Lesson 1: The Relationship of Addition and Subtraction

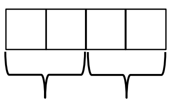
Classwork

Opening Exercise

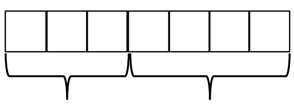
a. Draw a tape diagram to represent the following expression: $5 + 4$.

b. Write an expression for each tape diagram.

i.



ii.



Exercises

1. Predict what will happen when a tape diagram has a large number of squares, some squares are removed, and then the same amount of squares are added back on.

2. Build a tape diagram with 10 squares.

   a. Remove six squares. Write an expression to represent the tape diagram.

   b. Add six squares onto the tape diagram. Alter the original expression to represent the current tape diagram.

c. Evaluate the expression.

3. Write an equation, using variables, to represent the identities we demonstrated with tape diagrams.

4. Using your knowledge of identities, fill in each of the blanks.

a. $4 + 5 -$ _____ $= 4$

b. $25 -$ _____ $+ 10 = 25$

c. _____ $+16 - 16 = 45$

d. $56 - 20 + 20 =$ _____

5. Using your knowledge of identities, fill in each of the blanks.

a. $a + b -$ _____ $= a$

b. $c - d + d =$ _____

c. $e +$ _____ $- f = e$

d. _____ $- h + h = g$

Problem Set

1. Fill in each blank.

   a. $\_\_\_\_\_ + 15 - 15 = 21$

   b. $450 - 230 + 230 = \_\_\_\_\_$

   c. $1289 - \_\_\_\_\_\_ + 856 = 1289$

2. Why are the equations $w - x + x = w$ and $w + x - x = w$ called identities?

Lesson 2: The Relationship of Multiplication and Division

Classwork

Opening Exercise

Draw a pictorial representation of the division and multiplication problems using a tape diagram.

a. $8 \div 2$

b. 3×2

Exploratory Challenge

Work in pairs or small groups to determine equations to show the relationship between multiplication and division. Use tape diagrams to provide support for your findings.

1. Create two equations to show the relationship between multiplication and division. These equations should be identities and include variables. Use the squares to develop these equations.

2. Write your equations on large paper. Show a series of tape diagrams to defend each of your equations.

Use the following rubric to critique other posters.

1. Name of group you are critiquing
2. Equation you are critiquing
3. Whether or not you believe the equations are true and reasons why

Problem Set

1. Fill in each blank to make the equation true.

   a. $132 \div 3 \times 3 =$ ____

   b. ____ $\div 25 \times 25 = 225$

   c. $56 \times$ ____ $\div 8 = 56$

   d. $452 \times 12 \div$ ____ $= 452$

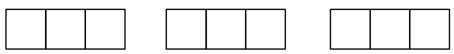
2. How is the relationship of addition and subtraction similar to the relationship of multiplication and division?

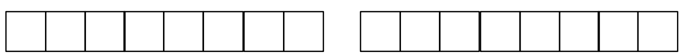
Lesson 3: The Relationship of Multiplication and Addition

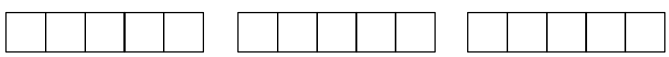
Classwork

Opening Exercise

Write two different expressions that can be depicted by the tape diagram shown. One expression should include addition, while the other should include multiplication.

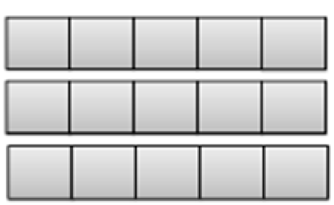
a. 

b. 

c. 

Exercises

1. Write the addition sentence that describes the model and the multiplication sentence that describes the model.



2. Write an equivalent expression to demonstrate the relationship of multiplication and addition.

   a. $6 + 6$

   b. $3 + 3 + 3 + 3 + 3 + 3$

   c. $4 + 4 + 4 + 4 + 4$

   d. 6×2

   e. 4×6

   f. 3×9

   g. $h + h + h + h + h$

   h. $6y$

3. Roberto is not familiar with tape diagrams and believes that he can show the relationship of multiplication and addition on a number line. Help Roberto demonstrate that the expression 3×2 is equivalent to $2 + 2 + 2$ on a number line.

4. Tell whether the following equations are true or false. Then, explain your reasoning.
   a. $x + 6g - 6g = x$

   b. $2f - 4e + 4e = 2f$

5. Write an equivalent expression to demonstrate the relationship between addition and multiplication.

   a. $6 + 6 + 6 + 6 + 4 + 4 + 4$

   b. $d + d + d + w + w + w + w + w$

   c. $a + a + b + b + b + c + c + c + c$

Problem Set

Write an equivalent expression to show the relationship of multiplication and addition.

1. $10 + 10 + 10$

2. $4 + 4 + 4 + 4 + 4 + 4 + 4 + 4$

3. 8×2

4. 3×9

5. $6m$

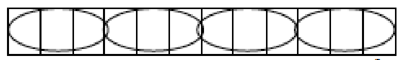
6. $d + d + d + d + d$

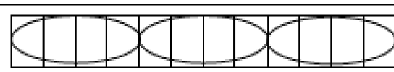
Lesson 4: The Relationship of Division and Subtraction

Classwork

Exercise 1

Build subtraction equations using the indicated equations. The first example has been completed for you.

Division Equation	Divisor Indicates the Size of the Unit	Tape Diagram	What is x, y, z?
$12 \div x = 4$	$12 - x - x - x - x = 0$	 $\mathbf{12 - 3 - 3 - 3 - 3 = 0; \mathit{x} = 3}$ ***units in each group***	$x = 3$
$18 \div x = 3$			
$35 \div y = 5$			
$42 \div z = 6$			

Division Equation	Divisor Indicates the Number of Units	Tape Diagram	What is x, y, z?
$12 \div x = 4$	$12 - 4 - 4 - 4 = 0$	 $\mathbf{12 - 4 - 4 - 4 = 0; \mathit{x} = 3}$ ***groups***	$x = 3$
$18 \div x = 3$			
$35 \div y = 5$			
$42 \div z = 6$			

Exercise 2

Answer each question using what you have learned about the relationship of division and subtraction.

a. If $12 \div x = 3$, how many times would x have to be subtracted from 12 in order for the answer to be zero? What is the value of x?

b. $36 - f - f - f - f = 0$. Write a division sentence for this repeated subtraction sentence. What is the value of f?

c. If $24 \div b = 12$, which number is being subtracted 12 times in order for the answer to be zero?

Problem Set

Build subtraction equations using the indicated equations.

	Division Equation	Divisor Indicates the Size of the Unit	Tape Diagram	What is x, y, z?
1.	$24 \div x = 4$			
2.	$36 \div x = 6$			
3.	$28 \div y = 7$			
4.	$30 \div y = 5$			
5.	$16 \div z = 4$			

	Division Equation	Divisor Indicates the Number of Units	Tape Diagram	What is x, y, z?
1.	$24 \div x = 4$			
2.	$36 \div x = 6$			
3.	$28 \div y = 7$			
4.	$30 \div y = 5$			
5.	$16 \div z = 4$			

Lesson 5: Exponents

Classwork

Opening Exercise

As you evaluate these expressions, pay attention to how you arrived at your answers.

$4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4$

$9 + 9 + 9 + 9 + 9$

$10 + 10 + 10 + 10 + 10$

Examples 1–5

Write each expression in exponential form.

1. $5 \times 5 \times 5 \times 5 \times 5 =$

2. $2 \times 2 \times 2 \times 2 =$

Write each expression in expanded form.

3. $8^3 =$

4. $10^6 =$

5. $g^3 =$

Go back to Examples 1–4, and use a calculator to evaluate the expressions.

What is the difference between $3g$ and g^3?

Examples 6–8

6. Write the expression in expanded form, and then evaluate.
   $(3.8)^4 =$

7. Write the expression in exponential form, and then evaluate.
   $2.1 \times 2.1 =$

8. Write the expression in exponential form, and then evaluate.
   $0.75 \times 0.75 \times 0.75 =$

The base number can also be a fraction. Convert the decimals to fractions in Examples 7 and 8 and evaluate. Leave your answer as a fraction. Remember how to multiply fractions!

Examples 9–10

9. Write the expression in exponential form, and then evaluate.

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} =$

10. Write the expression in expanded form, and then evaluate.

$\left(\frac{2}{3}\right)^2 =$

Exercises

Fill in the chart, supplying the missing expression.

1. Fill in the missing expression for each row. For whole number and decimal bases, use a calculator to find the standard form of the number. For fraction bases, leave your answer as a fraction.

Exponential Form	Expanded Form	Standard Form
3^2	3×3	9
	$2 \times 2 \times 2 \times 2 \times 2 \times 2$	
4^5		
	$\frac{3}{4} \times \frac{3}{4}$	
	1.5×1.5	

2. Write five cubed in all three forms: exponential form, expanded form, and standard form.

3. Write *fourteen and seven-tenths squared* in all three forms.

4. One student thought two to the third power was equal to six. What mistake do you think he made, and how would you help him fix his mistake?

Lesson Summary

EXPONENTIAL NOTATION FOR WHOLE NUMBER EXPONENTS: Let m be a nonzero whole number. For any number a, the expression a^m is the product of m factors of a, i.e.,

$$a^m = \underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ times}}.$$

The number a is called the *base,* and m is called the *exponent* or *power* of a.

When m is 1, "the product of one factor of a" just means a, i.e., $a^1 = a$. Raising any nonzero number a to the power of 0 is defined to be 1, i.e., $a^0 = 1$ for all $a \neq 0$.

Problem Set

1. Complete the table by filling in the blank cells. Use a calculator when needed.

Exponential Form	Expanded Form	Standard Form
3^5		
	$4 \times 4 \times 4$	
$(1.9)^2$		
$\left(\frac{1}{2}\right)^5$		

2. Why do whole numbers raised to an exponent get greater, while fractions raised to an exponent get smaller?

3. The powers of 2 that are in the range 2 through $1{,}000$ are 2, 4, 8, 16, 32, 64, 128, 256, and 512. Find all the powers of 3 that are in the range 3 through $1{,}000$.

4. Find all the powers of 4 in the range 4 through $1{,}000$.

5. Write an equivalent expression for $n \times a$ using only addition.

6. Write an equivalent expression for w^b using only multiplication.
   a. Explain what w is in this new expression.
   b. Explain what b is in this new expression.

7. What is the advantage of using exponential notation?

8. What is the difference between $4x$ and x^4? Evaluate both of these expressions when $x = 2$.

Lesson 6: The Order of Operations

Classwork

Example 1: Expressions with Only Addition, Subtraction, Multiplication, and Division

What operations are evaluated first?

What operations are always evaluated last?

Exercises 1–3

1. $4 + 2 \times 7$

2. $36 \div 3 \times 4$

3. $20 - 5 \times 2$

Example 2: Expressions with Four Operations and Exponents

$$4 + 9^2 \div 3 \times 2 - 2$$

What operation is evaluated first?

What operations are evaluated next?

What operations are always evaluated last?

What is the final answer?

Exercises 4–5

4. $90 - 5^2 \times 3$

5. $4^3 + 2 \times 8$

Example 3: Expressions with Parentheses

Consider a family of 4 that goes to a soccer game. Tickets are $\$5.00$ each. The mom also buys a soft drink for $\$2.00$. How would you write this expression?

How much will this outing cost?

Consider a different scenario: The same family goes to the game as before, but each of the family members wants a drink. How would you write this expression?

Why would you add the 5 and 2 first?

How much will this outing cost?

How many groups are there?

What is each group comprised of?

Exercises 6–7

6. $2 + (9^2 - 4)$

7. $2 \cdot \big(13 + 5 - 14 \div (3 + 4)\big)$

Example 4: Expressions with Parentheses and Exponents

$$2 \times (3 + 4^2)$$

Which value will we evaluate first within the parentheses? Evaluate.

Evaluate the rest of the expression.

What do you think will happen when the exponent in this expression is outside of the parentheses?

$$2 \times (3 + 4)^2$$

Will the answer be the same?

Which should we evaluate first? Evaluate.

What happens differently here than in our last example?

What should our next step be?

Evaluate to find the final answer.

What do you notice about the two answers?

What was different between the two expressions?

What conclusions can you draw about evaluating expressions with parentheses and exponents?

Exercises 8–9

8. $7 + (12 - 3^2)$

9. $7 + (12 - 3)^2$

Lesson Summary

NUMERICAL EXPRESSION: A *numerical expression* is a number, or it is any combination of sums, differences, products, or divisions of numbers that evaluates to a number.

Statements like "$3+$" or "$3 \div 0$" are not numerical expressions because neither represents a point on the number line. Note: Raising numbers to whole number powers are considered numerical expressions as well since the operation is just an abbreviated form of multiplication: $2^3 = 2 \cdot 2 \cdot 2$.

VALUE OF A NUMERICAL EXPRESSION: The *value of a numerical expression* is the number found by evaluating the expression.

For example: $\frac{1}{3} \cdot (2+4) + 7$ is a numerical expression, and its value is 9.

Problem Set

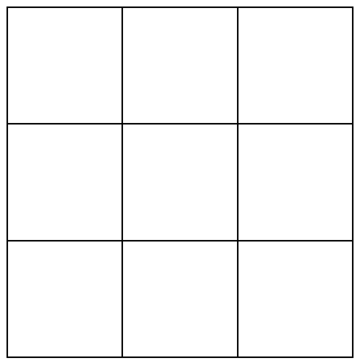
Evaluate each expression.

1. $3 \times 5 + 2 \times 8 + 2$
2. $(\$1.75 + 2 \times \$0.25 + 5 \times \$0.05) \times 24$
3. $(2 \times 6) + (8 \times 4) + 1$
4. $\big((8 \times 1.95) + (3 \times 2.95) + 10.95\big) \times 1.06$
5. $\big((12 \div 3)^2 - (18 \div 3^2)\big) \times (4 \div 2)$

Lesson 7: Replacing Letters with Numbers

Classwork

Example 1



What is the length of one side of this square?

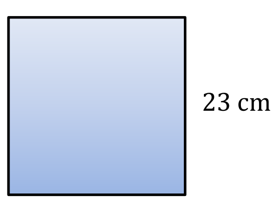
What is the formula for the area of a square?

What is the square's area as a multiplication expression?

What is the square's area?

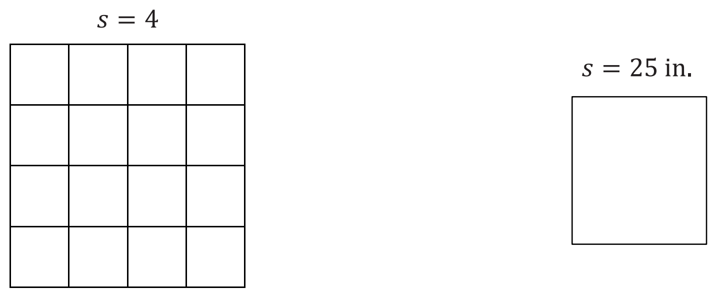
We can count the units. However, look at this other square. Its side length is 23 cm. That is just too many tiny units to draw. What expression can we build to find this square's area?

What is the area of the square? Use a calculator if you need to.



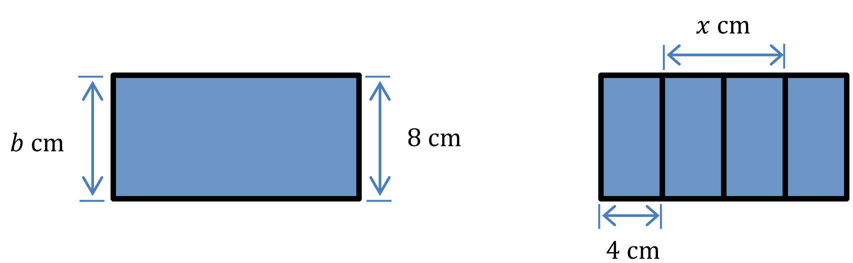
Exercise 1

Complete the table below for both squares. Note: These drawings are not to scale.



Length of One Side of the Square	Square's Area Written as an Expression	Square's Area Written as a Number

Example 2



What does the letter b represent in this blue rectangle?

With a partner, answer the following question: Given that the second rectangle is divided into four equal parts, what number does the x represent?

How did you arrive at this answer?

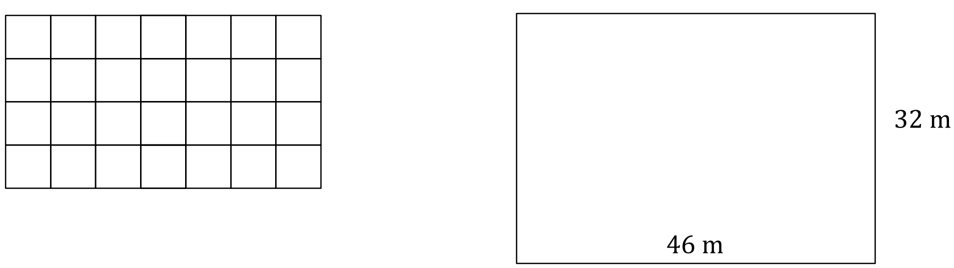
What is the total length of the second rectangle? Tell a partner how you know.

If the two large rectangles have equal lengths and widths, find the area of each rectangle.

Discuss with your partner how the formulas for the area of squares and rectangles can be used to evaluate area for a particular figure.

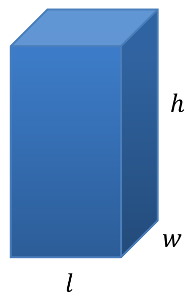
Exercise 2

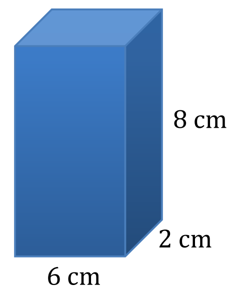
Complete the table below for both rectangles. Note: These drawings are not to scale. Using a calculator is appropriate.



Length of Rectangle	Width of Rectangle	Rectangle's Area Written as an Expression	Rectangle's Area Written as a Number

Example 3





What does the l represent in the first diagram?

What does the w represent in the first diagram?

What does the h represent in the first diagram?

Since we know the formula to find the volume is $V = l \times w \times h$, what number can we substitute for the l in the formula? Why?

What other number can we substitute for the l?

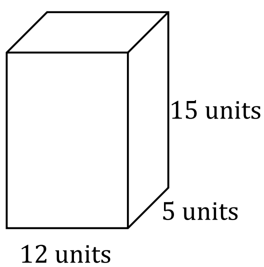
What number can we substitute for the w in the formula? Why?

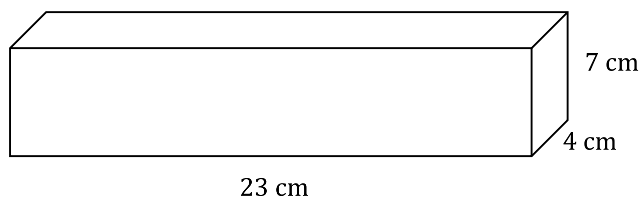
What number can we substitute for the h in the formula?

Determine the volume of the second right rectangular prism by substituting the letters in the formula with their appropriate numbers.

Exercise 3

Complete the table for both figures. Using a calculator is appropriate.





Length of Rectangular Prism	Width of Rectangular Prism	Height of Rectangular Prism	Rectangular Prism's Volume Written as an Expression	Rectangular Prism's Volume Written as a Number

Lesson Summary

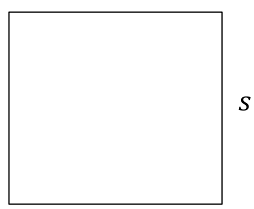
EXPRESSION: An *expression* is a numerical expression, or it is the result of replacing some (or all) of the numbers in a numerical expression with variables.

There are two ways to build expressions:

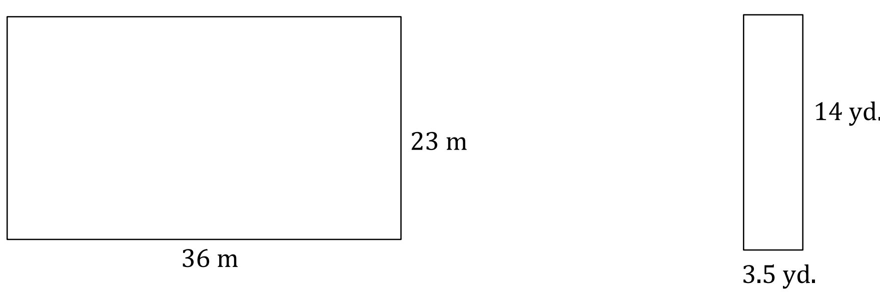
1. We can start out with a numerical expression, like $\frac{1}{3} \cdot (2+4)+7$, and replace some of the numbers with letters to get $\frac{1}{3} \cdot (x+y)+z$.
2. We can build such expressions from scratch, as in $x + x(y - z)$, and note that if numbers were placed in the expression for the variables x, y, and z, the result would be a numerical expression.

Problem Set

1. Replace the side length of this square with 4 in. and find the area.



2. Complete the table for each of the given figures.



Length of Rectangle	Width of Rectangle	Rectangle's Area Written as an Expression	Rectangle's Area as a Number

3. Find the perimeter of each quadrilateral in Problems 1 and 2.

4. Using the formula $V = l \times w \times h$, find the volume of a right rectangular prism when the length of the prism is 45 cm, the width is 12 cm, and the height is 10 cm.

Lesson 8: Replacing Numbers with Letters

Classwork

Opening Exercise

$$4+0=4$$

$$4\times 1=4$$

$$4\div 1=4$$

$$4\times 0=0$$

$$1\div 4=\frac{1}{4}$$

How many of these statements are true?

How many of those statements would be true if the number 4 was replaced with the number 7 in each of the number sentences?

Would the number sentences be true if we were to replace the number 4 with any other number?

What if we replaced the number 4 with the number 0? Would each of the number sentences be true?

What if we replace the number 4 with a letter g? Please write all 4 expressions below, replacing each 4 with a g.

Are these all true (except for $g = 0$) when dividing?

Example 1: Additive Identity Property of Zero

$$g + 0 = g$$

Remember a letter in a mathematical expression represents a number. Can we replace g with any number?

Choose a value for g, and replace g with that number in the equation. What do you observe?

Repeat this process several times, each time choosing a different number for g.

Will all values of g result in a true number sentence?

Write the mathematical language for this property below.

Example 2: Multiplicative Identity Property of One

$$g \times 1 = g$$

Remember a letter in a mathematical expression represents a number. Can we replace g with any number?

Choose a value for g, and replace g with that number in the equation. What do you observe?

Will all values of g result in a true number sentence? Experiment with different values before making your claim.

Write the mathematical language for this property below.

Example 3: Commutative Property of Addition and Multiplication

$$3 + 4 = 4 + 3$$
$$3 \times 4 = 4 \times 3$$

Replace the 3's in these number sentences with the letter a.

Choose a value for a, and replace a with that number in each of the equations. What do you observe?

Will all values of a result in a true number sentence? Experiment with different values before making your claim.

Now write the equations again, this time replacing the number 4 with a variable, b.

Will all values of a and b result in true number sentences for the first two equations? Experiment with different values before making your claim.

Write the mathematical language for this property below.

Example 4

$$3 + 3 + 3 + 3 = 4 \times 3$$

$$3 \div 4 = \frac{3}{4}$$

Replace the 3's in these number sentences with the letter a.

Choose a value for a and replace a with that number in each of the equations. What do you observe?

Will all values of a result in a true number sentence? Experiment with different values before making your claim.

Now write the equations again, this time replacing the number 4 with a variable, b.

Will all values of a and b result in true number sentences for the equations? Experiment with different values before making your claim.

Problem Set

1. State the commutative property of addition using the variables a and b.

2. State the commutative property of multiplication using the variables a and b.

3. State the additive property of zero using the variable b.

4. State the multiplicative identity property of one using the variable b.

5. Demonstrate the property listed in the first column by filling in the third column of the table.

Commutative Property of Addition	$25 + c =$	
Commutative Property of Multiplication	$l \times w =$	
Additive Property of Zero	$h + 0 =$	
Multiplicative Identity Property of One	$v \times 1 =$	

6. Why is there no commutative property for subtraction or division? Show examples.

Lesson 9: Writing Addition and Subtraction Expressions

Classwork

Example 1

Create a bar diagram to show 3 plus 5.

How would this look if you were asked to show 5 plus 3?

Are these two expressions equivalent?

Example 2

How can we show a number increased by 2?

Can you prove this using a model? If so, draw the model.

Example 3

Write an expression to show the sum of m and k.

Which property can be used in Examples 1–3 to show that both expressions given are equivalent?

Example 4

How can we show 10 minus 6?

- Draw a bar diagram to model this expression.

- What expression would represent this model?

- Could we also use $6 - 10$?

Example 5

How can we write an expression to show 3 less than a number?

- Start by drawing a diagram to model the subtraction. Are we taking away from the 3 or the unknown number?

- What expression would represent this model?

Example 6

How would we write an expression to show the number c being subtracted from the sum of a and b?

- Start by writing an expression for "the sum of a and b."

- Now show c being subtracted from the sum.

Example 7

Write an expression to show the number c minus the sum of a and b.

Why are the parentheses necessary in this example and not the others?

Replace the variables with numbers to see if $c - (a + b)$ is the same as $c - a + b$.

Exercises

1. Write an expression to show the sum of 7 and 1.5.

2. Write two expressions to show w increased by 4. Then draw models to prove that both expressions represent the same thing.

3. Write an expression to show the sum of a, b, and c.

4. Write an expression and a model showing 3 less than p.

5. Write an expression to show the difference of 3 and p.

6. Write an expression to show 4 less than the sum of g and 5.

7. Write an expression to show 4 decreased by the sum of g and 5.

8. Should Exercises 6 and 7 have different expressions? Why or why not?

Problem Set

1. Write two expressions to show a number decreased by 11. Then draw models to prove that both expressions represent the same thing.

2. Write an expression to show the sum of x and y.

3. Write an expression to show h decreased by 13.

4. Write an expression to show k less than 3.5.

5. Write an expression to show the sum of g and h reduced by 11.

6. Write an expression to show 5 less than y, plus g.

7. Write an expression to show 5 less than the sum of y and g.

Lesson 10: Writing and Expanding Multiplication Expressions

Classwork

Example 1

Write each expression using the fewest number of symbols and characters. Use math terms to describe the expressions and parts of the expression.

a. $6 \times b$

b. $4 \cdot 3 \cdot h$

c. $2 \times 2 \times 2 \times a \times b$

d. $5 \times m \times 3 \times p$

e. $1 \times g \times w$

Example 2

To expand multiplication expressions we will rewrite the expressions by including the " $\cdot$ " back into the expressions.

a. $5g$

b. $7abc$

c. $12g$

d. $3h \cdot 8$

e. $7g \cdot 9h$

Example 3

a. Find the product of $4f \cdot 7g$.

b. Multiply $3de \cdot 9yz$.

c. Double the product of $6y$ and $3bc$.

Lesson Summary

AN EXPRESSION IN EXPANDED FORM: An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form.

Problem Set

1. Rewrite the expression in standard form (use the fewest number of symbols and characters possible).
   a. $5 \cdot y$
   b. $7 \cdot d \cdot e$
   c. $5 \cdot 2 \cdot 2 \cdot y \cdot z$
   d. $3 \cdot 3 \cdot 2 \cdot 5 \cdot d$

2. Write the following expressions in expanded form.
   a. $3g$
   b. $11mp$
   c. $20yz$
   d. $15abc$

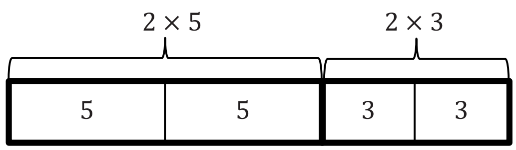
3. Find the product.
   a. $5d \cdot 7g$
   b. $12ab \cdot 3cd$

Lesson 11: Factoring Expressions

Classwork

Example 1

a. Use the model to answer the following questions.



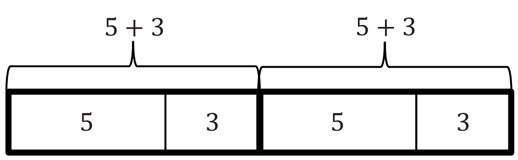
How many fives are in the model?

How many threes are in the model?

What does the expression represent in words?

What expression could we write to represent the model?

b. Use the new model and the previous model to answer the next set of questions.



How many fives are in the model?

How many threes are in the model?

What does the expression represent in words?

What expression could we write to represent the model?

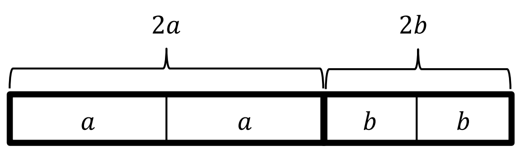
c. Is the model in part (a) equivalent to the model in part (b)?

d. What relationship do we see happening on either side of the equal sign?

e. In Grade 5 and in Module 2 of this year, you have used similar reasoning to solve problems. What is the name of the property that is used to say that $2(5+3)$ is the same as $2 \times 5 + 2 \times 3$?

Example 2

Now, we will take a look at an example with variables. Discuss the questions with your partner.



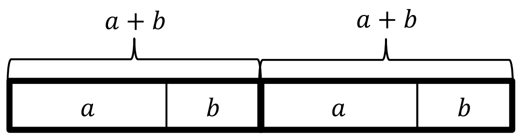
What does the model represent in words?

What does $2a$ mean?

How many a's are in the model?

How many b's are in the model?

What expression could we write to represent the model?



How many a's are in the expression?

How many b's are in the expression?

What expression could we write to represent the model?

Are the two expressions equivalent?

Example 3

Use GCF and the distributive property to write equivalent expressions.

1. $3f + 3g =$ __________________

   What is the question asking us to do?

   How would Problem 1 look if we expanded each term?

   What is the GCF in Problem 1?

   How can we use the GCF to rewrite this?

2. $6x + 9y =$ ______________________

What is the question asking us to do?

How would Problem 2 look if we expanded each term?

What is the GCF in Problem 2?

How can we use the GCF to rewrite this?

3. $3c + 11c =$ ______________________

Is there a GCF in Problem 3?

Rewrite the expression using the distributive property.

4. $24b + 8 =$ ______________________

Explain how you used GCF and the distributive property to rewrite the expression in Problem 4.

Why is there a 1 in the parentheses?

How is this related to the first two examples?

Exercises

1. Apply the distributive property to write equivalent expressions.
   a. $7x + 7y$
   b. $15g + 20h$
   c. $18m + 42n$
   d. $30a + 39b$
   e. $11f + 15f$
   f. $18h + 13h$
   g. $55m + 11$
   h. $7 + 56y$

2. Evaluate each of the expressions below.
   a. $6x + 21y$ and $3(2x + 7y)$ $\quad$ $x = 3$ and $y = 4$
   b. $5g + 7g$ and $g(5 + 7)$ $\quad$ $g = 6$

c. $14x + 2$ and $2(7x + 1)$ $x = 10$

d. Explain any patterns that you notice in the results to parts (a)–(c).

e. What would happen if other values were given for the variables?

Closing

How can you use your knowledge of GCF and the distributive property to write equivalent expressions?

Find the missing value that makes the two expressions equivalent.

$4x + 12y$ _____$(x + 3y)$

$35x + 50y$ _____$(7x + 10y)$

$18x + 9y$ _____$(2x + y)$

$32x + 8y$ _____$(4x + y)$

$100x + 700y$ _____$(x + 7y)$

Explain how you determine the missing number.

Lesson Summary

AN EXPRESSION IN FACTORED FORM: An expression that is a product of two or more expressions is said to be in *factored form*.

Problem Set

1. Use models to prove that $3(a+b)$ is equivalent to $3a+3b$.

2. Use GCF and the distributive property to write equivalent expressions in factored form for the following expressions.
   a. $4d+12e$
   b. $18x+30y$
   c. $21a+28y$
   d. $24f+56g$

Lesson 12: Distributing Expressions

Classwork

Opening Exercise

a. Create a model to show 2×5.

b. Create a model to show $2 \times b$, or $2b$.

Example 1

Write an expression that is equivalent to $2(a + b)$.

Create a model to represent $(a + b)$.

The expression $2(a + b)$ tells us that we have 2 of the $(a + b)$'s. Create a model that shows 2 groups of $(a + b)$.

How many a's and how many b's do you see in the diagram?

How would the model look if we grouped together the a's and then grouped together the b's?

What expression could we write to represent the new diagram?

What conclusion can we draw from the models about equivalent expressions?

Let $a = 3$ and $b = 4$.

What happens when we double $(a + b)$?

Example 2

Write an expression that is equivalent to double $(3x + 4y)$.

How can we rewrite double $(3x + 4y)$?

Is this expression in factored form, expanded form, or neither?

Let's start this problem the same way that we started the first example. What should we do?

How can we change the model to show $2(3x + 4y)$?

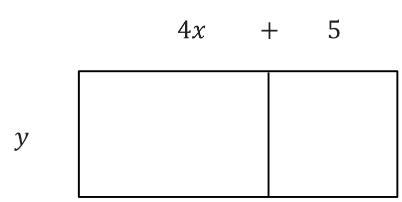
Are there terms that we can combine in this example?

What is an equivalent expression that we can use to represent $2(3x + 4y)$?

Summarize how you would solve this question without the model.

Example 3

Write an expression in expanded form that is equivalent to the model below.



What factored expression is represented in the model?

How can we rewrite this expression in expanded form?

Example 4

Write an expression in expanded form that is equivalent to $3(7d + 4e)$.

Exercises

Create a model for each expression below. Then write another equivalent expression using the distributive property.

1. $3(x + y)$

2. $4(2h + g)$

Apply the distributive property to write an equivalent expression in expanded form.

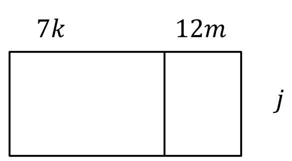
3. $8(h + 3)$

4. $3(2h + 7)$

5. $5(3x + 9y)$

6. $4(11h + 3g)$

7.



8. $a(9b + 13)$

Problem Set

1. Use the distributive property to write the following expressions in expanded form.
   a. $4(x + y)$
   b. $8(a + 3b)$
   c. $3(2x + 11y)$
   d. $9(7a + 6b)$
   e. $c(3a + b)$
   f. $y(2x + 11z)$

2. Create a model to show that $2(2x + 3y) = 4x + 6y$.

Lesson 13: Writing Division Expressions

Classwork

Example 1

Write an expression showing $1 \div 2$ without the use of the division symbol.

What can we determine from the model?

Example 2

Write an expression showing $a \div 2$ without the use of the division symbol.

What can we determine from the model?

When we write division expressions using the division symbol we represent ________________________________.

How would this look when we write division expressions using a fraction?

Example 3

a. Write an expression showing $a \div b$ without the use of the division symbol.

b. Write an expression for g divided by the quantity h plus 3.

c. Write an expression for the quotient of the quantity m reduced by 3 and 5.

Exercises

Write each expression two ways: using the division symbol and as a fraction.

a. 12 divided by 4.

b. 3 divided by 5.

c. a divided by 4.

d. The quotient of 6 and m.

e. Seven divided by the quantity x plus y.

f. y divided by the quantity x minus 11.

g. The sum of the quantity h and 3 divided by 4.

h. The quotient of the quantity k minus 10 and m.

Problem Set

1. Rewrite the expressions using the division symbol and as a fraction.
   a. Three divided by 4.
   b. The quotient of m and 11.
   c. 4 divided by the sum of h and 7.
   d. The quantity x minus 3 divided by y.

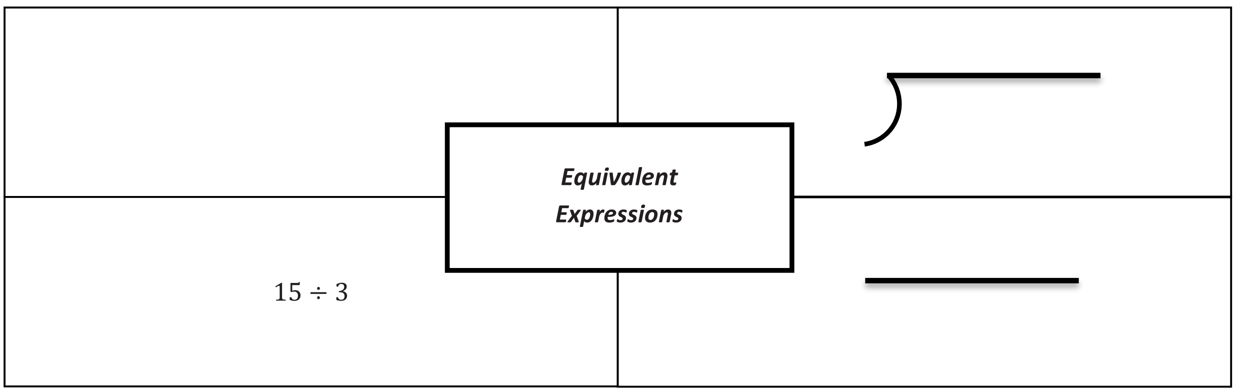
2. Draw a model to show that $x \div 3$ is the same as $\frac{x}{3}$.

Lesson 14: Writing Division Expressions

Classwork

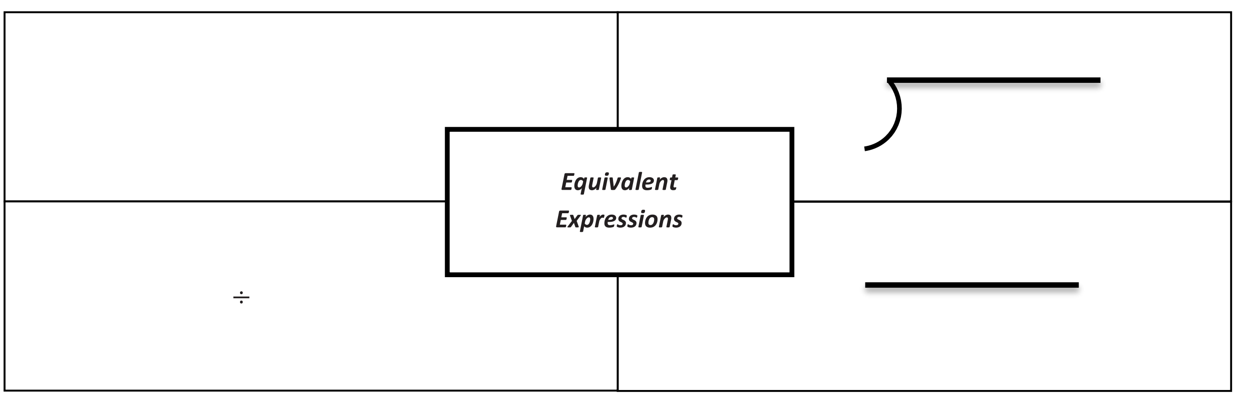
Example 1

Fill in the three remaining squares so that all the squares contain equivalent expressions.



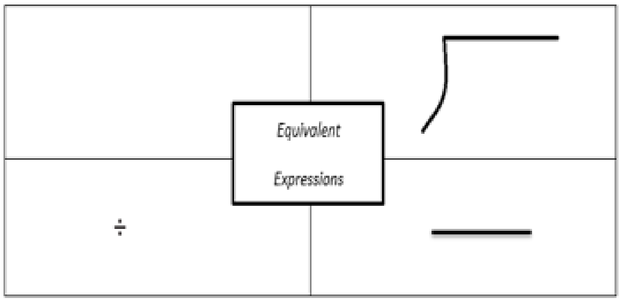
Example 2

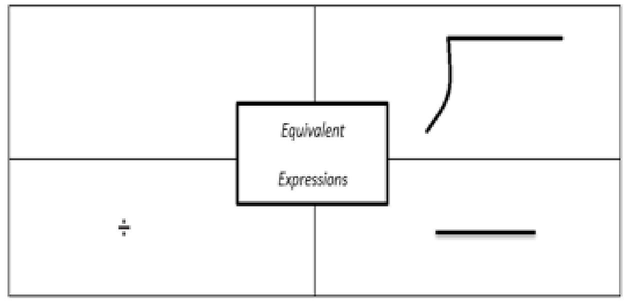
Fill in a blank copy of the four boxes using the words *dividend* and *divisor* so that it is set up for any example.

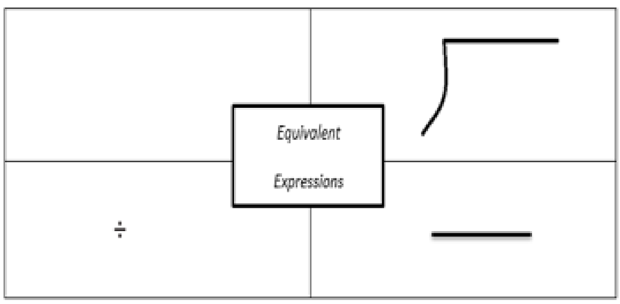


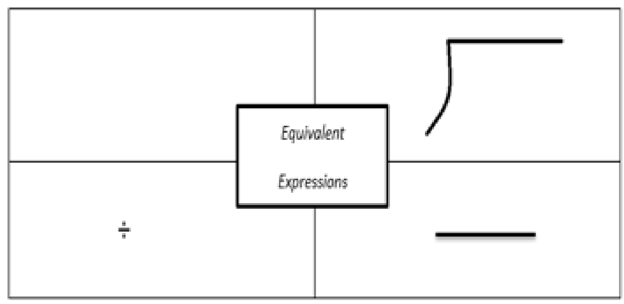
Exercises

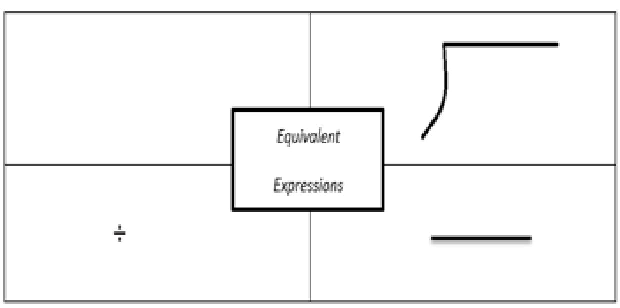
Complete the missing spaces in each rectangle set.

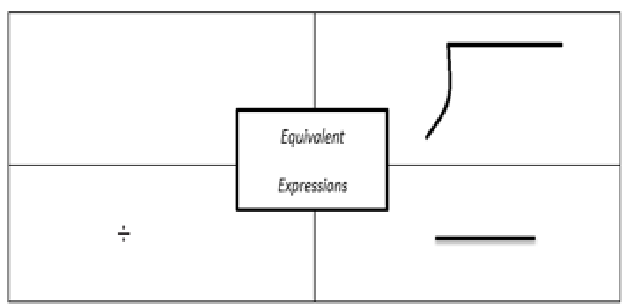


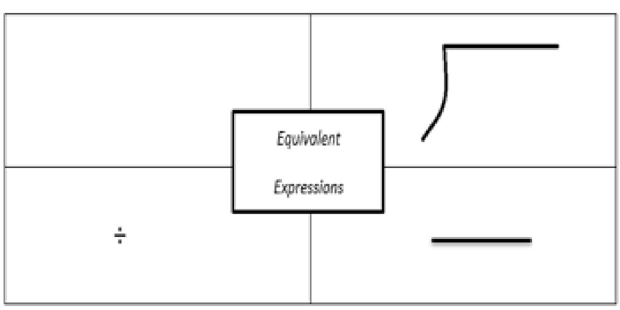


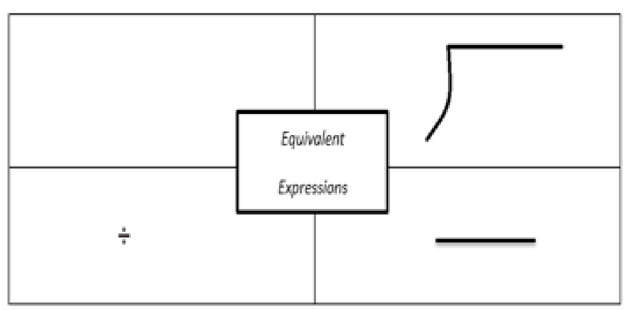






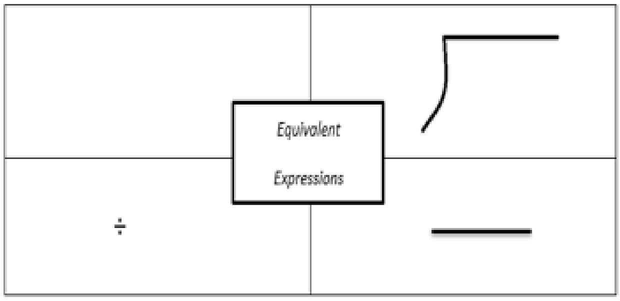


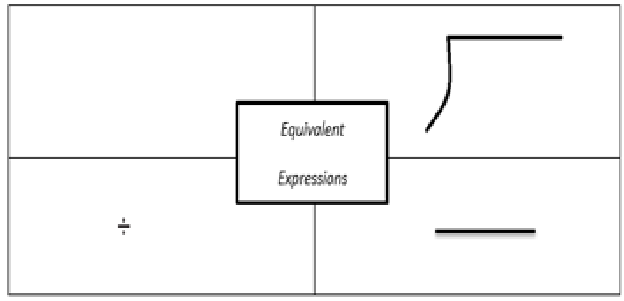


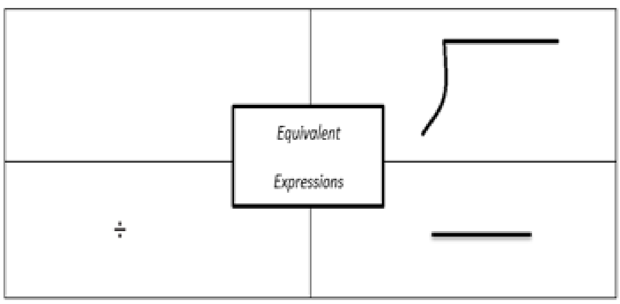


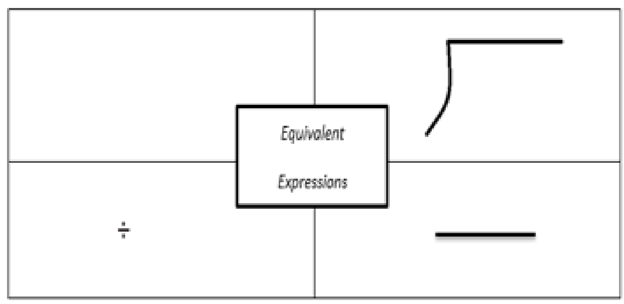
Problem Set

Complete the missing spaces in each rectangle set.









Lesson 15: Read Expressions in Which Letters Stand for Numbers

Classwork

Opening Exercise

Complete the graphic organizer with mathematical words that indicate each operation. Some words may indicate more than one operation.

ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION	EXPONENTS

Example 1

Write an expression using words.

a. $a - b$

b. xy

c. $4f + p$

d. $d - b^3$

e. $5(u - 10) + h$

f. $\frac{3}{d+f}$

Exercises

Circle all the vocabulary words that could be used to describe the given expression.

1. $6h - 10$

   ADDITION SUBTRACTION MULTIPLICATION DIVISION

2. $\frac{5d}{6}$

   SUM DIFFERENCE PRODUCT QUOTIENT

3. $5(2 + d) - 8$

   ADD SUBTRACT MULTIPLY DIVIDE

4. abc

   MORE THAN LESS THAN TIMES EACH

Write an expression using vocabulary to represent each given expression.

5. $8 - 2g$

6. $15(a + c)$

7. $\frac{m+n}{5}$

8. $b^3 - 18$

9. $f - \frac{d}{2}$

10. $\frac{u}{x}$

Problem Set

1. List five different vocabulary words that could be used to describe each given expression.
   a. $a - d + c$
   b. $20 - 3c$
   c. $\frac{b}{d+2}$

2. Write an expression using math vocabulary for each expression below.
   a. $5b - 18$
   b. $\frac{n}{2}$
   c. $a + (d - 6)$
   d. $10 + 2b$

Lesson 16: Write Expressions in Which Letters Stand for Numbers

Classwork

Opening Exercise

Underline the key words in each statement.

a. The sum of twice b and 5.

b. The quotient of c and d.

c. a raised to the fifth power then increased by the product of 5 and c.

d. The quantity of a plus b divided by 4.

e. 10 less than the product of 15 and c.

f. 5 times d then increased by 8.

Mathematical Modeling Exercise 1

Model how to change the expressions given in the opening exercise from words to variables and numbers.

a. The sum of twice b and 5.

b. The quotient of c and d.

c. a raised to the fifth power then increased by the product of 5 and c.

d. The quantity of a plus b divided by 4.

e. 10 less than the product of 15 and c.

f. 5 times d then increased by 8.

Mathematical Modeling Exercise 2

Model how to change each real-world scenario to an expression using variables and numbers. Underline the text to show the key words before writing the expression.

Marcus has 4 more dollars than Yaseen. If y is the amount of money Yaseen has, write an expression to show how much money Marcus has.

Mario is missing half of his assignments. If a represents the number of assignments, write an expression to show how many assignments Mario is missing.

Kamilah's weight has tripled since her first birthday. If w represents the amount Kamilah weighed on her first birthday, write an expression to show how much Kamilah weighs now.

Nathan brings cupcakes to school and gives them to his five best friends who share them equally. If c represents the number of cupcakes Nathan brings to school, write an expression to show how many cupcakes each of his friends receive.

Mrs. Marcus combines her atlases and dictionaries and then divides them among 10 different tables. If a represents the number of atlases and d represents the number of dictionaries Mrs. Marcus has, write an expression to show how many books would be on each table.

To improve in basketball, Ivan's coach told him that he needs to take four times as many free throws and four times as many jump shots every day. If f represents the number of free throws and j represents the number of jump shots Ivan shoots daily, write an expression to show how many shots he will need to take in order to improve in basketball.

Exercises

Mark the text by underlining key words, and then write an expression using variables and/or numbers for each statement.

1. b decreased by c squared.

2. 24 divided by the product of 2 and a.

3. 150 decreased by the quantity of 6 plus b.

4. The sum of twice c and 10.

5. Marlo had $\$35$ but then spent $\$m$.

6. Samantha saved her money and was able to quadruple the original amount, m.

7. Veronica increased her grade, g, by 4 points, and then doubled it.

8. Adbell had m pieces of candy and ate 5 of them. Then, he split the remaining candy equally among 4 friends.

9. To find out how much paint is needed, Mr. Jones must square the side length, s, of the gate, and then subtract 15.

10. Luis brought x cans of cola to the party, Faith brought d cans of cola, and De'Shawn brought h cans of cola. How many cans of cola did they bring altogether?

Problem Set

Mark the text by underlining key words, and then write an expression using variables and numbers for each of the statements below.

1. Justin can type w words per minute. Melvin can type 4 times as many words as Justin. Write an expression that represents the rate at which Melvin can type.

2. Yohanna swam y yards yesterday. Sheylin swam 5 yards less than half the amount of yards as Yohanna. Write an expression that represents the number of yards Sheylin swam yesterday.

3. A number is d decreased by 5 and then doubled.

4. Nahom had n baseball cards and Semir had s baseball cards. They combined their baseball cards and then sold 10 of them.

5. The sum of 25 and h is divided by f cubed.

Lesson 17: Write Expressions in Which Letters Stand for Numbers

Classwork

Exercises

Station One	1. The sum of a and b.
	2. Five more than twice a number c.
	3. Martha bought d number of apples and then ate 6 of them.
Station Two	1. 14 decreased by p.
	2. The total of d and f, divided by 8.
	3. Rashod scored 6 less than 3 times as many baskets as Mike. Mike scores b baskets.
Station Three	1. The quotient of c and 6.
	2. Triple the sum of x and 17.
	3. Gabrielle had b buttons but then lost 6. Gabrielle took the remaining buttons and split them equally among her 5 friends.

Station Four	1. d doubled.
	2. Three more than 4 times a number x.
	3. Mali has c pieces of candy. She doubles the amount of candy she has then gives away 15 pieces.
Station Five	1. f cubed.
	2. The quantity of 4 increased by a, and then the sum is divided by 9.
	3. Tai earned 4 points fewer than double Oden's points. Oden earned p points.
Station Six	1. The difference between d and 8.
	2. 6 less than the sum of d and 9.
	3. Adalyn has x pants and s shirts. She combined them and sold half of them. How many items did Adalyn sell?

Problem Set

Write an expression using letters and/or numbers for each problem below.

1. 4 less than the quantity of 8 times n.

2. 6 times the sum of y and 11.

3. The square of m reduced by 49.

4. The quotient when the quantity of 17 plus p is divided by 8.

5. Jim earned j in tips, and Steve earned s in tips. They combine their tips then split them equally.

6. Owen has c collector cards. He quadruples the number of cards he has, and then combines them with Ian, who has i collector cards.

7. Rae ran 4 times as many miles as Madison and Aaliyah combined. Madison ran m miles and Aaliyah ran a miles.

8. By using coupons, Mary Jo was able to decrease the retail price of her groceries, g, by $\$125$.

9. To calculate the area of a triangle, you find the product of the base and height and then divide by 2.

10. The temperature today was 10 degrees colder than twice yesterday's temperature, t.

Lesson 18: Writing and Evaluating Expressions—Addition and Subtraction

Classwork

Opening Exercise

How can we show a number increased by 2?

Can you prove this using a model?

Example 1: The Importance of Being Specific in Naming Variables

When naming variables in expressions, it is important to be very clear about what they represent. The units of measure must be included if something is measured.

Exercises

1. Read the variable in the table, and improve the description given, making it more specific.

Variable	Incomplete Description	Complete Description with Units
Joshua's speed (J)	Let J = Joshua's speed	
Rufus's height (R)	Let R = Rufus's height	
Milk sold (M)	Let M = the amount of milk sold	
Colleen's time in the 40 meter hurdles (C)	Let C = Colleen's time	
Sean's age (S)	Let S = Sean's age	

2. Read each variable in the table and improve the description given, making it more specific.

Variable	Incomplete Description	Complete Description with Units
Karolyn's CDs (K)	Let K = Karolyn's CDs	Let K = the number of CDs Karolyn has
Joshua's merit badges (J)	Let J = Joshua's merit badges	
Rufus's trading cards (R)	Let R = Rufus's trading cards	
Milk money (M)	Let M = the amount of milk money	

Example 2: Writing and Evaluating Addition and Subtraction Expressions

Read each story problem. Identify the unknown quantity, and write an addition or subtraction expression that is described. Finally, evaluate your expression using the information given in column four.

Story Problem	Description with Units	Expression	Evaluate the Expression If:	Show Your Work and Evaluate
Gregg has two more dollars than his brother Jeff. Write an expression for the amount of money Gregg has.	Let j = Jeff's money in dollars	$j + 2$	Jeff has $12.	$j + 2$ $12 + 2$ 14 Gregg has $14.
Gregg has two more dollars than his brother Jeff. Write an expression for the amount of money Jeff has.	Let g = Gregg's money in dollars	$g - 2$	Gregg has $14.	$g - 2$ $14 - 2$ 12 Jeff has $12.
Abby read 8 more books than Kristen in the first marking period. Write an expression for the number of books Abby read.			Kristen read 9 books in the first marking period.	

Abby read 6 more books than Kristen in the second marking period. Write an expression for the number of books Kristen read.			Abby read 20 books in the second marking period.	
Daryl has been teaching for one year longer than Julie. Write an expression for the number of years that Daryl has been teaching.			Julie has been teaching for 28 years.	
Ian scored 4 fewer goals than Julia in the first half of the season. Write an expression for the number of goals Ian scored.			Julia scored 13 goals.	
Ian scored 3 fewer goals than Julia in the second half of the season. Write an expression for the number of goals Julia scored.			Ian scored 8 goals.	
Johann visited Niagara Falls 3 times fewer than Arthur. Write an expression for the number of times Johann visited Niagara Falls.			Arthur visited Niagara Falls 5 times.	

Problem Set

1. Read the story problem. Identify the unknown quantity and write an addition or subtraction expression that is described. Finally, evaluate your expression using the information given in column four.

Story Problem	Description with Units	Expression	Evaluate the Expression If:	Show Your Work and Evaluate
Sammy has two more baseballs than his brother Ethan.	Let e = the number of balls Ethan has	$e + 2$	Ethan has 7 baseballs.	$e + 2$ $7 + 2$ 9 Sammy has 9 baseballs.
Ella wrote 8 more stories than Anna in the fifth grade.			Anna wrote 10 stories in the fifth grade.	
Lisa has been dancing for 3 more years than Danika.			Danika has been dancing for 6 years.	
The New York Rangers scored 2 fewer goals than the Buffalo Sabres last night.			The Rangers scored 3 goals last night.	
George has gone camping 3 times fewer than Dave.			George has gone camping 8 times.	

2. If George went camping 15 times, how could you figure out how many times Dave went camping?

Lesson 19: Substituting to Evaluate Addition and Subtraction Expressions

Classwork

Opening Exercise

My older sister is exactly two years older than I am. Sharing a birthday is both fun and annoying. Every year on our birthday we have a party, which is fun, but she always brags that she is two years older than I am, which is annoying. Shown below is a table of our ages, starting when I was born:

My Age (in years)	My Sister's Age (in years)
0	2
1	3
2	4
3	5
4	6

a. Looking at the table, what patterns do you see? Tell a partner.

b. On the day I turned 8 years old, how old was my sister?

c. How do you know?

d. On the day I turned 16 years old, how old was my sister?

e. How do you know?

f. Do we need to extend the table to calculate these answers?

Example 1

My Age (in years)	My Sister's Age (in years)
0	2
1	3
2	4
3	5
4	6

a. What if you don't know how old I am? Let's use a variable for my age. Let $Y =$ my age in years. Can you develop an expression to describe how old my sister is?

b. Please add that to the last row of the table.

Example 2

My Age (in years)	My Sister's Age (in years)
0	2
1	3
2	4
3	5
4	6

a. How old was I when my sister was 6 years old?

b. How old was I when my sister was 15 years old?

c. How do you know?

d. Look at the table in Example 2. If you know my sister's age, can you determine my age?

e. If we use the variable G for my sister's age in years, what expression would describe my age in years?

f. Fill in the last row of the table with the expressions.

g. With a partner, calculate how old I was when my sister was 22, 23, and 24 years old.

Exercises

1. Noah and Carter are collecting box tops for their school. They each bring in 1 box top per day starting on the first day of school. However, Carter had a head start because his aunt sent him 15 box tops before school began. Noah's grandma saved 10 box tops, and Noah added those on his first day.

   a. Fill in the missing values that indicate the total number of box tops each boy brought to school.

School Day	Number of Box Tops Noah Has	Number of Box Tops Carter Has
1	11	16
2		
3		
4		
5		

   b. If we let D be the number of days since the new school year began, on day D of school, how many box tops will Noah have brought to school?

   c. On day D of school, how many box tops will Carter have brought to school?

   d. On day 10 of school, how many box tops will Noah have brought to school?

   e. On day 10 of school, how many box tops will Carter have brought to school?

2. Each week the Primary School recycles 200 pounds of paper. The Intermediate School also recycles the same amount but had another 300 pounds left over from summer school. The Intermediate School custodian added this extra 300 pounds to the first recycle week.

   a. Number the weeks and record the amount of paper recycled by both schools.

Week	Total Amount of Paper Recycled by the Primary School This School Year in Pounds	Total Amount of Paper Recycled by the Intermediate School This School Year in Pounds

   b. If this trend continues, what will be the total amount collected for each school on Week 10?

3. Shelly and Kristen share a birthday, but Shelly is 5 years older.

   a. Make a table showing their ages every year, beginning when Kristen was born.

   b. If Kristen is 16 years old, how old is Shelly?

   c. If Kristen is K years old, how old is Shelly?

   d. If Shelly is S years old, how old is Kristen?

Problem Set

1. Suellen and Tara are in sixth grade, and both take dance lessons at Twinkle Toes Dance Studio. This is Suellen's first year, while this is Tara's fifth year of dance lessons. Both girls plan to continue taking lessons throughout high school.
   a. Complete the table showing the number of years the girls will have danced at the studio.

Grade	Suellen's Years of Experience Dancing	Tara's Years of Experience Dancing
Sixth		
Seventh		
Eighth		
Ninth		
Tenth		
Eleventh		
Twelfth		

   b. If Suellen has been taking dance lessons for Y years, how many years has Tara been taking lessons?

2. Daejoy and Damian collect fossils. Before they went on a fossil-hunting trip, Daejoy had 25 fossils in her collection, and Damian had 16 fossils in his collection. On a 10-day fossil hunting trip, they each collected 2 new fossils each day.
   a. Make a table showing how many fossils each person had in their collection at the end of each day.

   b. If this pattern of fossil finding continues, how many fossils does Damian have when Daejoy has F fossils?
   c. If this pattern of fossil finding continues, how many fossils does Damian have when Daejoy has 55 fossils?

3. A train consists of three types of cars: box cars, an engine, and a caboose. The relationship between the types of cars is demonstrated in the table below.

Number of Box Cars	Number of Cars in the Train
0	2
1	3
2	4
10	12
100	102

   a. Tom wrote an expression for the relationship depicted in the table as $B + 2$. Theresa wrote an expression for the same relationship as $C - 2$. Is it possible to have two different expressions to represent one relationship? Explain.

   b. What do you think the variable in each student's expression represents? How would you define them?

4. David was 3 when Marieka was born. Complete the table.

Marieka's Age in Years	David's Age in Years
5	8
6	9
7	10
8	11
10	
	20
32	
M	
	D

5. Caitlin and Michael are playing a card game. In the first round, Caitlin scored 200 points and Michael scored 175 points. In each of the next few rounds, they each scored 50 points. Their score sheet is below.

Caitlin's points	Michael's points
200	175
250	225
300	275
350	325

   a. If this trend continues, how many points will Michael have when Caitlin has 600 points?

   b. If this trend continues, how many points will Michael have when Caitlin has C points?

   c. If this trend continues, how many points will Caitlin have when Michael has 975 points?

   d. If this trend continues, how many points will Caitlin have when Michael has M points?

6. The high school marching band has 15 drummers this year. The band director insists that there are to be 5 more trumpet players than drummers at all times.
   a. How many trumpet players are in the marching band this year?
   b. Write an expression that describes the relationship of the number of trumpet players (T) and the number of drummers (D).
   c. If there are only 14 trumpet players interested in joining the marching band next year, how many drummers will the band director want in the band?

Lesson 20: Writing and Evaluating Expressions—Multiplication and Division

Classwork

Example 1

1. The farmers' market is selling bags of apples. In every bag, there are 3 apples.

   a. Complete the table.

Number of Bags	Total Number of Apples
1	3
2	
3	
4	
B	

   b. What if the market had 25 bags of apples to sell? How many apples is that in all?

   c. If a truck arrived that had some number, a, more apples on it, then how many bags would the clerks use to bag up the apples?

   d. If a truck arrived that had 600 more apples on it, how many bags would the clerks use to bag up the apples?

   e. How is part (d) different from part (b)?

Exercises

1. In New York State, there is a five-cent deposit on all carbonated beverage cans and bottles. When you return the empty can or bottle, you get the five cents back.
   a. Complete the table.

Number of Containers Returned	Refund in Dollars
1	
2	
3	
4	
10	
50	
100	
C	

   b. If we let C represent the number of cans, what is the expression that shows how much money is returned?

   c. Use the expression to find out how much money Brett would receive if he returned 222 cans.

   d. If Gavin needs to earn $\$4.50$ for returning cans, how many cans does he need to collect and return?

   e. How is part (d) different from part (c)?

2. The fare for a subway or a local bus ride is $\$2.50$.

   a. Complete the table.

Number of Rides	Cost of Rides in Dollars
1	
2	
3	
4	
5	
10	
30	
R	

   b. If we let R represent the number of rides, what is the expression that shows the cost of the rides?

   c. Use the expression to find out how much money 60 rides would cost.

   d. If a commuter spends $\$175.00$ on subway or bus rides, how many trips did the commuter take?

   e. How is part (d) different from part (c)?

Challenge Problem

3. A pendulum swings though a certain number of cycles in a given time. Owen made a pendulum that swings 12 times every 15 seconds.

   a. Construct a table showing the number of cycles through which a pendulum swings. Include data for up to one minute. Use the last row for C cycles, and write an expression for the time it takes for the pendulum to make C cycles.

   b. Owen and his pendulum team set their pendulum in motion and counted 16 cycles. What was the elapsed time?

   c. Write an expression for the number of cycles a pendulum swings in S seconds.

   d. In a different experiment, Owen and his pendulum team counted the cycles of the pendulum for 35 seconds. How many cycles did they count?

Problem Set

1. A radio station plays 12 songs each hour. They never stop for commercials, news, weather, or traffic reports.
   a. Write an expression describing how many songs are played by the radio station in H hours.
   b. How many songs will be played in an entire day (24 hours)?
   c. How long does it take the radio station to play 60 consecutive songs?

2. A ski area has a high speed lift that can move $2{,}400$ skiers to the top of the mountain each hour.
   a. Write an expression describing how many skiers can be lifted in H hours.
   b. How many skiers can be moved to the top of the mountain in 14 hours?
   c. How long will it take to move $3{,}600$ skiers to the top of the mountain?

3. Polly writes a magazine column, for which she earns $\$35$ per hour. Create a table of values that shows the relationship between the number of hours that Polly works, H, and the amount of money Polly earns in dollars, E.

   a. If you know how many hours Polly works, can you determine how much money she earned? Write the corresponding expression.
   b. Use your expression to determine how much Polly earned after working for $3\frac{1}{2}$ hours.
   c. If you know how much money Polly earned, can you determine how long she worked? Write the corresponding expression.
   d. Use your expression to determine how long Polly worked if she earned $\$52.50$.

4. Mitchell delivers newspapers after school, for which he earns $\$0.09$ per paper. Create a table of values that shows the relationship between the number of papers that Mitchell delivers, P, and the amount of money Mitchell earns in dollars, E.

a. If you know how many papers Mitchell delivered, can you determine how much money he earned? Write the corresponding expression.

b. Use your expression to determine how much Mitchell earned by delivering 300 newspapers.

c. If you know how much money Mitchell earned, can you determine how many papers he delivered? Write the corresponding expression.

d. Use your expression to determine how many papers Mitchell delivered if he earned $\$58.50$ last week.

5. Randy is an art dealer who sells reproductions of famous paintings. Copies of the *Mona Lisa* sell for $\$475$.

a. Last year Randy sold $\$9,975$ worth of *Mona Lisa* reproductions. How many did he sell?

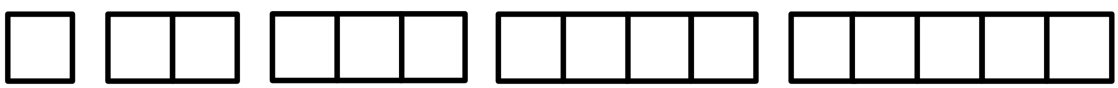
b. If Randy wants to increase his sales to at least $\$15,000$ this year, how many copies will he need to sell (without changing the price per painting)?

Lesson 21: Writing and Evaluating Expressions—Multiplication and Addition

Classwork

Mathematical Modeling Exercise

The Italian Villa Restaurant has square tables that the servers can push together to accommodate the customers. Only one chair fits along the side of the square table. Make a model of each situation to determine how many seats will fit around various rectangular tables.



Number of Square Tables	Number of Seats at the Table
1	
2	
3	
4	
5	
50	
200	
T	

Are there any other ways to think about solutions to this problem?

It is impractical to make a model of pushing 50 tables together to make a long rectangle. If we did have a rectangle that long, how many chairs would fit on the long sides of the table?

How many chairs fit on the ends of the long table?

How many chairs fit in all? Record it on your table.

Work with your group to determine how many chairs would fit around a very long rectangular table if 200 square tables were pushed together.

If we let T represent the number of square tables that make one long rectangular table, what is the expression for the number of chairs that will fit around it?

Example 1

Look at Example 1 with your group. Determine the cost for various numbers of pizzas, and also determine the expression that describes the cost of having P pizzas delivered.

a. Pizza Queen has a special offer on lunch pizzas: $\$4.00$ each. They charge $\$2.00$ to deliver, regardless of how many pizzas are ordered. Determine the cost for various numbers of pizzas, and also determine the expression that describes the cost of having P pizzas delivered.

Number of Pizzas Delivered	Total Cost in Dollars
1	
2	
3	
4	
10	
50	
P	

What mathematical operations did you need to perform to find the total cost?

Suppose our principal wanted to buy a pizza for everyone in our class? Determine how much this would cost.

b. If the booster club had $\$400$ to spend on pizza, what is the greatest number of pizzas they could order?

c. If the pizza price was raised to $\$5.00$ and the delivery price was raised to $\$3.00$, create a table that shows the total cost (pizza plus delivery) of 1, 2, 3, 4, and 5 pizzas. Include the expression that describes the new cost of ordering P pizzas.

Number of Pizzas Delivered	Total Cost in Dollars
1	
2	
3	
4	
5	
P	

Problem Set

1. Compact discs (CDs) cost $\$12$ each at the Music Emporium. The company charges $\$4.50$ for shipping and handling, regardless of how many compact discs are purchased.
   a. Create a table of values that show the relationship between the number of compact discs that Mickey buys, D, and the amount of money Mickey spends, C, in dollars.

Number of CDs Mickey Buys (D)	Total Cost in Dollars (C)
1	
2	
3	

   b. If you know how many CDs Mickey orders, can you determine how much money he spends? Write the corresponding expression.
   c. Use your expression to determine how much Mickey spent buying 8 CDs.

2. Mr. Gee's class orders paperback books from a book club. The books cost $\$2.95$ each. Shipping charges are set at $\$4.00$, regardless of the number of books purchased.
   a. Create a table of values that show the relationship between the number of books that Mr. Gee's class buys, B, and the amount of money they spend, C, in dollars.

Number of Books Ordered (B)	Amount of Money Spent in Dollars (C)
1	
2	
3	

   b. If you know how many books Mr. Gee's class orders, can you determine how much money they spend? Write the corresponding expression.
   c. Use your expression to determine how much Mr. Gee's class spent buying 24 books.

3. Sarah is saving money to take a trip to Oregon. She received $\$450$ in graduation gifts and saves $\$120$ per week working.

   a. Write an expression that shows how much money Sarah has after working W weeks.

   b. Create a table that shows the relationship between the amount of money Sarah has (M) and the number of weeks she works (W).

Amount of Money Sarah Has (M)	**Number of Weeks Worked (W)**
	1
	2
	3
	4
	5
	6
	7
	8

   c. The trip will cost $\$1{,}200$. How many weeks will Sarah have to work to earn enough for the trip?

4. Mr. Gee's Language Arts class keeps track of how many words per minute are read aloud by each of the students. They collect this Oral Reading Fluency data each month. Below is the data they collected for one student in the first four months of school.

   a. Assume this increase in Oral Reading Fluency continues throughout the rest of the school year. Complete the table to project the reading rate for this student for the rest of the year.

Month	**Number of Words Read Aloud in One Minute**
September	126
October	131
November	136
December	141
January	
February	
March	
April	
May	
June	

   b. If this increase in Oral Reading Fluency continues throughout the rest of the school year, when would this student achieve the goal of reading 165 words per minute?

   c. The expression for this student's Oral Reading Fluency is $121 + 5m$, where m represents the number of months during the school year. Use this expression to determine how many words per minute the student would read after 12 months of instructions.

5. When corn seeds germinate, they tend to grow 5 inches in the first week, then 3 inches per week for the remainder of the season. The relationship between height (H) and number of weeks since germination (W) is shown below.

   a. Complete the missing values in the table.

Number of Weeks Since Germination (W)	**Height of Corn Plant (H)**
1	5
2	8
3	11
4	14
5	
6	

   b. The expression for this height is $2 + 3W$. How tall will the corn plant be after 15 weeks of growth?

6. The Honeymoon Charter Fishing Boat Company only allows newlywed couples on their sunrise trips. There is a captain, a first mate, and a deck hand manning the boat on these trips.

   a. Write an expression that shows the number of people on the boat when there are C couples booked for the trip.

   b. If the boat can hold a maximum of 20 people, how many couples can go on the sunrise fishing trip?

Lesson 22: Writing and Evaluating Expressions—Exponents

Classwork

Example 1: Folding Paper

Exercises

1. Predict how many times you can fold a piece of paper in half.

   My Prediction: ____________

2. Before any folding (zero folds), there is only one layer of paper. This is recorded in the first row of the table. Fold your paper in half. Record the number of layers of paper that result. Continue as long as possible.

Number of Folds	Number of Paper Layers That Result	Number of Paper Layers Written as a Power of 2
0	1	2^0
1		
2		
3		
4		
5		
6		
7		
8		

   a. Are you able to continue folding the paper indefinitely? Why or why not?

   b. How could you use a calculator to find the next number in the series?

c. What is the relationship between the number of folds and the number of layers?

d. How is this relationship represented in exponential form of the numerical expression?

e. If you fold a paper f times, write an expression to show the number of paper layers.

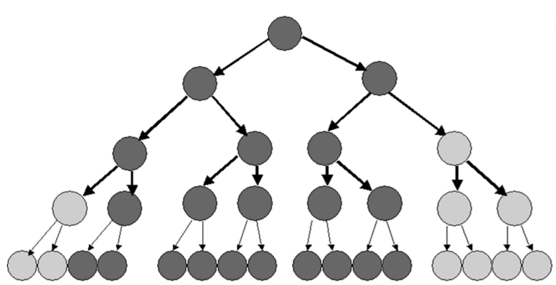
3. If the paper were to be cut instead of folded, the height of the stack would double at each successive stage, and it would be possible to continue.

   a. Write an expression that describes how many layers of paper result from 16 cuts.

   b. Evaluate this expression by writing it in standard form.

Example 2: Bacterial Infection

Bacteria are microscopic single-celled organisms that reproduce in a couple of different ways, one of which is called binary fission. In binary fission, a bacterium increases its size until it is large enough to split into two parts that are identical. These two grow until they are both large enough to split into two individual bacteria. This continues as long as growing conditions are favorable.



a. Record the number of bacteria that result from each generation.

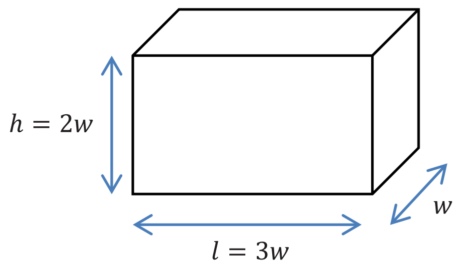
Generation	Number of bacteria	Number of bacteria written as a power of 2
1	2	2^1
2	4	2^2
3	8	2^3
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		

b. How many generations would it take until there were over one million bacteria present?

c. Under the right growing conditions, many bacteria can reproduce every 15 minutes. Under these conditions, how long would it take for one bacterium to reproduce itself into more than one million bacteria?

d. Write an expression for how many bacteria would be present after g generations.

Example 3: Volume of a Rectangular Solid



This box has a width, w. The height of the box, h, is twice the width. The length of the box, l, is three times the width. That is, the width, height, and length of a rectangular prism are in the ratio of $1:2:3$.

For rectangular solids like this, the volume is calculated by multiplying length times width times height.

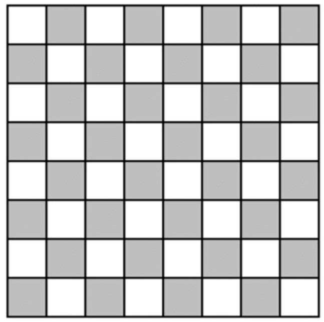
$$V = l \cdot w \cdot h$$
$$V = 3w \cdot w \cdot 2w$$
$$V = 3 \cdot 2 \cdot w \cdot w \cdot w$$
$$V = 6\,w^3$$

Follow the above example to calculate the volume of these rectangular solids, given the width, w.

Width in centimeters (cm)	Volume in cubic centimeters (cm^3)
1	
2	
3	
4	
w	

Problem Set

1. A checkerboard has 64 squares on it.



   a. If one grain of rice is put on the first square, 2 grains of rice on the second square, 4 grains of rice on the third square, 8 grains of rice on the fourth square, etc. (doubling each time), complete the table to show how many grains of rice are on each square. Write your answers in exponential form on the table below.

Checkerboard Square	Grains of Rice	Checkerboard Square	Grains of Rice	Checkerboard Square	Grains of Rice	Checkerboard Square	Grains of Rice
1		17		33		49	
2		18		34		50	
3		19		35		51	
4		20		36		52	
5		21		37		53	
6		22		38		54	
7		23		39		55	
8		24		40		56	
9		25		41		57	
10		26		42		58	
11		27		43		59	
12		28		44		60	
13		29		45		61	
14		30		46		62	
15		31		47		63	
16		32		48		64	

   b. How many grains of rice would be on the last square? Represent your answer in exponential form and standard form. Use the table above to help solve the problem.

   c. Would it have been easier to write your answer to part (b) in exponential form or standard form?

2. If an amount of money is invested at an annual interest rate of 6%, it doubles every 12 years. If Alejandra invests \$500, how long will it take for her investment to reach \$2,000 (assuming she doesn't contribute any additional funds)?

3. The athletics director at Peter's school has created a phone tree that is used to notify team players in the event a game has to be canceled or rescheduled. The phone tree is initiated when the director calls two captains. During the second stage of the phone tree, the captains each call two players. During the third stage of the phone tree, these players each call two other players. The phone tree continues until all players have been notified. If there are 50 players on the teams, how many stages will it take to notify all of the players?

Lesson 23: True and False Number Sentences

Classwork

Opening Exercise

Determine what each symbol stands for and provide an example.

Symbol	What the Symbol Stands For	Example
$=$		
$>$		
$<$		
$\leq$		
$\geq$		

Example 1

For each equation or inequality your teacher displays, write the equation or inequality, and then substitute 3 for every x. Determine if the equation or inequality results in a true number sentence or a false number sentence.

Exercises

Substitute the indicated value into the variable, and state (in a complete sentence) whether the resulting number sentence is true or false. If true, find a value that would result in a false number sentence. If false, find a value that would result in a true number sentence.

1. $4 + x = 12$. Substitute 8 for x.

2. $3g > 15$. Substitute $4\frac{1}{2}$ for g.

3. $\frac{f}{4} < 2$. Substitute 8 for f.

4. $14.2 \leq h - 10.3$. Substitute 25.8 for h.

5. $4 = \frac{8}{h}$. Substitute 6 for h.

6. $3 > k + \frac{1}{4}$. Substitute $1\frac{1}{2}$ for k.

7. $4.5 - d > 2.5$. Substitute 2.5 for d.

8. $8 \geq 32p$. Substitute $\frac{1}{2}$ for p.

9. $\frac{w}{2} < 32$. Substitute 16 for w.

10. $18 \leq 32 - b$. Substitute 14 for b.

Lesson Summary

Number Sentence: A *number sentence* is a statement of equality (or inequality) between two numerical expressions.

Truth Values of a Number Sentence: A number sentence that is an equation is said to be *true* if both numerical expressions evaluate to the same number; it is said to be *false* otherwise. True and false are called *truth values*.

Number sentences that are inequalities also have truth values. For example, $3 < 4$, $6 + 8 > 15 - 12$, and $(15 + 3)^2 < 1000 - 32$ are all true number sentences, while the sentence $9 > 3(4)$ is false.

Problem Set

Substitute the value into the variable, and state (in a complete sentence) whether the resulting number sentence is true or false. If true, find a value that would result in a false number sentence. If false, find a value that would result in a true number sentence.

1. $3\frac{5}{6} = 1\frac{2}{3} + h$. Substitute $2\frac{1}{6}$ for h.

2. $39 > 156g$. Substitute $\frac{1}{4}$ for g.

3. $\frac{f}{4} \leq 3$. Substitute 12 for f.

4. $121 - 98 \geq r$. Substitute 23 for r.

5. $\frac{54}{q} = 6$. Substitute 10 for q.

Create a number sentence using the given variable and symbol. The number sentence you write must be true for the given value of the variable.

6. Variable: d Symbol: $\geq$ The sentence is true when 5 is substituted for d.

7. Variable: y Symbol: $\neq$ The sentence is true when 10 is substituted for y.

8. Variable: k Symbol: $<$ The sentence is true when 8 is substituted for k.

9. Variable: a Symbol: $\leq$ The sentence is true when 9 is substituted for a.

Lesson 24: True and False Number Sentences

Classwork

Opening Exercise

State whether each number sentence is true or false. If the number sentence is false, explain why.

a. $4 + 5 > 9$

b. $3 \cdot 6 = 18$

c. $32 > \frac{64}{4}$

d. $78 - 15 < 68$

e. $22 \geq 11 + 12$

Example 1

Write true or false if the number substituted for g results in a true or false number sentence.

Substitute g with	$4g = 32$	$g = 8$	$3g \geq 30$	$g \geq 10$	$\frac{g}{2} > 2$	$g > 4$	$30 \geq 38 - g$	$g \geq 8$
8								
4								
2								
0								
10								

Example 2

State when the following equations/inequalities will be true and when they will be false.

a. $r + 15 = 25$

b. $6 - d > 0$

c. $\frac{1}{2}f = 15$

d. $\frac{y}{3} < 10$

e. $7g \geq 42$

f. $a - 8 \leq 15$

Exercises

Complete the following problems in pairs. State when the following equations and inequalities will be true and when they will be false.

1. $15c > 45$

2. $25 = d - 10$

3. $56 \geq 2e$

4. $\frac{h}{5} \geq 12$

5. $45 > h + 29$

6. $4a \leq 16$

7. $3x = 24$

Identify all equality and inequality signs that can be placed into the blank to make a true number sentence.

8. $15 + 9$ ______ 24

9. $8 \cdot 7$ _______ 50

10. $\frac{15}{2}$ _______ 10

11. 34 _______ $17 \cdot 2$

12. 18 _______ $24.5 - 6$

Problem Set

State when the following equations and inequalities will be true and when they will be false.

1. $36 = 9k$

2. $67 > f - 15$

3. $\frac{v}{9} = 3$

4. $10 + b > 42$

5. $d - 8 \geq 35$

6. $32f < 64$

7. $10 - h \leq 7$

8. $42 + 8 \geq g$

9. $\frac{m}{3} = 14$

Lesson 25: Finding Solutions to Make Equations True

Classwork

Opening Exercise

Identify a value for the variable that would make each equation or inequality into a true number sentence. Is this the only possible answer? State when the equation or inequality is true using equality and inequality symbols.

a. $3 + g = 15$

b. $30 > 2d$

c. $\frac{15}{f} < 5$

d. $42 \leq 50 - m$

Example 1

Each of the following numbers, if substituted for the variable, makes one of the equations below into a true number sentence. Match the number to that equation: 3, 6, 15, 16, 44.

a. $n + 26 = 32$

b. $n - 12 = 32$

c. $17n = 51$

d. $4^2 = n$

e. $\frac{n}{3} = 5$

Lesson Summary

VARIABLE: A *variable* is a symbol (such as a letter) that represents a number (i.e., it is a placeholder for a number).

A variable is a placeholder for "a number" that does not "vary."

EXPRESSION: An *expression* is a numerical expression or a result of replacing some (or all) of the numbers in a numerical expression with variables.

EQUATION: An *equation* is a statement of equality between two expressions.

If A and B are two expressions in the variable x, then $A = B$ is an equation in the variable x.

Problem Set

Find the solution to each equation.

1. $4^3 = y$
2. $8a = 24$
3. $32 = g - 4$
4. $56 = j + 29$
5. $\frac{48}{r} = 12$
6. $k = 15 - 9$
7. $x \cdot \frac{1}{5} = 60$
8. $m + 3.45 = 12.8$
9. $a = 1^5$

Lesson 26: One-Step Equations—Addition and Subtraction

Classwork

Exercise 1

Solve each equation. Use both tape diagrams and algebraic methods for each problem. Use substitution to check your answers.

a. $b + 9 = 15$

b. $12 = 8 + c$

Exercise 2

Given the equation $d - 5 = 7$:

a. Demonstrate how to solve the equation using tape diagrams.

b. Demonstrate how to solve the equation algebraically.

c. Check your answer.

Exercise 3

Solve each problem, and show your work. You may choose which method (tape diagrams or algebraically) you prefer. Check your answers after solving each problem.

a. $e + 12 = 20$

b. $f - 10 = 15$

c. $g - 8 = 9$

Problem Set

1. Find the solution to the equation below using tape diagrams. Check your answer.

$$m - 7 = 17$$

2. Find the solution of the equation below algebraically. Check your answer.

$$n + 14 = 25$$

3. Find the solution of the equation below using tape diagrams. Check your answer.

$$p + 8 = 18$$

4. Find the solution to the equation algebraically. Check your answer.

$$g - 62 = 14$$

5. Find the solution to the equation using the method of your choice. Check your answer.

$$m + 108 = 243$$

6. Identify the mistake in the problem below. Then, correct the mistake.

$$p - 21 = 34$$
$$p - 21 - 21 = 34 - 21$$
$$p = 13$$

7. Identify the mistake in the problem below. Then, correct the mistake.

$$q + 18 = 22$$
$$q + 18 - 18 = 22 + 18$$
$$q = 40$$

8. Match the equation with the correct solution on the right.

$r + 10 = 22$	$r = 10$
$r - 15 = 5$	$r = 20$
$r - 18 = 14$	$r = 12$
$r + 5 = 15$	$r = 32$

Lesson 27: One-Step Equations—Multiplication and Division

Classwork

Example 1

Solve $3z = 9$ using tape diagrams and algebraically. Then, check your answer.

First, draw two tape diagrams, one to represent each side of the equation.

If 9 had to be split into three groups, how big would each group be?

Demonstrate the value of z using tape diagrams.

How can we demonstrate this algebraically?

How does this get us the value of z?

How can we check our answer?

Example 2

Solve $\frac{y}{4} = 2$ using tape diagrams and algebraically. Then, check your answer.

First, draw two tape diagrams, one to represent each side of the equation.

If the first tape diagram shows the size of $y \div 4$, how can we draw a tape diagram to represent y?

Draw this tape diagram.

What value does each $y \div 4$ section represent? How do you know?

How can you use a tape diagram to show the value of y?

How can we demonstrate this algebraically?

How does this help us find the value of y?

How can we check our answer?

Exercises

1. Use tape diagrams to solve the following problem: $3m = 21$.

2. Solve the following problem algebraically: $15 = \frac{n}{5}$.

3. Calculate the solution of the equation using the method of your choice: $4p = 36$.

4. Examine the tape diagram below, and write an equation it represents. Then, calculate the solution to the equation using the method of your choice.

70

q	q	q	q	q	q	q

5. Write a multiplication equation that has a solution of 12. Use tape diagrams to prove that your equation has a solution of 12.

6. Write a division equation that has a solution of 12. Prove that your equation has a solution of 12 using algebraic methods.

Problem Set

1. Use tape diagrams to calculate the solution of $30 = 5w$. Then, check your answer.

2. Solve $12 = \frac{x}{4}$ algebraically. Then, check your answer.

3. Use tape diagrams to calculate the solution of $\frac{y}{5} = 15$. Then, check your answer.

4. Solve $18z = 72$ algebraically. Then, check your answer.

5. Write a division equation that has a solution of 8. Prove that your solution is correct by using tape diagrams.

6. Write a multiplication equation that has a solution of 8. Solve the equation algebraically to prove that your solution is correct.

7. When solving equations algebraically, Meghan and Meredith each got a different solution. Who is correct? Why did the other person not get the correct answer?

Meghan	**Meredith**
$\frac{y}{2} = 4$	$\frac{y}{2} = 4$
$\frac{y}{2} \cdot 2 = 4 \cdot 2$	$\frac{y}{2} \div 2 = 4 \div 2$
$y = 8$	$y = 2$

Lesson 28: Two-Step Problems—All Operations

Classwork

Mathematical Modeling Exercise

Juan has gained 20 lb. since last year. He now weighs 120 lb. Rashod is 15 lb. heavier than Diego. If Rashod and Juan weighed the same amount last year, how much does Diego weigh? Let j represent Juan's weight last year in pounds, and let d represent Diego's weight in pounds.

Draw a tape diagram to represent Juan's weight.

Draw a tape diagram to represent Rashod's weight.

Draw a tape diagram to represent Diego's weight.

What would combining all three tape diagrams look like?

Write an equation to represent Juan's tape diagram.

Write an equation to represent Rashod's tape diagram.

How can we use the final tape diagram or the equations above to answer the question presented?

Calculate Diego's weight.

We can use identities to defend our thought that $d + 35 - 35 = d$.

Does your answer make sense?

Example 1

Marissa has twice as much money as Frank. Christina has $\$20$ more than Marissa. If Christina has $\$100$, how much money does Frank have? Let f represent the amount of money Frank has in dollars and m represent the amount of money Marissa has in dollars.

Draw a tape diagram to represent the amount of money Frank has.

Draw a tape diagram to represent the amount of money Marissa has.

Draw a tape diagram to represent the amount of money Christina has.

Which tape diagram provides enough information to determine the value of the variable m?

Write and solve the equation.

The identities we have discussed throughout the module solidify that $m + 20 - 20 = m$.

What does the 80 represent?

Now that we know Marissa has $\$80$, how can we use this information to find out how much money Frank has?

Write an equation.

Solve the equation.

Once again, the identities we have used throughout the module can solidify that $2f \div 2 = f$.

What does the 40 represent?

Does 40 make sense in the problem?

<u>Station One</u>: Use tape diagrams to solve the problem.

Raeana is twice as old as Madeline, and Laura is 10 years older than Raeana. If Laura is 50 years old, how old is Madeline? Let m represent Madeline's age in years, and let r represent Raeana's age in years.

Station Two: Use tape diagrams to solve the problem.

Carli has 90 apps on her phone. Braylen has half the amount of apps as Theiss. If Carli has three times the amount of apps as Theiss, how many apps does Braylen have? Let b represent the number of Braylen's apps and t represent the number of Theiss's apps.

<u>Station Three</u>: Use tape diagrams to solve the problem.

Reggie ran for 180 yards during the last football game, which is 40 more yards than his previous personal best. Monte ran 50 more yards than Adrian during the same game. If Monte ran the same amount of yards Reggie ran in one game for his previous personal best, how many yards did Adrian run? Let r represent the number of yards Reggie ran during his previous personal best and a represent the number of yards Adrian ran.

<u>Station Four</u>: Use tape diagrams to solve the problem.

Lance rides his bike downhill at a pace of 60 miles per hour. When Lance is riding uphill, he rides 8 miles per hour slower than on flat roads. If Lance's downhill speed is 4 times faster than his flat road speed, how fast does he travel uphill? Let f represent Lance's pace on flat roads in miles per hour and u represent Lance's pace uphill in miles per hour.

Problem Set

Use tape diagrams to solve each problem.

1. Dwayne scored 55 points in the last basketball game, which is 10 points more than his previous personal best. Lebron scored 15 points more than Chris in the same game. Lebron scored the same number of points as Dwayne's previous personal best. Let d represent the number of points Dwayne scored during his previous personal best and c represent the number of Chris's points.
   a. How many points did Chris score during the game?
   b. If these are the only three players who scored, what was the team's total number of points at the end of the game?

2. The number of customers at Yummy Smoothies varies throughout the day. During the lunch rush on Saturday, there were 120 customers at Yummy Smoothies. The number of customers at Yummy Smoothies during dinner time was 10 customers fewer than the number during breakfast. The number of customers at Yummy Smoothies during lunch was 3 times more than during breakfast. How many people were at Yummy Smoothies during breakfast? How many people were at Yummy Smoothies during dinner? Let d represent the number of customers at Yummy Smoothies during dinner and b represent the number of customers at Yummy Smoothies during breakfast.

3. Karter has 24 t-shirts. Karter has 8 fewer pairs of shoes than pairs of pants. If the number of t-shirts Karter has is double the number of pants he has, how many pairs of shoes does Karter have? Let p represent the number of pants Karter has and s represent the number of pairs of shoes he has.

4. Darnell completed 35 push-ups in one minute, which is 8 more than his previous personal best. Mia completed 6 more push-ups than Katie. If Mia completed the same amount of push-ups as Darnell completed during his previous personal best, how many push-ups did Katie complete? Let d represent the number of push-ups Darnell completed during his previous personal best and k represent the number of push-ups Katie completed.

5. Justine swims freestyle at a pace of 150 laps per hour. Justine swims breaststroke 20 laps per hour slower than she swims butterfly. If Justine's freestyle speed is three times faster than her butterfly speed, how fast does she swim breaststroke? Let b represent Justine's butterfly speed in laps per hour and r represent Justine's breaststroke speed in laps per hour.

Lesson 29: Multi-Step Problems—All Operations

Classwork

Example 1

The school librarian, Mr. Marker, knows the library has $1{,}400$ books but wants to reorganize how the books are displayed on the shelves. Mr. Marker needs to know how many fiction, nonfiction, and resource books are in the library. He knows that the library has four times as many fiction books as resource books and half as many nonfiction books as fiction books. If these are the only types of books in the library, how many of each type of book are in the library?

Draw a tape diagram to represent the total number of books in the library.

Draw two more tape diagrams, one to represent the number of fiction books in the library and one to represent the number of resource books in the library.

- Resource Books:
- Fiction Books:

What variable should we use throughout the problem?

Write the relationship between resource books and fiction books algebraically.

Draw a tape diagram to represent the number of nonfiction books.

How did you decide how many sections this tape diagram would have?

Represent the number of nonfiction books in the library algebraically.

Use the tape diagrams we drew to solve the problem.

Write an equation that represents the tape diagram.

Determine the value of r.

How many fiction books are in the library?

How many nonfiction books are in the library?

Set up a table with four columns and label each column.

How many fiction books are in the library?

How many nonfiction books are in the library?

How many resource books are in the library?

Does the library have four times as many fiction books as resource books?

Does the library have half as many nonfiction books as fiction books?

Does the library have $1{,}400$ books?

Exercises 1–4

Solve each problem below using tables and algebraic methods. Then, check your answer with the word problem.

1. Indiana Ridge Middle School wanted to add a new school sport, so they surveyed the students to determine which sport is most popular. Students were able to choose among soccer, football, lacrosse, or swimming. The same number of students chose lacrosse and swimming. The number of students who chose soccer was double the number of students who chose lacrosse. The number of students who chose football was triple the number of students who chose swimming. If 434 students completed the survey, how many students chose each sport?

2. At Prairie Elementary School, students are asked to pick their lunch ahead of time so the kitchen staff will know what to prepare. On Monday, 6 times as many students chose hamburgers as chose salads. The number of students who chose lasagna was one third the number of students who chose hamburgers. If 225 students ordered lunch, how many students chose each option if hamburger, salad, and lasagna were the only three options?

3. The art teacher, Mr. Gonzalez, is preparing for a project. In order for students to have the correct supplies, Mr. Gonzalez needs 10 times more markers than pieces of construction paper. He needs the same number of bottles of glue as pieces of construction paper. The number of scissors required for the project is half the number of pieces of construction paper. If Mr. Gonzalez collected 400 items for the project, how many of each supply did he collect?

4. The math teacher, Ms. Zentz, is buying appropriate math tools to use throughout the year. She is planning on buying twice as many rulers as protractors. The number of calculators Ms. Zentz is planning on buying is one quarter of the number of protractors. If Ms. Zentz buys 65 items, how many protractors does Ms. Zentz buy?

Problem Set

Solve the problems, and then check your answers with the word problem.

1. On average, a baby uses three times the number of large diapers as small diapers and double the number of medium diapers as small diapers.
   a. If the average baby uses $2,940$ diapers, size large and small, how many of each size would be used?
   b. Support your answer with equations.

2. Tom has three times as many pencils as pens but has a total of 100 writing utensils.
   a. How many pencils does Tom have?
   b. How many more pencils than pens does Tom have?

3. Serena's mom is planning her birthday party. She bought balloons, plates, and cups. Serena's mom bought twice as many plates as cups. The number of balloons Serena's mom bought was half the number of cups.
   a. If Serena's mom bought 84 items, how many of each item did she buy?
   b. Tammy brought 12 balloons to the party. How many total balloons were at Serena's birthday party?
   c. If half the plates and all but four cups were used during the party, how many plates and cups were used?

4. Elizabeth has a lot of jewelry. She has four times as many earrings as watches but half the number of necklaces as earrings. Elizabeth has the same number of necklaces as bracelets.
   a. If Elizabeth has 117 pieces of jewelry, how many earrings does she have?
   b. Support your answer with an equation.

5. Claudia was cooking breakfast for her entire family. She made double the amount of chocolate chip pancakes as she did regular pancakes. She only made half as many blueberry pancakes as she did regular pancakes. Claudia also knows her family loves sausage, so she made triple the amount of sausage as blueberry pancakes.
   a. How many of each breakfast item did Claudia make if she cooked 90 items in total?
   b. After everyone ate breakfast, there were 4 chocolate chip pancakes, 5 regular pancakes, 1 blueberry pancake, and no sausage left. How many of each item did the family eat?

6. During a basketball game, Jeremy scored triple the number of points as Donovan. Kolby scored double the number of points as Donovan.
   a. If the three boys scored 36 points, how many points did each boy score?
   b. Support your answer with an equation.

Lesson 30: One-Step Problems in the Real World

Classwork

Opening Exercise

Draw an example of each term, and write a brief description.

Acute

Obtuse

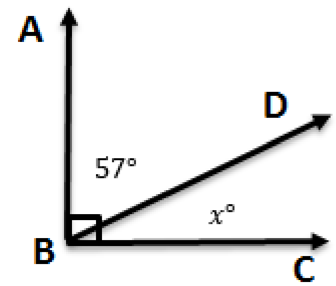
Right

Straight

Reflex

Example 1

$\angle ABC$ measures $90°$. The angle has been separated into two angles. If one angle measures $57°$, what is the measure of the other angle?



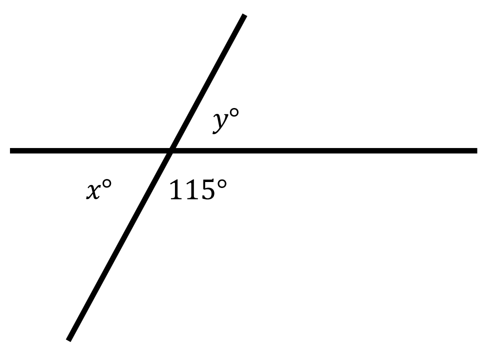
How are these two angles related?

What equation could we use to solve for x?

Now let's solve.

Example 2

Michelle is designing a parking lot. She has determined that one of the angles should be $115°$. What is the measure of angle x and angle y?



How is angle x related to the $115°$ angle?

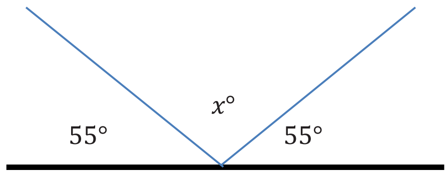
What equation would we use to show this?

How would you solve this equation?

How is angle y related to the angle that measures $115°$?

Example 3

A beam of light is reflected off a mirror. Below is a diagram of the reflected beam. Determine the missing angle measure.



How are the angles in this question related?

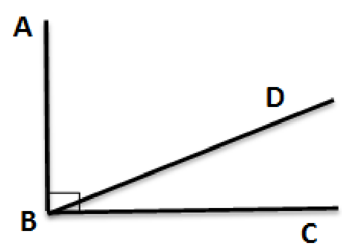
What equation could we write to represent the situation?

How would you solve an equation like this?

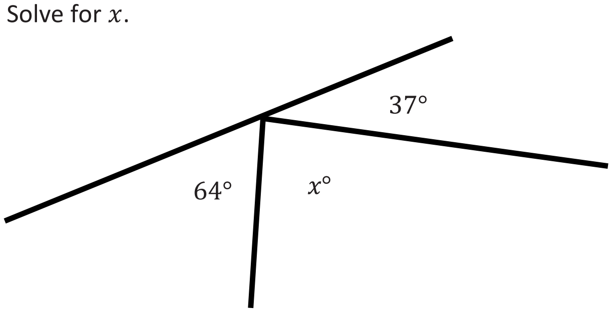
Exercises 1–5

Write and solve an equation in each of the problems.

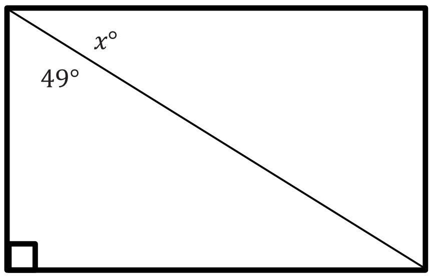
1. $\angle ABC$ measures $90°$. It has been split into two angles, $\angle ABD$ and $\angle DBC$. The measure of the two angles is in a ratio of $2:1$. What are the measures of each angle?



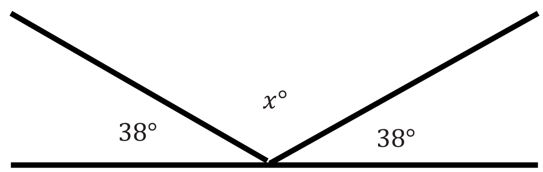
2. Solve for x.



3. Candice is building a rectangular piece of a fence according to the plans her boss gave her. One of the angles is not labeled. Write an equation and use it to determine the measure of the unknown angle.



4. Rashid hit a hockey puck against the wall at a $38°$ angle. The puck hit the wall and traveled in a new direction. Determine the missing angle in the diagram.

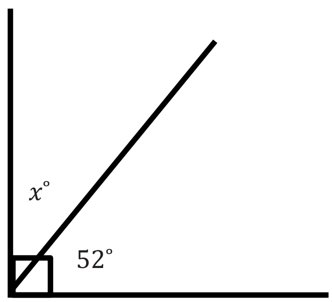


5. Jaxon is creating a mosaic design on a rectangular table. He has added two pieces to one of the corners. The first piece has an angle measuring $38°$ and is placed in the corner. A second piece has an angle measuring $27°$ and is also placed in the corner. Draw a diagram to model the situation. Then, write an equation and use it to determine the measure of the unknown angle in a third piece that could be added to the corner of the table.

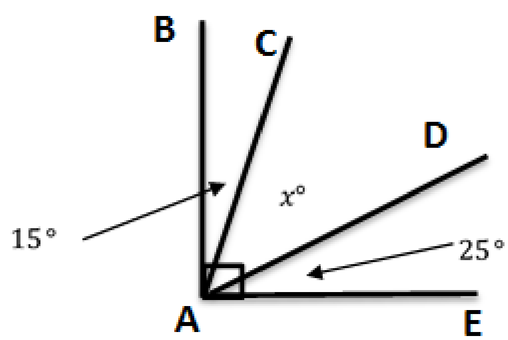
Problem Set

Write and solve an equation for each problem.

1. Solve for x.

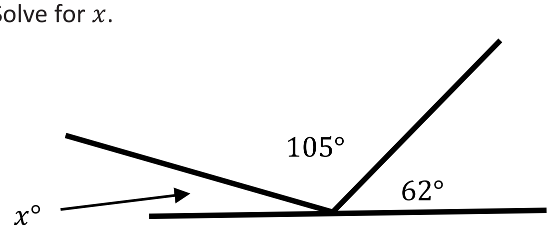


2. $\angle BAE$ measures $90°$. Solve for x.

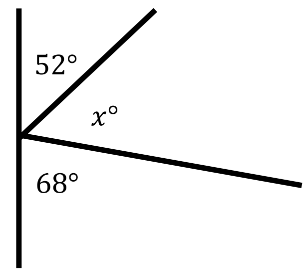


3. Thomas is putting in a tile floor. He needs to determine the angles that should be cut in the tiles to fit in the corner. The angle in the corner measures $90°$. One piece of the tile will have a measure of $24°$. Write an equation, and use it to determine the measure of the unknown angle.

4. Solve for x.



5. Aram has been studying the mathematics behind pinball machines. He made the following diagram of one of his observations. Determine the measure of the missing angle.



6. The measures of two angles have a sum of $90°$. The measures of the angles are in a ratio of $2:1$. Determine the measures of both angles.

7. The measures of two angles have a sum of $180°$. The measures of the angles are in a ratio of $5:1$. Determine the measures of both angles.

Lesson 31: Problems in Mathematical Terms

Classwork

Example 1

Marcus reads for 30 minutes each night. He wants to determine the total number of minutes he will read over the course of a month. He wrote the equation $t = 30d$ to represent the total amount of time that he has spent reading, where t represents the total number of minutes read and d represents the number of days that he read during the month. Determine which variable is independent and which is dependent. Then, create a table to show how many minutes he has read in the first seven days.

Independent variable ___________________________

Dependent variable ___________________________

Example 2

Kira designs websites. She can create three different websites each week. Kira wants to create an equation that will give her the total number of websites she can design given the number of weeks she works. Determine the independent and dependent variables. Create a table to show the number of websites she can design over the first 5 weeks. Finally, write an equation to represent the number of websites she can design when given any number of weeks.

Independent variable ______________________________

Dependent variable ______________________________

Equation ______________________________

Example 3

Priya streams movies through a company that charges her a $\$5$ monthly fee plus $\$1.50$ per movie. Determine the independent and dependent variables, write an equation to model the situation, and create a table to show the total cost per month given that she might stream between 4 and 10 movies in a month.

Independent variable ______________________________

Dependent variable ______________________________

Equation ______________________________

Exercises 1–4

1. Sarah is purchasing pencils to share. Each package has 12 pencils. The equation $n = 12p$, where n is the total number of pencils and p is the number of packages, can be used to determine the total number of pencils Sarah purchased. Determine which variable is dependent and which is independent. Then, make a table showing the number of pencils purchased for 3–7 packages.

2. Charlotte reads 4 books each week. Let b be the number of books she reads each week, and let w be the number of weeks that she reads. Determine which variable is dependent and which is independent. Then, write an equation to model the situation, and make a table that shows the number of books read in under 6 weeks.

3. A miniature golf course has a special group rate. You can pay $\$20$ plus $\$3$ per person when you have a group of 5 or more friends. Let f be the number of friends and c be the total cost. Determine which variable is independent and which is dependent, and write an equation that models the situation. Then, make a table to show the cost for 5 to 12 friends.

4. Carlos is shopping for school supplies. He bought a pencil box for $\$3$, and he also needs to buy notebooks. Each notebook is $\$2$. Let t represent the total cost of the supplies and n be the number of notebooks Carlos buys. Determine which variable is independent and which is dependent, and write an equation that models the situation. Then, make a table to show the cost for 1 to 5 notebooks.

Problem Set

1. Jaziyah sells 3 houses each month. To determine the number of houses she can sell in any given number of months she uses the equation $t = 3m$, where t is the total number of houses sold and m is the number of months. Name the independent and dependent variables. Then, create a table to show how many houses she sells in fewer than 6 months.

2. Joshua spends 25 minutes of each day reading. Let d be the number of days that he reads, and let m represent the total minutes of reading. Determine which variable is independent and which is dependent. Then, write an equation that will model the situation. Make a table showing the number of minutes spent reading over 7 days.

3. Each package of hot dog buns contains 8 buns. Let p be the number of packages of hot dog buns and b be the total number of buns. Determine which variable is independent and which is dependent. Then, write an equation that will model the situation, and make a table showing the number of hot dog buns in 3 to 8 packages.

4. Emma was given 5 seashells. Each week she collected 3 more. Let w be the number of weeks Emma collects seashells and s be the number of seashells she has total. Which variable is independent and which is dependent? Write an equation to model the relationship, and make a table to show how many seashells she has from week 4 to week 10.

5. Emilia is shopping for fresh produce at a farmers' market. She bought a watermelon for $\$5$, and she also wants to buy peppers. Each pepper is $\$0.75$. Let t represent the total cost of the produce and n be the number of peppers bought. Determine which variable is independent and which is dependent, and write an equation that models the situation. Then, make a table to show the cost for 1 to 5 peppers.

6. A taxicab service charges a flat fee of $\$7$ plus an additional $\$1.25$ per mile driven. Show the relationship between the total cost and the number of miles driven. Which variable is independent and which is dependent? Write an equation to model the relationship, and make a table to show the cost of 4 to 10 miles.

Lesson 32: Multi-Step Problems in the Real World

Classwork

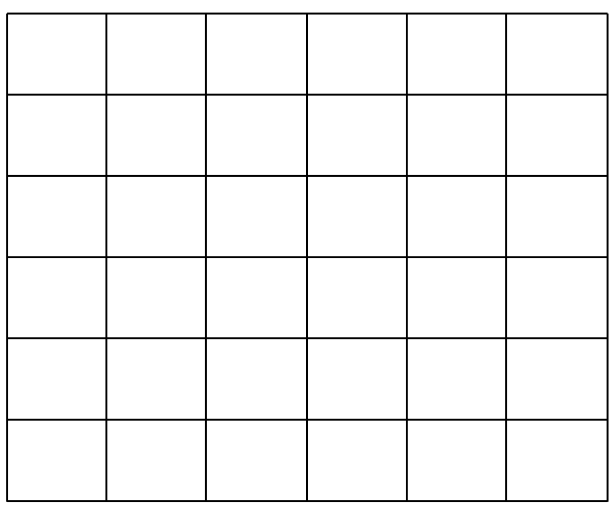
Opening Exercise

Xin is buying beverages for a party that come in packs of 8. Let p be the number of packages Xin buys and t be the total number of beverages. The equation $t = 8p$ can be used to calculate the total number of beverages when the number of packages is known. Determine the independent and dependent variable in this scenario. Then, make a table using whole number values of p less than 6.

Number of Packages (p)	Total Number of Beverages ($t = 8p$)
0	
1	
2	
3	
4	
5	

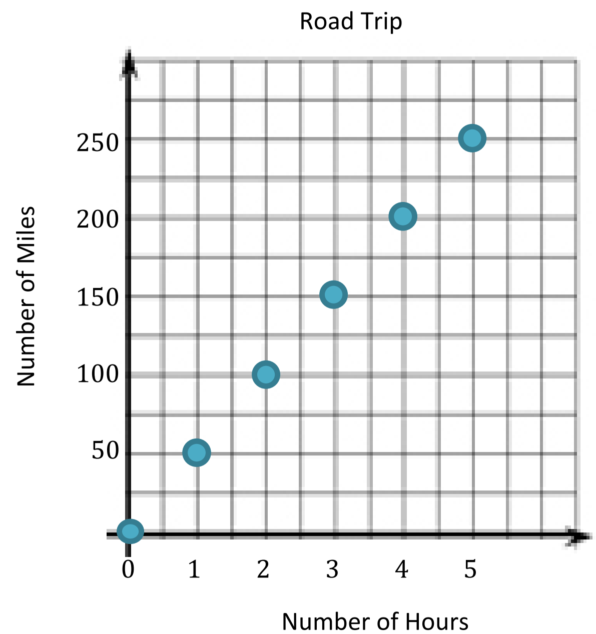
Example 1

Make a graph for the table in the Opening Exercise.



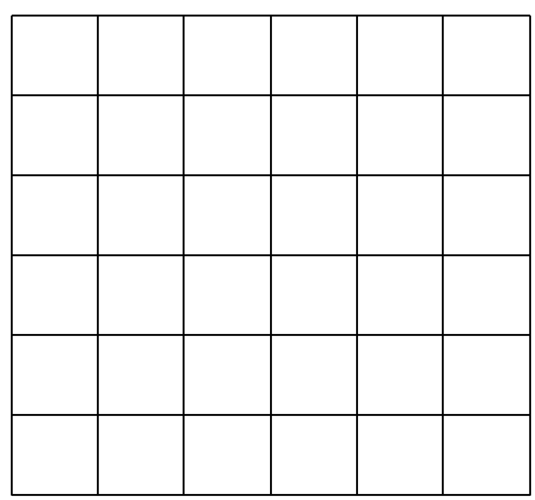
Example 2

Use the graph to determine which variable is the independent variable and which is the dependent variable. Then, state the relationship between the quantities represented by the variables.

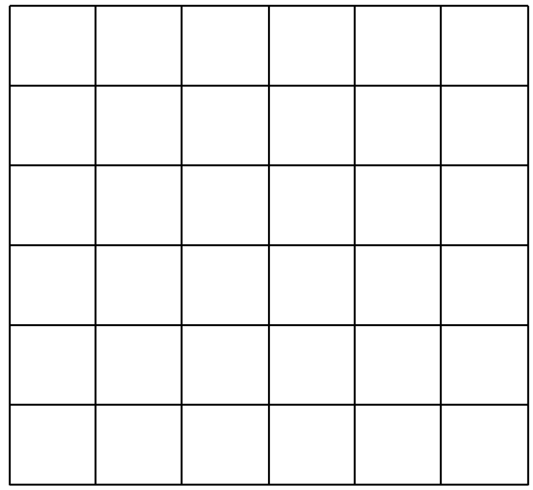


Exercises

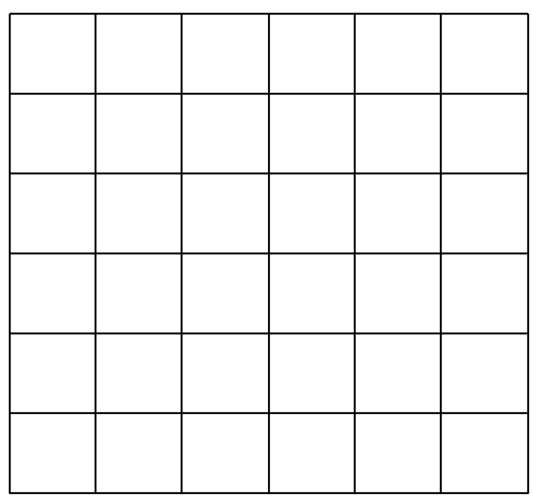
1. Each week Quentin earns $\$30$. If he saves this money, create a graph that shows the total amount of money Quentin has saved from week 1 through week 8. Write an equation that represents the relationship between the number of weeks that Quentin has saved his money, w, and the total amount of money in dollars that he has saved, s. Then, name the independent and dependent variables. Write a sentence that shows this relationship.



2. Zoe is collecting books to donate. She started with 3 books and collects two more each week. She is using the equation $b = 2w + 3$, where b is the total number of books collected and w is the number of weeks she has been collecting books. Name the independent and dependent variables. Then, create a graph to represent how many books Zoe has collected when w is 5 or less.



3. Eliana plans to visit the fair. She must pay $5 to enter the fair grounds and an additional $3 per ride. Write an equation to show the relationship between r, the number of rides, and t, the total cost. State which variable is dependent and which is independent. Then, create a graph that models the equation.



Problem Set

1. Caleb started saving money in a cookie jar. He started with $\$25$. He adds $\$10$ to the cookie jar each week. Write an equation where w is the number of weeks Caleb saves his money and t is the total amount in dollars in the cookie jar. Determine which variable is the independent variable and which is the dependent variable. Then, graph the total amount in the cookie jar for w being less than 6 weeks.

2. Kevin is taking a taxi from the airport to his home. There is a $\$6$ flat fee for riding in the taxi. In addition, Kevin must also pay $\$1$ per mile. Write an equation where m is the number of miles and t is the total cost in dollars of the taxi ride. Determine which variable is independent and which is dependent. Then, graph the total cost for m being less than 6 miles.

3. Anna started with $\$10$. She saved an additional $\$5$ each week. Write an equation that can be used to determine the total amount saved in dollars saved, t, after a given number of weeks, w. Determine which variable is independent and which is dependent. Then, graph the total amount saved for the first 8 weeks.

4. Aliyah is purchasing produce at the farmers' market. She plans to buy $\$10$ worth of potatoes and some apples. The apples cost $\$1.50$ per pound. Write an equation to show the total cost of the produce, where T is the total cost in dollars, and a is the number of pounds of apples. Determine which variable is dependent and independent. Then, graph the equation.

Lesson 33: From Equations to Inequalities

Classwork

Example 1

What value(s) does the variable have to represent for the equation or inequality to result in a true number sentence? What value(s) does the variable have to represent for the equation or inequality to result in a false number sentence?

a. $y + 6 = 16$

b. $y + 6 > 16$

c. $y + 6 \geq 16$

d. $3g = 15$

e. $3g < 15$

f. $3g \leq 15$

Example 2

Which of the following number(s), if any, make the equation or inequality true: $\{0, 3, 5, 8, 10, 14\}$?

a. $m + 4 = 12$

b. $m + 4 < 12$

c. $f - 4 = 2$

d. $f - 4 > 2$

e. $\frac{1}{2}h = 8$

f. $\frac{1}{2}h \geq 8$

Exercises 1–8

Choose the number(s), if any, that make the equation or inequality true from the following set of numbers: $\{0, 1, 5, 8, 11, 17\}$.

1. $m + 5 = 6$

2. $m + 5 \leq 6$

3. $5h = 40$

4. $5h > 40$

5. $\frac{1}{2}y = 5$

6. $\frac{1}{2}y \leq 5$

7. $k - 3 = 20$

8. $k - 3 > 20$

Problem Set

Choose the number(s), if any, that make the equation or inequality true from the following set of numbers: $\{0, 3, 4, 5, 9, 13, 18, 24\}$.

1. $h - 8 = 5$

2. $h - 8 < 5$

3. $4g = 36$

4. $4g \geq 36$

5. $\frac{1}{4}y = 7$

6. $\frac{1}{4}y > 7$

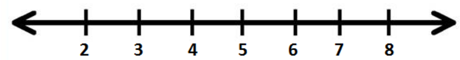
7. $m - 3 = 10$

8. $m - 3 \leq 10$

Lesson 34: Writing and Graphing Inequalities in Real-World Problems

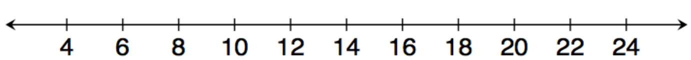
Classwork

Example 1

Statement	Inequality	Graph
a. Caleb has at least $5.	______________	
b. Tarek has more than $5.	______________	
c. Vanessa has at most $5.	______________	
d. Li Chen has less than $5.	______________	

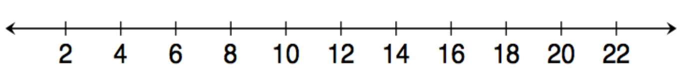
Example 2

Kelly works for Quick Oil Change. If customers have to wait longer than 20 minutes for the oil change, the company does not charge for the service. The fastest oil change that Kelly has ever done took 6 minutes. Show the possible customer wait times in which the company charges the customer.



Example 3

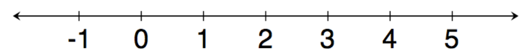
Gurnaz has been mowing lawns to save money for a concert. Gurnaz will need to work for at least six hours to save enough money, but he must work fewer than 16 hours this week. Write an inequality to represent this situation, and then graph the solution.



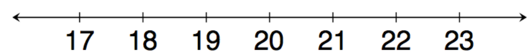
Exercises 1–5

Write an inequality to represent each situation. Then, graph the solution.

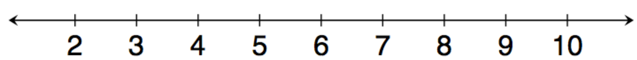
1. Blayton is at most 2 meters above sea level.



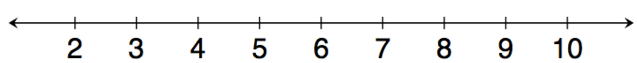
2. Edith must read for a minimum of 20 minutes.



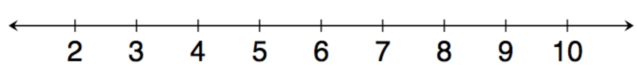
3. Travis milks his cows each morning. He has never gotten fewer than 3 gallons of milk; however, he always gets fewer than 9 gallons of milk.



4. Rita can make 8 cakes for a bakery each day. So far she has orders for more than 32 cakes. Right now, Rita needs more than four days to make all 32 cakes.

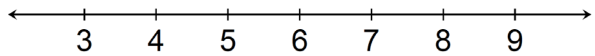


5. Rita must have all the orders placed right now done in 7 days or fewer. How will this change your inequality and your graph?

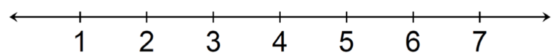


Possible Extension Exercises 6–10

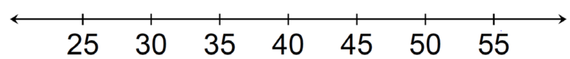
6. Kasey has been mowing lawns to save up money for a concert. He earns $\$15$ per hour and needs at least $\$90$ to go to the concert. How many hours should he mow?



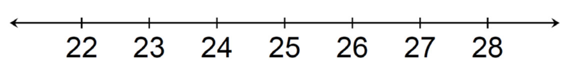
7. Rachel can make 8 cakes for a bakery each day. So far, she has orders for more than 32 cakes. How many days will it take her to complete the orders?



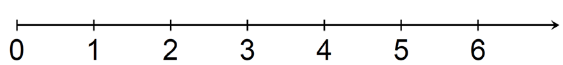
8. Ranger saves $\$70$ each week. He needs to save at least $\$2{,}800$ to go on a trip to Europe. How many weeks will he need to save?



9. Clara has less than $\$75$. She wants to buy 3 pairs of shoes. What price shoes can Clara afford if all the shoes are the same price?



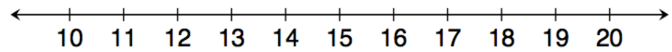
10. A gym charges $\$25$ per month plus $\$4$ extra to swim in the pool for an hour. If a member only has $\$45$ to spend each month, at most how many hours can the member swim?



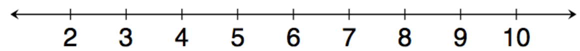
Problem Set

Write and graph an inequality for each problem.

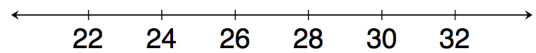
1. At least 13.



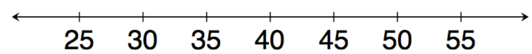
2. Less than 7.



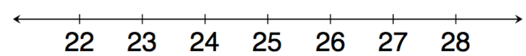
3. Chad will need at least 24 minutes to complete the 5K race. However, he wants to finish in under 30 minutes.



4. Eva saves $\$60$ each week. Since she needs to save at least $\$2{,}400$ to go on a trip to Europe, she will need to save for at least 40 weeks.



5. Clara has $\$100$. She wants to buy 4 pairs of the same pants. Due to tax, Clara can afford pants that are less than $\$25$.



6. A gym charges $\$30$ per month plus $\$4$ extra to swim in the pool for an hour. Because a member has just $\$50$ to spend at the gym each month, the member can swim at most 5 hours.