Finding Limits Algebraically - Classwork

We are going to now determine limits without benefit of looking at a graph, that is $\lim\limits_{x\to a} f(x)$.

There are three steps to remember:
1) plug in a
2) Factor/cancel and go back to step 1
3) ∞, $-\infty$, or DNE

Example 1) find $\lim\limits_{x\to -2} x^2 - 4x + 1$

You can do this by plugging in.

Example 2) find $\lim\limits_{x\to -2} \dfrac{2x-6}{x-2}$

You can also do this by plugging in.

Example 3) find $\lim\limits_{x\to -2} \dfrac{x^2-2x-8}{x^2-4}$

Plug in and you get $\dfrac{0}{0}$ - no good

So attempt to factor and cancel

Example 4) find $\lim\limits_{x\to 1} \dfrac{x^2-2x+1}{x^3-1}$

Plug in and you get $\dfrac{0}{0}$ - no good

So attempt to factor and cancel

If steps 1 and 2 do not work (you have a zero in the denominator, your answer is one of the following:

∞	$-\infty$	**Does Not Exist (DNE)**

To determine which, you must split your limit into two separate limits.: $\lim\limits_{x\to a^-} f(x)$ and $\lim\limits_{x\to a+} f(x)$. Make a sign chart by plugging in a number close to a on the left side and determining its sign. You will also plug in a number close to a on the right side and determine its sign. **Each of these will be some form of ∞, either positive or negative.** Only if they are the same will the limit be ∞ or $-\infty$.

What this says is that in this case, $\lim\limits_{x\to a^-} f(x) =$ some form of ∞ and $\lim\limits_{x\to a+} f(x) =$ some form of ∞

You need to check whether they are the same.

Example 5) find $\lim\limits_{x\to 2} \dfrac{2x+5}{x-2}$

Step 1) Plug in - $\dfrac{9}{0}$ - no good Step 2) - No factoring/cancel So your answer is ∞, $-\infty$ or DNE

Example 6) find $\lim_{x\to 0}\frac{4}{x^2}$

Step 1) Plug in - $\frac{4}{0}$ - no good Step 2) - No factoring/cancel So your answer is ∞, -∞ or DNE

Example 7) find $\lim_{x\to -3}\frac{x^2+2x-3}{x^2+6x+9}$

Example 8) find $\lim_{x\to 2}\frac{2x-4}{x^3-6x^2+12x-8}$

Example 9) $f(x)=\begin{cases} x^2-4, x\geq 1 \\ -2x-1, x<1 \end{cases}$ find $\lim_{x\to 1}f(x)$

Example 10) $f(x)=\begin{cases} \frac{x}{x-2}, x\geq 2 \\ \frac{x-3}{x-2}, x<2 \end{cases}$ find $\lim_{x\to 2}f(x)$

Example 11) $\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$

Finally, we are interested also in problems of the type: $\lim_{x\to\pm\infty}f(x)$. Here are the rules:

Write $f(x)$ as a fraction. 1) If the highest power of x appears in the denominator (bottom heavy), $\lim_{x\to\pm\infty}f(x)=0$

2) If the highest power of x appears in the numerator (top heavy), $\lim_{x\to\pm\infty}f(x)=\pm\infty$

plug in very large or small numbers and determine the sign of the answer

3) If the highest power of x appears both in the numerator and denominator

(powers equal), $\lim_{x\to\pm\infty}f(x)=\frac{\text{coefficient of numerator's highest power}}{\text{coefficient of denominator's highest power}}$

Example 12) $\lim_{x\to\infty}\frac{4x^2+50}{x^3-85}$

Example 13) $\lim_{x\to\infty}\frac{4x^3-5x^2+3x-1}{5x^3-7x-25}$

Example 14) $\lim_{x\to\infty}\frac{3x^3-23}{4x-1}$

Example 15) $\lim_{x\to -\infty}\frac{4x-5x^2+3}{\frac{1}{x}}$

Example 16) $\lim_{x\to\infty}\frac{\sqrt{x^2-3x}}{2x+1}$

Example 17) $\lim_{x\to -\infty}\frac{\sqrt{x^2-3x}}{2x+1}$

Finding Limits Algebraically - Homework

1) $\lim\limits_{x \to 5} 12$

2) $\lim\limits_{x \to 0} \pi$

3) $\lim\limits_{x \to 2} 4x$

4) $\lim\limits_{x \to 5} 3x^2 - 4x - 1$

5) $\lim\limits_{x \to 0^-} 5x^3 - 7x^2 + 2^x - 2$

6) $\lim\limits_{y \to -1} 3y^4 - 6y^3 - 2y$

7) $\lim\limits_{x \to 4} \dfrac{2x-4}{x-1}$

8) $\lim\limits_{x \to -2} \dfrac{x^2+4x+4}{x^2}$

9) $\lim\limits_{x \to 1} \dfrac{2x-2}{x-1}$

10) $\lim\limits_{x \to 4} \dfrac{x^2-16}{x-4}$

11) $\lim\limits_{t \to -2} \dfrac{t^3+8}{t+2}$

12) $\lim\limits_{x \to 2} \dfrac{x^2-4x+4}{x^2+x-6}$

13) $\lim\limits_{x \to -1} \dfrac{x^2+6x+5}{x^2-3x-4}$

14) $\lim\limits_{x \to 1} \dfrac{x^3+x^2-5x+3}{x^3-3x+2}$

15) $\lim\limits_{x \to 3} \dfrac{x}{x-3}$

16) $\lim\limits_{x \to 5} \dfrac{x}{x^2-25}$

17) $\lim\limits_{y \to 6} \dfrac{y+6}{y^2-36}$

18) $\lim\limits_{x \to 4} \dfrac{3-x}{x^2-2x-8}$

19) $\lim\limits_{x \to 1} \dfrac{4}{x^2-2x+1}$

20) $\lim\limits_{x \to 5} \dfrac{x}{|x-5|}$

21) $\lim\limits_{x \to 3} \dfrac{-x^2}{x^2-6x+9}$

22) $f(x)=\begin{cases} x-1, x\ge 3 \\ 2x-3, x<3 \end{cases}$ find $\lim\limits_{x\to 3} f(x)$

23) $f(x)=\begin{cases} x^3-1, x\ge -1 \\ 2x, x<-1 \end{cases}$ find $\lim\limits_{x\to -1} f(x)$

24) $f(x)=\begin{cases} \dfrac{x-2}{x-1}, x\ge 1 \\ \dfrac{x}{x-1}, x<1 \end{cases}$ find $\lim\limits_{x\to 1} f(x)$

25) $\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$

26) Let $f(x)=\begin{cases} x^2-2x-3, x\ne 2 \\ k-3, x=2 \end{cases}$

find k such that $\lim\limits_{x\to 2} f(x)=f(2)$

27) $f(x)=\begin{cases} \dfrac{x^2-49}{x-7}, x\ne 7 \\ k^2-2, x=7 \end{cases}$

find k such that $\lim\limits_{x\to 7} f(x)=f(7)$

28) $\lim\limits_{x\to\infty} 6$

29) $\lim\limits_{x\to\infty} (-2x+11)$

30) $\lim\limits_{x\to-\infty} \left(3x^4-3x^3+5x^2+8x-3\right)$

31) $\lim\limits_{x\to\infty} \dfrac{2x-3}{4x+5}$

32) $\lim\limits_{x\to-\infty} \dfrac{7-3x^3}{2x^3+1}$

33) $\lim\limits_{x\to\infty} \dfrac{2}{5x-3}$

34) $\lim\limits_{x\to-\infty} \dfrac{2x+30}{6x^{12}-5}$

35) $\lim\limits_{x\to\infty} \dfrac{4x^4}{6x^3-19}$

36) $\lim\limits_{x\to-\infty} \dfrac{4x^2-3x-2-5x^3}{9x^2+9x+7}$

37) $\lim\limits_{x\to\infty} \dfrac{x}{\sqrt{x^2+4}}$

38) $\lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2+4}}$

39) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{3x^2+x}}{x^2-1}$