Techniques of Differentiation - Classwork

Taking derivatives is a a process that is vital in calculus. In order to take derivatives, there are rules that will make the process simpler than having to use the definition of the derivative.

1. The constant rule: The derivative of a constant function is 0. That is, if c is a real number, then $\frac{d}{dx}[c]=0$.

a) $y=7$ $\quad y'=$

b) $f(x)=0$ $\quad f'(x)=$

c) $s(t)=-8$ $\quad s'(t)=$

d) $y=a\pi^3$ $\quad \frac{dy}{dx}=$

2. The single variable rule: The derivative of x is 1. $\frac{d}{dx}[x]=1$. This is consistent with the fact that the slope of the line $y=x$ is 1.

a) $y=x$ $\quad y'=$

b) $f(x)=x$ $\quad f'(x)=$

c) $s(t)=t$ $\quad s'(t)=$

3. The power rule: If n is a rational number then the function x^n is differentiable and $\frac{d}{dx}[x^n]=nx^{n-1}$.

Take the derivatives of the following. Use correct notation.

a) $y=x^2$

b) $f(x)=x^6$

c) $s(t)=t^{30}$

d) $y=\sqrt{x}$

e) $y=\frac{1}{x}$

f) $f(x)=\frac{1}{x^3}$

g) $s(t)=\frac{1}{\sqrt[3]{t}}$

h) $y=\frac{1}{x^{3/4}}$

4) The constant multiple rule: If f is a differentiable function and c is a real number, then $\frac{d}{dx}[cf(x)]=cf'(x)$

Take the derivatives of the following. Use correct notation.

a) $y=\frac{2}{x^2}$

b) $f(x)=\frac{4x^3}{3}$

c) $s(t)=-t^5$

d) $y=4\sqrt{x}$

e) $y=\frac{-5}{3x^3}$

f) $f(x)=\frac{-5}{(3x)^3}$

g) $s(t)=\frac{4}{\sqrt{t}}$

h) $y=\frac{-12}{\sqrt[3]{x^5}}$

5. The sum and difference rules. The derivative of a sum or difference is the sum or difference of the derivatives.

$$\frac{d}{dx}\left[f(x)+g(x)\right]=f'(x)+g'(x) \quad \text{and} \quad \frac{d}{dx}\left[f(x)-g(x)\right]=f'(x)-g'(x)$$

Take the derivatives of the following. Use correct notation.

a) $y=x^2+5x-3$

b) $f(x)=x^4-\frac{3}{2}x^3+2x^2+x-6$

c) $y=\frac{4}{x}-\frac{4}{x^2}+\frac{4}{x^3}$

d) $y=6\sqrt{x}\left(\sqrt[3]{x}-2x+6\right)$

e) $f(x)=(2x-3)^2$

f) $y=\left(x^2-x+1\right)^2$

6. <u>The Product Rule:</u> The derivative of the product of two functions is the first times the derivative of the second plus the second times the derivative of the first. $\frac{d}{dx}\left[f(x)\cdot g(x)\right]=f(x)\cdot g'(x)+g(x)\cdot f'(x)$

a) Find y' if $y=\left(4x-2x^2\right)(3x-5)$

without product rule

with product rule

b) Find y' if $y=\left(x^2-x+1\right)^2$

c) Find $f'(x)$ if $f(x)=\left(3x^2-2x+5\right)\left(-5x^4+2x^3-7x^2+x+2\right)$

7. <u>The Quotient Rule</u>: The derivative of the quotient of two functions *f* and *g* can be found using the following:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\cdot f'(x)-f(x)\cdot g'(x)}{\left[g(x)\right]^2}$$

a) Find $\frac{d}{dx}\left[\frac{5x+2}{x^2-1}\right]$

b) Find $\frac{d}{dx}\left[\frac{5x+3}{x^2+4x-2}\right]$

c) Find $\frac{d}{dx}\left[\frac{x^3-4x^2+4x-2}{2x}\right]$

d) Find $\frac{d}{dx}\left[\frac{2x}{x^3-4x^2+4x-2}\right]$

Find an equation of the tangent line to the graph of f at the indicated point and then use your calculator to confirm the results.

a) $y = (x^2 - 4x + 2)(4x - 1)$ at $(1, -3)$

b) $y = \dfrac{x-4}{x^2+3}$ at $\left(2, \dfrac{-2}{7}\right)$

Determine the points at which the graph of the following function has a horizontal tangent.

a) $y = \dfrac{8}{3}x^3 + 5x^2 - 3x - 1$

b) $y = \dfrac{x^2-3}{x^2+1}$

c) $y = \dfrac{x-1}{x^2+3}$

Use the chart to find $f'(3)$

$g(3)$	$g'(3)$	$h(3)$	$h'(3)$
4	-2	3	π

a) $f(x) = 4g(x) - \dfrac{1}{2}h(x) + 1$

b) $f(x) = g(x)h(x)$

c) $f(x) = \dfrac{g(x)}{2h(x)}$

d) $f(x) = \dfrac{g(x) - h(x)}{g(x)}$

The 2nd derivative of a function $y = f(x)$ can be written as $\dfrac{d^2y}{dx^2}$ or $f''(x)$. The 3rd derivative is $\dfrac{d^3y}{dx^3}$ or $f'''(x)$.
The 2nd derivative of a function is the derivative of the derivative of the function.
For each of the following, find $f''(x)$.

a) $f(x) = \dfrac{8}{3}x^3 - 5x^2 - 7x - 1$

b) $f(x) = \dfrac{x^2 + 4x - 2}{x}$

c) $f(x) = \dfrac{x}{x+1}$

d) $f(x) = 4\sqrt{x} - \dfrac{2}{\sqrt{x}}$

Techniques of Differentiation - Homework

For the following functions, find $f'(x)$ and $f'(c)$ at the indicated value of *c*.

1) $f(x) = x^2 - 6x + 1 \quad c = 0$

2) $f(x) = \dfrac{1}{x} - \dfrac{3}{x^2} + \dfrac{4}{x^3} \quad c = 1$

3) $f(x) = 3\sqrt{x} - \dfrac{1}{\sqrt[3]{x}} \quad c = 1$

For the following functions, find the derivative using the power rule.

4) $y = \dfrac{8}{3x^2}$

5) $y = \dfrac{-9}{\left(3x^2\right)^3}$

6) $y = \dfrac{6x^{3/2}}{x}$

7) $y = \dfrac{4x^2 - 5x + 6}{3}$

8) $y = \dfrac{x^2 - 6x + 2}{2x}$

9) $y = \dfrac{x^3 + 8}{x + 2}$

10) $y = x^4 - \dfrac{3}{2}x^3 + 5x^2 - 6x - 2$

11) $y = \dfrac{x^3 - 3x^2 + 10x - 5}{x^2}$

12) $y = \left(x^2 + 4x\right)\left(2x - 1\right)$

13) $y = \left(x - 2\right)^3$

14) $y = \sqrt[3]{x} - \sqrt[3]{x^2}$

15) $y = \dfrac{\left(x^2 - x + 2\right)^2}{x}$

For the following functions, find the derivatives.

16) $y = \left(x^2 - 4x - 6\right)\left(x^3 - 5x^2 - 3x\right)$

17) $y = \dfrac{3x - 2}{2x + 3}$

18) $y = \dfrac{x^2 - 4x - 2}{x^2 - 1}$

19) $y=\dfrac{x-1}{\sqrt{x}}$

20) $y=\dfrac{x^2-x+1}{\sqrt[3]{x}}$

21) $y=\left(\dfrac{x-3}{x+4}\right)(3x-2)$

22) $y=\dfrac{x-1}{x^2+2x+2}$

23) $y=\dfrac{x^2+k^2}{x^2-k^2}$, k is a constant

24) $y=\dfrac{x^2-k^2}{x^2+k^2}$, k a constant

Find an equation of the tangent line to the graph of f at the indicated point and then use your calculator to confirm the results.

25) $f(x)=\dfrac{x^2}{x-1}$ at $(2,4)$

26) $f(x)=(x-2)(x^2-3x-1)$ at $(-1,-9)$

27) $f(x)=\dfrac{x^2-4x+2}{2x-1}$ at $\left(2,-\dfrac{2}{3}\right)$

28) $y=\left(\dfrac{x+3}{x+1}\right)(4x+1)$ at $\left(-\dfrac{1}{2},-5\right)$

Determine the point(s) at which the graph of the following function has a horizontal tangent.

29) $f(x) = \dfrac{x^2}{x^2 - 4}$

30) $f(x) = \dfrac{4x}{x^2 + 4}$

Use the chart to find $h'(4)$

$f(4)$	$f'(4)$	$g(4)$	$g'(4)$
-8	3	3π	4

31) $h(x) = 5f(x) - \dfrac{2}{3}g(x)$

32) $h(x) = 3 + 8f(x)$

33) $h(x) = f(x)g(x)$

34) $h(x) = \dfrac{f(x)}{g(x)}$

35) $h(x) = \dfrac{g(x)}{f(x)}$

36) $h(x) = \dfrac{f(x) + 2}{-3g(x)}$

For each of the following, find $f''(x)$.

37) $f(x) = \dfrac{x^3 - 3x^2 - 4x - 1}{2x}$

38) $f(x) = \dfrac{x}{x - 4}$

39) $f(x) = \sqrt{x} - 4\sqrt[3]{x} + \dfrac{6}{5\sqrt[4]{x}}$

40) Find an equation of the line that is tangent to $f(x) = x^2 - 6x + 7$ and

a) parallel to the line $y = 2x + 4$

b) perpendicular to the line $y = 2x + 4$