Differentiation by the Chain Rule - Classwork

Suppose you were asked to take the derivatives of the following. Could you do so?

a) $f(x) = (2x+5)^2$ b) $f(x) = (2x+5)^3$ c. $f(x) = (2x+5)^6$ d) $f(x) = \sqrt{2x+5}$

a) causes no problem. b) is also not a problem but multiplying it out so you can take the derivative is a bit of a pain. You are capable of doing c) but clearly do not wish to. But d) can't be done with the knowledge you have.

We now introduce a method of taking derivatives of more complicated expressions. It is called the **chain rule**. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentable function of x, then $y = f(g(x))$ is a differentable function of x and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ or equivalently, $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$.

Example 1) If $f(x) = (2x+5)^2$, find $f'(x)$ without and with the chain rule. Show they are equivalent.

a) without chain rule b) with chain rule

Example 2) If $f(x) = (2x+5)^3$, find $f'(x)$ without and with the chain rule. Show they are equivalent.

a) without chain rule b) with chain rule

Example 3) If $f(x) = (2x+5)^{10}$, find $f'(x)$

Example 4) If $f(x) = \sqrt{2x+5}$, find $f'(x)$

Example 5) Find y' if $y = \frac{1}{4x-3}$

Example 6) Find y' if $y = \left(3x^2 - 2x + 1\right)^3$

Find the derivatives of the following:

7) $y = \left(7 - 4x^2\right)^{2/3}$

8) $y = -5\sqrt{x^2 - 4x + 1}$

9) $y = \dfrac{-2}{\sqrt[4]{6x - 1}}$

More difficult problems: We now have 3 basic rules. Power rule, product rule, and quotient rule. Note that the chain rule is not a basic rule of differentiation. The chain rule is always in effect. Even when you find the derivative of $y = 7x$, your answer is 7 times the derivative of x which is 7(1) = 7.

Find the derivatives of the following:

10) $y = x^2(2x - 3)^4$

11) $y = x\sqrt{4 - x^2}$

12) $y = \left(\dfrac{2x - 1}{2x + 1}\right)^5$

13) $y = \dfrac{x}{\sqrt{x^2 - 1}}$

14) $y = \sqrt{\dfrac{x}{4x - 1}}$

15) $y = \left(x^2 - 4\right)\sqrt{x + 2}$

Given that $f(2) = -3, f'(2) = 6, g(2) = 3, g'(2) = -2, f'(3) = 4$, find the derivatives of the following at $x = 2$.

16) $f(x) \cdot g(x)$

17) $\dfrac{f(x)}{g(x)}$

18) $\left[f(x)\right]^3$

19) $f\left(g(x)\right)$

Differentiation by the Chain Rule - Homework

Find the derivatives of the following:

1. $y = (3x - 8)^4$

2. $y = (3x^2 + 2)^5$

3. $y = 4(x^2 + x - 1)^{10}$

4. $y = -5(4 - 9x)^{3/2}$

5. $y = \dfrac{1}{3x - 2}$

6. $y = \dfrac{-1}{(x^2 - 5x - 6)^2}$

7. $y = \left(\dfrac{2}{2 - x}\right)^2$

8. $y = \dfrac{4x}{(x + 1)^2}$

9. $y = \dfrac{-3}{(x^3 - x^2 + 3)^3}$

10. $y = x^3(5x - 1)^4$

11. $y = \sqrt{1 - t}$

12. $y = \sqrt[3]{3x^3 - 4x + 2}$

13. $y = \dfrac{2}{\sqrt{2x + 3}}$

14. $y = \dfrac{-1}{\sqrt{x} + 1}$

15. $y = \sqrt{\dfrac{3x}{2x - 3}}$

16. $y = \sqrt{x}(1-2x)^2$

17. $y = \sqrt[3]{\dfrac{2t}{t^2-4}}$

18. $y = \left(x^2+2x-6\right)^2\left(1-x^3\right)^2$

For each of the following, find the equation of the tangent line at the indicated point. Verify by calculator.

19. $y = \sqrt{x^2+2x+8}$ at $(2,4)$

20. $y = \sqrt[5]{3x^3+4x}$ at $(2,2)$

21. $y = \sqrt{\dfrac{3x-1}{2x+1}}$ at $(-1,2)$

Given the following information, find the value of the derivative of the functions at $x = 3$. Be careful, not all the information is needed to calculate these. Answers are next to the problem.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	1	8	-3	-5
6	3	-2	4	5
8	-1	3	π	4
1	2	-6	5	0

22. $f(x)+g(x)$ (Ans: -8)

23. $f(x)g(x)$ (Ans: -29)

24. $\dfrac{f(x)}{g(x)}$ (Ans: $\dfrac{-19}{64}$)

25. $\dfrac{g(x)}{f(x)}$ (Ans: 19)

26. $\left(f(x)\right)^2$ (Ans: -6)

27. $\dfrac{1}{g(x)}$ (Ans: $\dfrac{5}{64}$)

28. $\sqrt{f(x)}$ (Ans: $\dfrac{-3}{2}$)

29. $\sqrt{f(x)+g(x)}$ (Ans: $\dfrac{-4}{3}$)

30 . $f^3(x)g(x)$ (Ans: -77)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	1	8	-3	-5
6	3	-2	4	5
8	-1	3	π	4
1	2	-6	5	0

31. $\dfrac{1}{\sqrt[3]{g(x)}}$ (Ans: $\dfrac{5}{48}$)

32. $\dfrac{f(x)}{f(x)+g(x)}$ (Ans: $\dfrac{-19}{81}$)

33. $f\big(g(x)\big)$ (Ans: -5π)

34. $g\big(f(x)\big)$ (Ans: 0)

35. $f\big(f(x)\big)$ (Ans: -15)

36. $g\big(g(x)\big)$ (Ans: -20)

37. The table below gives some values of the derivative of some function f. Complete the table by finding (if possible) the derivatives of each of the following transformations of f.

a) $g(x) = f(x) - 2$

b) $h(x) = 2f(x)$

c) $r(x) = f(-3x)$

d) $s(x) = f(2x+1)$

x	**-2**	**-1**	**0**	**1**	**2**	**3**
$f'(x)$	4	$2/3$	$-1/3$	-1	-2	-4
$g'(x)$						
$h'(x)$						
$r'(x)$						
$s'(x)$						