 foot. Therefore, each grid square has an area of one square foot. After designing the garden, Sherry decided to actually build the garden 75% of the size represented in the diagram.

- What are the outside dimensions shown in the blueprint?
- What will the overall dimensions be in the actual garden? Write an equation to find the dimensions. How does the problem relate to the scale factor?
- If Sherry plans to use a wire fence to divide each section of the garden, how much fence does she need?

- d. If the fence costs \$3.25 per foot plus 7% sales tax, how much would the fence cost in total?

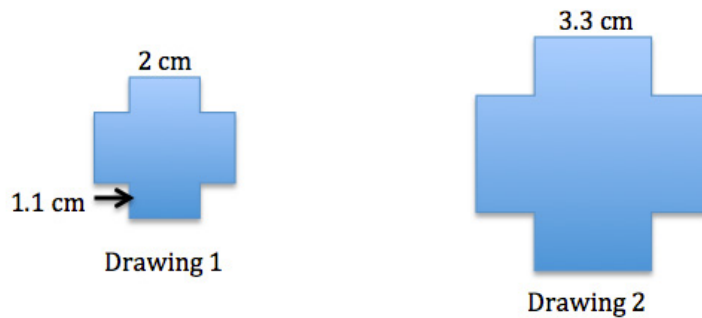
Example 3

Race Car #2 is a scale drawing of Race Car #1. The measurement from the front of Car #1 to the back of Car #1 is 12 feet, while the measurement from the front of Car #2 to the back of Car #2 is 39 feet. If the height of Car #1 is 4 feet, find the scale factor, and write an equation to find the height of Car #2. Explain what each part of the equation represents in the situation.



Exercise 2

Determine the scale factor, and write an equation that relates the vertical heights of each drawing to the scale factor. Explain how the equation illustrates the relationship.

**Exercise 3**

The length of a rectangular picture is 8 inches, and the picture is to be reduced to be $45\frac{1}{2}\%$ of the original picture. Write an equation that relates the lengths of each picture. Explain how the equation illustrates the relationship.

Lesson Summary

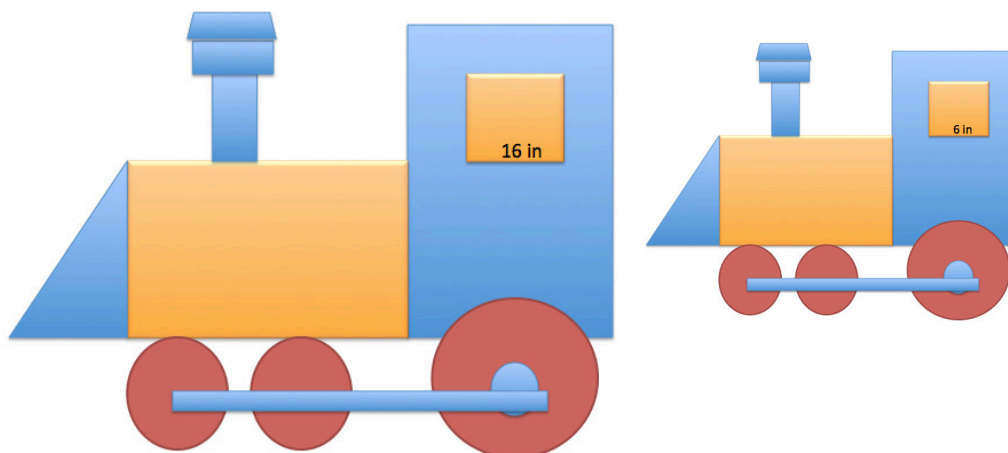
The scale factor is the number that determines whether the new drawing is an enlargement or a reduction of the original. If the scale factor is greater than 100%, then the resulting drawing will be an enlargement of the original drawing. If the scale factor is less than 100%, then the resulting drawing will be a reduction of the original drawing.

To compute actual lengths from a scale drawing, a scale factor must first be determined. To do this, use the relationship $\text{Quantity} = \text{Percent} \times \text{Whole}$, where the original drawing represents the whole and the scale drawing represents the quantity. Once a scale factor is determined, then the relationship

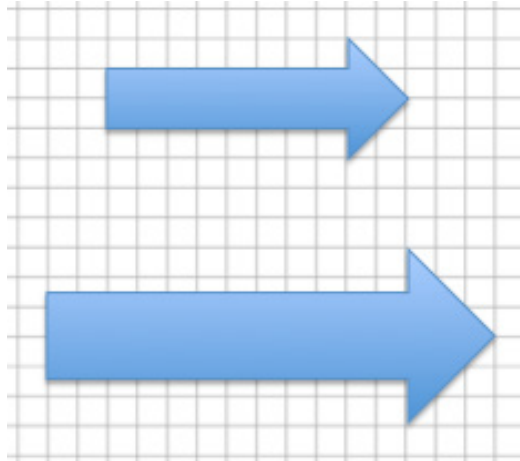
$\text{Quantity} = \text{Percent} \times \text{Whole}$ can be used again using the scale factor as the percent, the actual length from the original drawing as the whole, and the actual length of the scale drawing as the quantity.

Problem Set

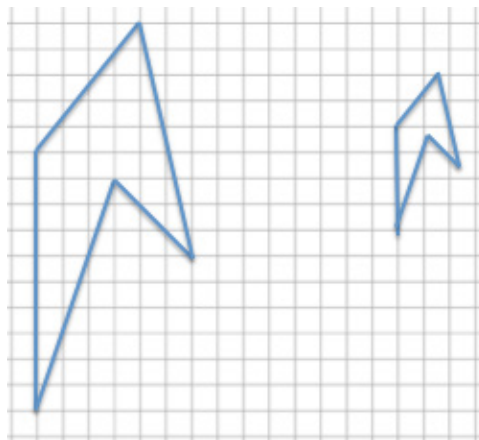
1. The smaller train is a scale drawing of the larger train. If the length of the tire rod connecting the three tires of the larger train, as shown below, is 36 inches, write an equation to find the length of the tire rod of the smaller train. Interpret your solution in the context of the problem.



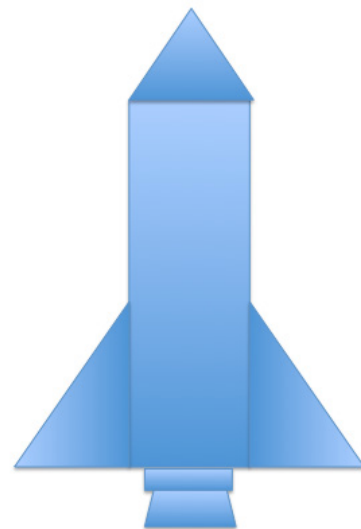
2. The larger arrow is a scale drawing of the smaller arrow. The distance around the smaller arrow is 28 units. What is the distance around the larger arrow? Use an equation to find the distance and interpret your solution in the context of the problem.



3. The smaller drawing below is a scale drawing of the larger. The distance around the larger drawing is 39.3 units. Using an equation, find the distance around the smaller drawing.



4. The figure is a diagram of a model rocket. The length of a model rocket is 2.5 feet, and the wing span is 1.25 feet. If the length of an actual rocket is 184 feet, use an equation to find the wing span of the actual rocket.

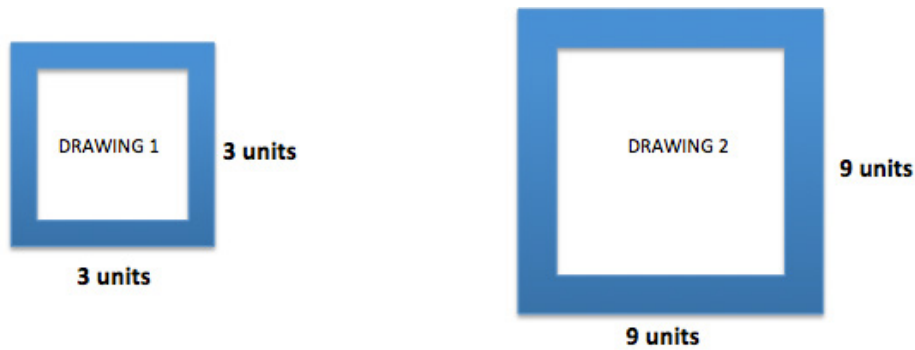


Lesson 15: Solving Area Problems Using Scale Drawings

Classwork

Opening Exercise

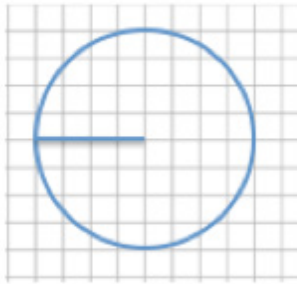
For each diagram, Drawing 2 is a scale drawing of Drawing 1. Complete the accompanying charts. For each drawing, identify the side lengths, determine the area, and compute the scale factor. Convert each scale factor into a fraction and percent, examine the results, and write a conclusion relating scale factors to area.



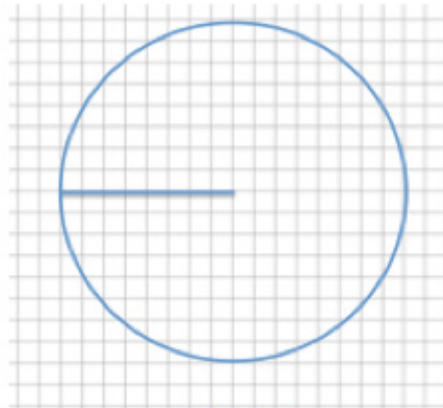
	Drawing 1	Drawing 2	Scale Factor as a Fraction and Percent
Side			
Area			

Scale Factor: _____

Quotient of Areas: _____



DRAWING 1



DRAWING 2

	Drawing 1	Drawing 2	Scale Factor as a Fraction and Percent
Radius			
Area			

Scale Factor: _____

Quotient of Areas: _____

The length of each side in Drawing 1 is 12 units, and the length of each side in Drawing 2 is 6 units.



Drawing 1



Drawing 2

	Drawing 1	Drawing 2	Scale Factor as a Fraction and Percent
Side			
Area			

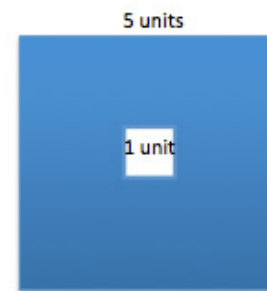
Scale Factor: _____

Quotient of Areas: _____

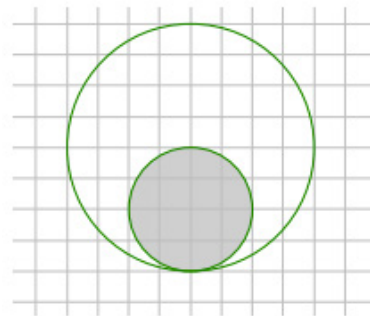
Conclusion:

Example 1

What percent of the area of the large square is the area of the small square?

**Example 2**

What percent of the area of the large disk lies outside the smaller disk?

**Example 3**

If the area of the shaded region in the larger figure is approximately 21.5 square inches, write an equation that relates the areas using scale factor and explain what each quantity represents. Determine the area of the shaded region in the smaller scale drawing.



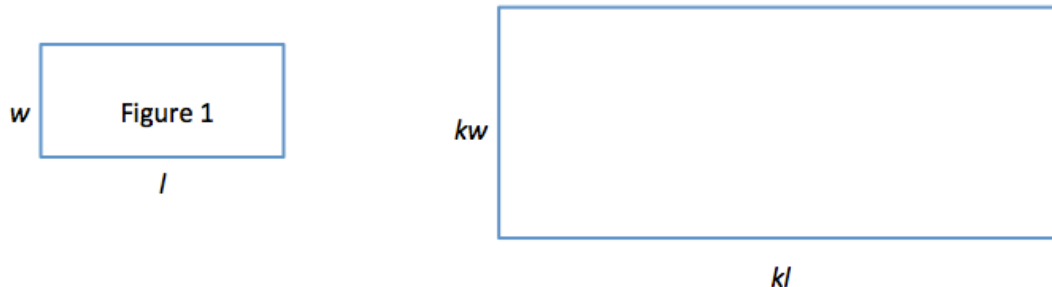
10 inches



6 inches

Example 4

Use Figure 1 below and the enlarged scale drawing to justify why the area of the scale drawing is k^2 times the area of the original figure.



Explain why the expressions $(kl)(kw)$ and k^2lw are equivalent. How do the expressions reveal different information about this situation?

Exercise 1

The Lake Smith basketball team had a team picture taken of the players, the coaches, and the trophies from the season. The picture was 4 inches by 6 inches. The team decided to have the picture enlarged to a poster, and then enlarged again to a banner measuring 48 inches by 72 inches.

- Sketch drawings to illustrate the original picture and enlargements.

- b. If the scale factor from the picture to the poster is 500%, determine the dimensions of the poster.
- c. What scale factor is used to create the banner from the picture?
- d. What percent of the area of the picture is the area of the poster? Justify your answer using the scale factor and by finding the actual areas.

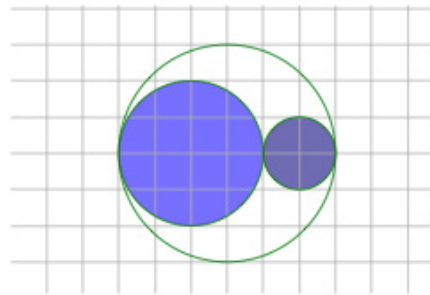
- e. Write an equation involving the scale factor that relates the area of the poster to the area of the picture.
- f. Assume you started with the banner and wanted to reduce it to the size of the poster. What would the scale factor as a percent be?
- g. What scale factor would be used to reduce the poster to the size of the picture?

Lesson Summary

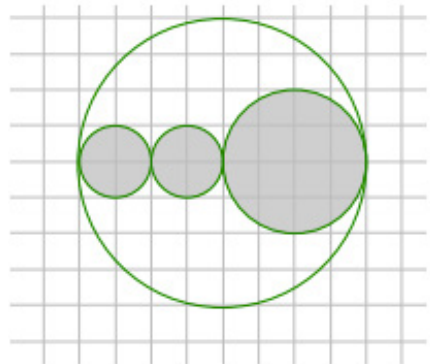
If the scale factor is represented by k , then the area of the scale drawing is k^2 times the corresponding area of the original drawing.

Problem Set

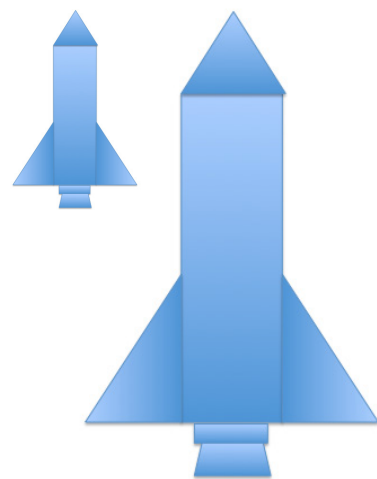
1. What percent of the area of the larger circle is shaded?
 - a. Solve this problem using scale factors.
 - b. Verify your work in part (a) by finding the actual areas.



2. The area of the large disk is 50.24 units^2 .
 - a. Find the area of the shaded region using scale factors. Use 3.14 as an estimate for π .
 - b. What percent of the large circular region is unshaded?



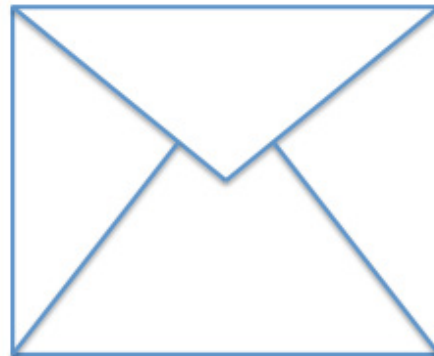
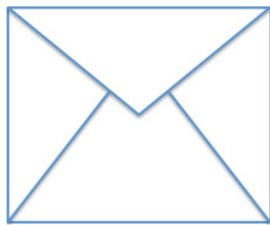
3. Ben cut the following rockets out of cardboard. The height from the base to the tip of the smaller rocket is 20 cm. The height from the base to the tip of the larger rocket is 120 cm. What percent of the area of the smaller rocket is the area of the larger rocket?



4. In the photo frame depicted below, three 5 inch by 5 inch squares are cut out for photographs. If these cut-out regions make up $\frac{3}{16}$ of the area of the entire photo frame, what are the dimensions of the photo frame?



5. Kelly was online shopping for envelopes for party invitations and saw these images on a website.



The website listed the dimensions of the small envelope as 6 in. by 8 in. and the medium envelope as 10 in. by $13\frac{1}{3}$ in.

- Compare the dimensions of the small and medium envelopes. If the medium envelope is a scale drawing of the small envelope, what is the scale factor?
- If the large envelope was created based on the dimensions of the small envelope using a scale factor of 250%, find the dimensions of the large envelope.
- If the medium envelope was created based on the dimensions of the large envelope, what scale factor was used to create the medium envelope?
- What percent of the area of the larger envelope is the area of the medium envelope?

Lesson 16: Population Problems

Classwork

Opening Exercise

Number of girls in classroom:	Number of boys in classroom:	Total number of students in classroom:
Percent of the total number of students that are girls:	Percent of the total number of students that are boys:	Percent of boys and girls in the classroom:
Number of girls whose names start with a vowel:	Number of boys whose names start with a vowel:	Number of students whose names start with a vowel:
Percent of girls whose names start with a vowel:	Percent of boys whose names start with a vowel:	
Percent of the total number of students that are girls whose names start with a vowel:	Percent of the total number of students that are boys whose names start with a vowel:	Percent of students whose names start with a vowel:

Example 1

A school has 60% girls and 40% boys. If 20% of the girls wear glasses and 40% of the boys wear glasses, what percent of all students wears glasses?

Exercise 1

How does the percent of students who wear glasses change if the percent of girls and boys remains the same (that is, 60% girls and 40% boys), but 20% of the boys wear glasses and 40% of the girls wear glasses?

Exercise 2

How would the percent of students who wear glasses change if the percent of girls is 40% of the school and the percent of boys is 60% of the school, and 40% of the girls wear glasses and 20% of the boys wear glasses? Why?

Example 2

The weight of the first of three containers is 12% more than the second, and the third container is 20% lighter than the second. By what percent is the first container heavier than the third container?

Exercise 3

Matthew's pet dog is 7% heavier than Harrison's pet dog, and Janice's pet dog is 20% lighter than Harrison's. By what percent is Matthew's dog heavier than Janice's?

Example 3

In one year's time, 20% of Ms. McElroy's investments increased by 5%, 30% of her investments decreased by 5%, and 50% of her investments increased by 3%. By what percent did the total of her investments increase?

Exercise 4

A concert had 6,000 audience members in attendance on the first night and the same on the second night. On the first night, the concert exceeded expected attendance by 20%, while the second night was below the expected attendance by 20%. What was the difference in percent of concert attendees and expected attendees for both nights combined?

Lesson Summary

When solving a percent population problem, you must first define the variable. This gives a reference of what the whole is. Then, multiply the sub-populations (such as girls and boys) by the given category (total students wearing glasses) to find the percent in the whole population.

Problem Set

1. One container is filled with a mixture that is 30% acid. A second container is filled with a mixture that is 50% acid. The second container is 50% larger than the first, and the two containers are emptied into a third container. What percent of acid is the third container?
2. The store's markup on a wholesale item is 40%. The store is currently having a sale, and the item sells for 25% off the retail price. What is the percent of profit made by the store?
3. During lunch hour at a local restaurant, 90% of customers order a meat entrée and 10% order a vegetarian entrée. Of the customers who order a meat entrée, 80% order a drink. Of the customers who order a vegetarian entrée, 40% order a drink. What is the percent of customers who order a drink with their entrée?
4. Last year's spell-a-thon spelling test for a first grade class had 15% more words with four or more letters than this year's spelling test. Next year, there will be 5% less than this year. What percent more words have four or more letters in last year's test than next year's?
5. An ice cream shop sells 75% less ice cream in December than in June. Twenty percent more ice cream is sold in July than in June. By what percent did ice cream sales increase from December to July?
6. The livestock on a small farm the prior year consisted of 40% goats, 10% cows, and 50% chickens. This year, there is a 5% decrease in goats, 9% increase in cows, and 15% increase in chickens. What is the percent increase or decrease of livestock this year?
7. In a pet shelter that is occupied by 55% dogs and 45% cats, 60% of the animals are brought in by concerned people who found these animals in the streets. If 90% of the dogs are brought in by concerned people, what is the percent of cats that are brought in by concerned people?
8. An artist wants to make a particular teal color paint by mixing a 75% blue hue and 25% yellow hue. He mixes a blue hue that has 85% pure blue pigment and a yellow hue that has 60% of pure yellow pigment. What is the percent of pure pigment that is in the resulting teal color paint?
9. On Mina's block, 65% of her neighbors do not have any pets, and 35% of her neighbors own at least one pet. If 25% of the neighbors have children but no pets, and 60% of the neighbors who have pets also have children, what percent of the neighbors have children?

Lesson 17: Mixture Problems

Classwork

Opening Exercise

Imagine you have two equally sized containers. One is pure water, and the other is 50% water and 50% juice. If you combined them, what percent of juice would be the result?

	1 st liquid	2 nd liquid	Resulting liquid
Amount of liquid (gallons)			
Amount of pure juice (gallons)			

If a 2-gallon container of pure juice is added to 3 gallons of water, what percent of the mixture is pure juice?

	1 st liquid	2 nd liquid	Resulting liquid
Amount of liquid (gallons)			
Amount of pure juice (gallons)			

If a 2-gallon container of juice mixture that is 40% pure juice is added to 3 gallons of water, what percent of the mixture is pure juice?

	1 st liquid	2 nd liquid	Resulting liquid
Amount of liquid (gallons)			
Amount of pure juice (gallons)			

If a 2-gallon juice cocktail that is 40% pure juice is added to 3 gallons of pure juice, what percent of the resulting mixture is pure juice?

	1 st liquid	2 nd liquid	Resulting liquid
Amount of liquid (gallons)			
Amount of pure juice (gallons)			

Example 1

A 5-gallon container of trail mix is 20% nuts. Another trail mix is added to it, resulting in a 12-gallon container of trail mix that is 40% nuts.

- Write an equation to describe the relationships in this situation.
- Explain in words how each part of the equation relates to the situation.
- What percent of the second trail mix is nuts?

Example 2

Soil that contains 30% clay is added to soil that contains 70% clay to create 10 gallons of soil containing 50% clay. How much of each of the soils was combined?

Exercise 2

The equation $(0.2)(x) + (0.8)(6 - x) = (0.4)(6)$ is used to model a mixture problem.

- How many units are in the total mixture?
- What percents relate to the two solutions that are combined to make the final mixture?
- The two solutions combine to make 6 units of what percent solution?
- When the amount of a resulting solution is given (for instance, 4 gallons) but the amounts of the mixing solutions are unknown, how are the amounts of the mixing solutions represented?

Lesson Summary

- Mixture problems deal with quantities of solutions and mixtures.
- The general structure of the expressions for mixture problems are

$$\text{Whole Quantity} = \text{Part} + \text{Part}.$$

- Using this structure makes the equation resemble the following:

$$(\% \text{ of resulting quantity})(\text{amount of resulting quantity}) = (\% \text{ of 1}^{\text{st}} \text{ quantity})(\text{amount of 1}^{\text{st}} \text{ quantity}) + (\% \text{ of 2}^{\text{nd}} \text{ quantity})(\text{amount of 2}^{\text{nd}} \text{ quantity}).$$

Problem Set

1. A 5-liter cleaning solution contains 30% bleach. A 3-liter cleaning solution contains 50% bleach. What percent of bleach is obtained by putting the two mixtures together?
2. A container is filled with 100 grams of bird feed that is 80% seed. How many grams of bird feed containing 5% seed must be added to get bird feed that is 40% seed?
3. A container is filled with 100 grams of bird feed that is 80% seed. Tom and Sally want to mix the 100 grams with bird feed that is 5% seed to get a mixture that is 40% seed. Tom wants to add 114 grams of the 5% seed, and Sally wants to add 115 grams of the 5% seed mix. What will be the percent of seed if Tom adds 114 grams? What will be the percent of seed if Sally adds 115 grams? How much do you think should be added to get 40% seed?
4. Jeanie likes mixing leftover salad dressings together to make new dressings. She combined 0.55 L of a 90% vinegar salad dressing with 0.45 L of another dressing to make 1 L of salad dressing that is 60% vinegar. What percent of the second salad dressing was vinegar?
5. Anna wants to make 30 mL of a 60% salt solution by mixing together a 72% salt solution and a 54% salt solution. How much of each solution must she use?
6. A mixed bag of candy is 25% chocolate bars and 75% other filler candy. Of the chocolate bars, 50% of them contains caramel. Of the other filler candy, 10% of them contain caramel. What percent of candy contains caramel?
7. A local fish market receives the daily catch of two local fishermen. The first fisherman's catch was 84% fish while the rest was other non-fish items. The second fisherman's catch was 76% fish while the rest was other non-fish items. If the fish market receives 75% of its catch from the first fisherman and 25% from the second, what was the percent of other non-fish items the local fish market bought from the fishermen altogether?

Lesson 18: Counting Problems

Classwork

Opening Exercise

You are about to switch out your books from your locker during passing period but forget the order of your locker combination. You know that there are the numbers 3, 16, and 21 in some order. What is the percent of locker combinations that start with 3?

Locker Combination Possibilities:

3, 16, 21

21, 16, 3

16, 21, 3

21, 3, 16

16, 3, 21

3, 21, 16

Example 1

All of the 3-letter passwords that can be formed using the letters A and B are as follows: AAA, AAB, ABA, ABB, BAA, BAB, BBA, BBB.

a. What percent of passwords contain at least two B's?

b. What percent of passwords contain no A's?

Exercises 1–2

1. How many 4-letter passwords can be formed using the letters A and B?

2. What percent of the 4-letter passwords contain

a. No A's?

b. Exactly one A?

c. Exactly two A's?

d. Exactly three A's?

e. Four A's?

f. The same number of A's and B's?

Example 2

In a set of 3-letter passwords, 40% of the passwords contain the letter B and two of another letter. Which of the two sets below meets the criteria? Explain how you arrived at your answer.

Set 1

BBB	AAA	CAC
CBC	ABA	CCC
BBC	CCB	CAB
AAB	AAC	BAA
ACB	BAC	BCC

Set 2

CEB	BBB
EBE	CCC
CCC	EEE
EEB	CBC
CCB	ECE

Exercises 3–4

3. Shana read the following problem:

“How many letter arrangements can be formed from the word *triangle* that have two vowels and two consonants (order does not matter)?”

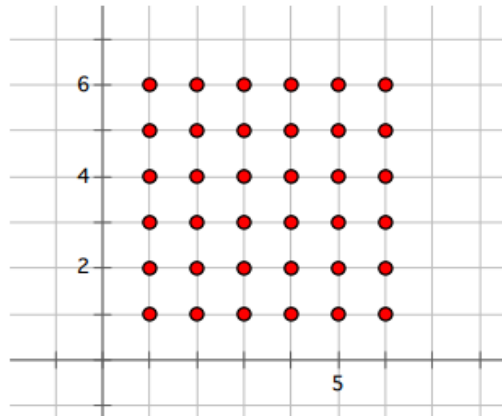
She answered that there are 30 letter arrangements.

Twenty percent of the letter arrangements that began with a vowel actually had an English definition. How many letter arrangements that begin with a vowel have an English definition?

4. Using three different keys on a piano, a songwriter makes the beginning of his melody with three notes, C, E, and G: CCE, EEE, EGC, GCE, CEG, GEE, CGE, GGE, EGG, EGE, GCG, EEC, ECC, ECG, GGG, GEC, CCG, CEE, CCC, GEG, CGC.
- a. From the list above, what is the percent of melodies with all three notes that are different?
- b. From the list above, what is the percent of melodies that have three of the same notes?

Example 3

Look at the 36 points on the coordinate plane with whole number coordinates between 1 and 6, inclusive.



- Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 7.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 6.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 5.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 4.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 3.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 2.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 8.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 9.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 10.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 11.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 12.
- What percent of the 36 points have coordinate sum 7?
- Write a numerical expression that could be used to determine the percent of the 36 points that have a coordinate sum of 7.
- What percent of the 36 points have coordinate sum 5 or less?
- What percent of the 36 points have coordinate sum 4 or 10?

Lesson Summary

To find the percent of possible outcomes for a counting problem you need to determine the total number of possible outcomes and the different favorable outcomes. The representation

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$

can be used where the quantity is the number of different favorable outcomes and the whole is the total number of possible outcomes.

Problem Set

- A six-sided die (singular for dice) is thrown twice. The different rolls are as follows:
 1 and 1, 1 and 2, 1 and 3, 1 and 4, 1 and 5, 1 and 6,
 2 and 1, 2 and 2, 2 and 3, 2 and 4, 2 and 5, 2 and 6,
 3 and 1, 3 and 2, 3 and 3, 3 and 4, 3 and 5, 3 and 6,
 4 and 1, 4 and 2, 4 and 3, 4 and 4, 4 and 5, 4 and 6,
 5 and 1, 5 and 2, 5 and 3, 5 and 4, 5 and 5, 5 and 6,
 6 and 1, 6 and 2, 6 and 3, 6 and 4, 6 and 5, 6 and 6.
 - What is the percent that both throws will be even numbers?
 - What is the percent that the second throw is a 5?
 - What is the percent that the first throw is lower than a 6?
- You have the ability to choose three of your own classes, art, language, and physical education. There are three art classes (A1, A2, A3), two language classes (L1, L2), and two P.E. classes (P1, P2) to choose from. The order does not matter and you must choose one from each subject.

A1, L1, P1	A2, L1, P1	A3, L1, P1
A1, L1, P2	A2, L1, P2	A3, L1, P2
A1, L2, P1	A2, L2, P1	A3, L2, P1
A1, L2, P2	A2, L2, P2	A3, L2, P2

Compare the percent of possibilities with A1 in your schedule to the percent of possibilities with L1 in your schedule.

3. Fridays are selected to show your school pride. The colors of your school are orange, blue, and white, and you can show your spirit by wearing a top, a bottom, and an accessory with the colors of your school. During lunch, 11 students are chosen to play for a prize on stage. The table charts what the students wore.

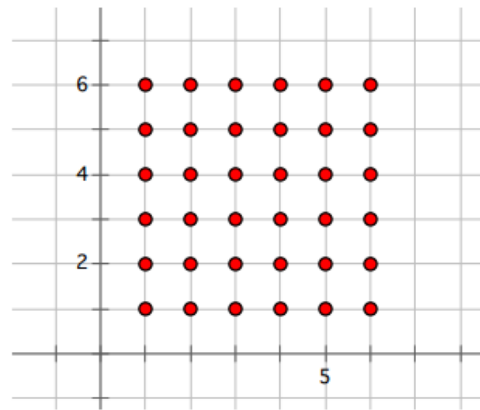
Top	W	O	W	O	B	W	B	B	W	W	W
Bottom	B	O	B	B	O	B	B	B	O	W	B
Accessory	W	O	B	W	B	O	B	W	O	O	O

- What is the percent of outfits that are one color?
 - What is the percent of outfits that include orange accessories?
4. Shana wears two rings (G represents gold, and S represents silver) at all times on her hand. She likes fiddling with them and places them on different fingers (pinky, ring, middle, index) when she gets restless. The chart is tracking the movement of her rings.

	Pinky Finger	Ring Finger	Middle Finger	Index Finger
Position 1		G	S	
Position 2			S	G
Position 3	G		S	
Position 4				S,G
Position 5	S	G		
Position 6	G	S		
Position 7	S		G	
Position 8	G		S	
Position 9		S,G		
Position 10		G	S	
Position 11			G	S
Position 12		S		G
Position 13	S,G			
Position 14			S,G	

- What percent of the positions shows the gold ring on her pinky finger?
- What percent of the positions shows she wears both rings on one finger?

5. Use the coordinate plane below to answer the following questions.



- a. What is the percent of the 36 points whose quotient of $\frac{x\text{-coordinate}}{y\text{-coordinate}}$ is greater than one?
- b. What is the percent of the 36 points whose coordinate quotient is equal to one?