# Lesson 1: An Experience in Relationships as Measuring Rate

## Classwork

### Example 1: How fast is our class?

Record the results from the paper-passing exercise in the table below.

| Trial | Number of Papers Passed | Time (in seconds) | Ratio of Number of Papers Passed to Time | Rate | Unit Rate |
|---|---|---|---|---|---|
| $1$ | | | | | |
| $2$ | | | | | |
| $3$ | | | | | |
| $4$ | | | | | |

**Key Terms from Grade 6 Ratios and Unit Rates**

A **ratio** is an ordered pair of non-negative numbers, which are not both zero. The ratio is denoted $A: B$ to indicate the order of the numbers: the number $A$ is first and the number $B$ is second.

Two ratios $A: B$ and $C: D$ are **equivalent ratios** if there is a positive number, $c$, such that $C = cA$ and $D = cB$.

A ratio of two quantities, such as $5$ miles per $2$ hours, can be written as another quantity called a **rate**.

The numerical part of the rate is called the **unit rate** and is simply the value of the ratio, in this case $2.5$. This means that in $1$ hour the car travels $2.5$ miles. The **unit** for the rate is miles/hour, read miles per hour.

## Example 2: Our Class by Gender

| | Number of boys | Number of girls | Ratio of boys to girls |
|---|---|---|---|
| Class 1 | | | |
| Class 2 | | | |
| Whole 7th Grade | | | |

Create a pair of equivalent ratios by making a comparison of quantities discussed in this Example.

### Exercise 1: Which is the Better Buy?

Value-Mart is advertising a Back-to-School sale on pencils. A pack of 30 sells for $7.97, whereas a 12-pack of the same brand cost for $4.77. Which is the better buy? How do you know?

**Lesson Summary**

**Unit rate** is often a useful means for comparing ratios and their associated rates when measured in different units. The unit rate allows us to compare varying sizes of quantities by examining the number of units of one quantity per one unit of the second quantity. This value of the ratio is the unit rate.

## Problem Set

1. Find each rate and unit rate.
   a. $420$ miles in $7$ hours
   b. $360$ customers in $30$ days
   c. $40$ meters in $16$ seconds
   d. $\$7.96$ for $5$ pounds

2. Write three ratios that are equivalent to the one given: The ratio of right-handed students to left-handed students is $18:4$.

3. Mr. Rowley has $16$ homework papers and $14$ exit tickets to return. Ms. Rivera has $64$ homework papers and $60$ exit tickets to return. For each teacher, write a ratio to represent the number of homework papers to number of exit tickets they have to return. Are the ratios equivalent? Explain.

4. Jonathan's parents told him that for every $5$ hours of homework or reading he completes, he will be able to play $3$ hours of video games. His friend Lucas's parents told their son that he can play $30$ minutes for every hour of homework or reading time he completes. If both boys spend the same amount of time on homework and reading this week, which boy gets more time playing video games? How do you know?

5. Of the $30$ girls who tried out for the lacrosse team at Euclid Middle School, $12$ were selected. Of the $40$ boys who tried out, $16$ were selected. Are the ratios of the number of students on the team to the number of students trying out the same for both boys and girls? How do you know?

6. Devon is trying to find the unit price on a 6-pack of drinks on sale for $2.99. His sister says that at that price, each drink would cost just over $\$2.00$. Is she correct, and how do you know? If she is not, how would Devon's sister find the correct price?

7. Each year Lizzie's school purchases student agenda books, which are sold in the school store. This year, the school purchased $350$ books at a cost of $\$1{,}137.50$. If the school would like to make a profit of $\$1{,}500$ to help pay for field trips and school activities, what is the least amount they can charge for each agenda book? Explain how you found your answer.

# Lesson 2: Proportional Relationships

## Classwork

### Example 1: Pay by the Ounce Frozen Yogurt!

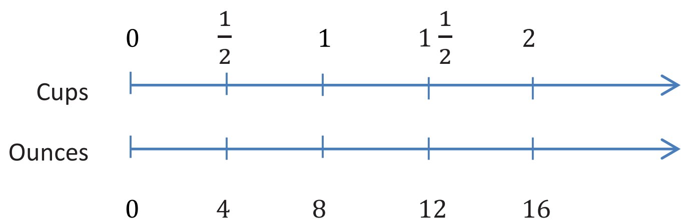
A new self-serve frozen yogurt store opened this summer that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed their dish and this is what they found. Determine if the cost is proportional to the weight.

| Weight (ounces) | 12.5 | 10 | 5 | 8 |
|---|---|---|---|---|
| Cost ($) | 5 | 4 | 2 | 3.20 |

The cost ________________________________________ the weight.

### Example 2: A Cooking Cheat Sheet!

In the back of a recipe book, a diagram provides easy conversions to use while cooking.

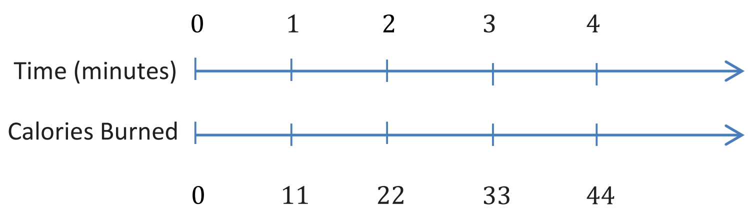



The ounces ________________________________________the cups.

**Exercise 1**

During Jose's physical education class today, students visited activity stations. Next to each station was a chart depicting how many calories (on average) would be burned by completing the activity.

Calories Burned while Jumping Rope




a. Is the number of calories burned proportional to time? How do you know?

b. If Jose jumped rope for 6.5 minutes, how many calories would he expect to burn?

## Example 3: Summer Job

Alex spent the summer helping out at his family's business. He was hoping to earn enough money to buy a new $220 gaming system by the end of the summer. Halfway through the summer, after working for 4 weeks, he had earned $112. Alex wonders, "If I continue to work and earn money at this rate, will I have enough money to buy the gaming system by the end of the summer?"

To determine if he will earn enough money, he decided to make a table. He entered his total money earned at the end of Week 1 and his total money earned at the end of Week 4.

| Week | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Total Earnings | | $28 | | | $112 | | | | |

a. Work with a partner to answer Alex's question.

b. Are Alex's total earnings proportional to the number of weeks he worked? How do you know?

**Lesson Summary**

Measures in one quantity are **proportional to** measures of a second quantity if there is a positive number $k$ so that for every measure $x$ of the first quantity, the corresponding quantity $y$ is given by $kx$. The equation $y = kx$ models this relationship.

A **proportional relationship** is one in which the measures of one quantity are proportional to the measures of the second quantity.

In the example given below, the distance *is proportional to* time since each measure of distance, $y$, can be calculated by multiplying each corresponding time, $t$, by the same value, $10$. This table illustrates a *proportional relationship* between time, $t$, and distance, $y$.

| Time (h), $t$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|
| Distance (km), $y$ | $0$ | $10$ | $20$ | $30$ |

## Problem Set

1. A cran-apple juice blend is mixed in a ratio of cranberry to apple of $3$ to $5$.
   a. Complete the table to show different amounts that are proportional.

| | | | |
|---|---|---|---|
| Amount of Cranberry | | | |
| Amount of Apple | | | |

   b. Why are these quantities proportional?

2. John is filling a bathtub that is $18$ inches deep. He notices that it takes two minutes to fill the tub with three inches of water. He estimates it will take ten more minutes for the water to reach the top of the tub if it continues at the same rate. Is he correct? Explain.

# Lesson 3: Identifying Proportional and Non-Proportional Relationships in Tables

## Classwork

### Example

You have been hired by your neighbors to babysit their children on Friday night. You are paid $8 per hour. Complete the table relating your pay to the number of hours you worked.

| Hours Worked | Pay |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| $4\frac{1}{2}$ | |
| 5 | |
| 6 | |
| 6.5 | |

Based on the table above, is the pay proportional to the hours worked? How do you know?

## Exercises

For Exercises 1–3, determine if $y$ is proportional to $x$. Justify your answer.

1. The table below represents the relationship of the amount of snowfall (in inches) in 5 counties to the amount of time (in hours) hours of a recent winter storm.

| $x$ Time (h) | $y$ Snowfall (in.) |
|---|---|
| 2 | 10 |
| 6 | 12 |
| 8 | 16 |
| 2.5 | 5 |
| 7 | 14 |

2. The table below shows the relationship between the cost of renting a movie (in dollars) to the number of days the movie is rented.

| $x$ Number of Days | $y$ Cost (dollars) |
|---|---|
| 6 | 2 |
| 9 | 3 |
| 24 | 8 |
| 3 | 1 |

3. The table below shows the relationship between the amount of candy bought (in pounds) and the total cost of the candy (in dollars).

| $x$ Amount of Candy (pounds) | $y$ Cost (dollars) |
|---|---|
| 5 | 10 |
| 4 | 8 |
| 6 | 12 |
| 8 | 16 |
| 10 | 20 |

4. Randy is planning to drive from New Jersey to Florida. Every time Randy stops for gas, he records the distance he traveled in miles and the total number of gallons used.

   Assume that the number of miles driven is proportional to the number of gallons consumed in order to complete the table.

| | | | | | | |
|---|---|---|---|---|---|---|
| Gallons Consumed | 2 | 4 | | 8 | 10 | 12 |
| Miles Driven | 54 | | 189 | 216 | | |

### Lesson Summary

One quantity is proportional to a second if a constant (number) exists such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.

Steps to determine if two quantities in a table are proportional to each other:

1. For each given measure of Quantity $A$ and Quantity $B$, find the value of $\frac{B}{A}$.
2. If the value of $\frac{B}{A}$ is the same for each pair of numbers, then the quantities are proportional to each other.

## Problem Set

In each table determine if y is proportional to x. Explain why or why not.

1.

| $x$ | $y$ |
|---|---|
| 3 | 12 |
| 5 | 20 |
| 2 | 8 |
| 8 | 32 |

2.

| $x$ | $y$ |
|---|---|
| 3 | 15 |
| 4 | 17 |
| 5 | 19 |
| 6 | 21 |

3.

| $x$ | $y$ |
|---|---|
| 6 | 4 |
| 9 | 6 |
| 12 | 8 |
| 3 | 2 |

4. Kayla made observations about the selling price of a new brand of coffee that sold in three different sized bags. She recorded those observations in the following table:

| Ounces of Coffee | 6 | 8 | 16 |
|---|---|---|---|
| Price in Dollars | $2.10 | $2.80 | $5.60 |

   a. Is the price proportional to the amount of coffee? Why or why not?

   b. Use the relationship to predict the cost of a 20 oz. bag of coffee?

5. You and your friends go to the movies. The cost of admission is $9.50 per person. Create a table showing the relationship between the number of people going to the movies and the total cost of admission.

   Explain why the cost of admission is proportional to the amount of people.

6. For every 5 pages Gil can read, his daughter can read 3 pages. Let $g$ represent the number of pages Gil reads and let $d$ represent the number of pages his daughter reads. Create a table showing the relationship between the number of pages Gil reads and the number of pages his daughter reads.

   Is the number of pages Gil's daughter reads proportional to the number of pages he reads? Explain why or why not.

7. The table shows the relationship between the number of parents in a household and the number of children in the same household. Is the number of children proportional to the number of parents in the household? Explain why or why not.

| Number of Parents | Number of Children |
|---|---|
| 0 | 0 |
| 1 | 3 |
| 1 | 5 |
| 2 | 4 |
| 2 | 1 |

8. The table below shows the relationship between the number of cars sold and the amount of money earned by the car salesperson. Is the amount of money earned, in dollars, proportional to the number of cars sold? Explain why or why not.

| Number of Cars Sold | Money Earned |
|---|---|
| 1 | 250 |
| 2 | 600 |
| 3 | 950 |
| 4 | 1,076 |
| 5 | 1,555 |

9. Make your own example of a relationship between two quantities that is NOT proportional. Describe the situation and create a table to model it. Explain why one quantity is not proportional to the other.

# Lesson 4: Identifying Proportional and Non-Proportional Relationships in Tables

## Classwork

### Example: Which Team Will Win the Race?

You have decided to walk in a long distance race. There are two teams that you can join. Team A walks at a constant rate of $2.5$ miles per hour. Team B walks $4$ miles the first hour and then $2$ miles per hour after that.

Task: Create a table for each team showing the distances that would be walked for times of $1$, $2$, $3$, $4$, $5$, and $6$ hours. Using your tables, answer the questions that follow.

| Team A | |
|---|---|
| Time (h) | Distance (miles) |
| | |
| | |
| | |
| | |
| | |
| | |

| Team B | |
|---|---|
| Time (h) | Distance (miles) |
| | |
| | |
| | |
| | |
| | |
| | |

a. For which team is distance proportional to time? Explain your reasoning.

b. Explain how you know distance for the other team is not proportional to time.

c. At what distance in the race would it be better to be on Team B than Team A? Explain.

d. If the members on each team ran for $10$ hours, how far would each member run on each team?

e. Will there always be a winning team, no matter what the length of the course? Why or why not?

f. If the race is $12$ miles long, which team should you choose to be on if you wish to win? Why would you choose this team?

g. How much sooner would you finish on that team compared to the other team?

**Exercises**

1. Bella types at a constant rate of $42$ words per minute. Is the number of words she can type proportional to the number of minutes she types? Create a table to determine the relationship.

| Minutes | $1$ | $2$ | $3$ | $6$ | $60$ |
|---|---|---|---|---|---|
| Number of Words | | | | | |

2. Mark recently moved to a new state. During the first month he visited five state parks. Each month after he visited two more. Complete the table below and use the results to determine if the number of parks visited is proportional to the number of months.

| Number of Months | Number of State Parks |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| | 23 |

3. The table below shows the relationship between the side length of a square and the area. Complete the table. Then determine if the length of the sides is proportional to the area.

| Side Length (inches) | Area (square inches) |
|---|---|
| 1 | 1 |
| 2 | 4 |
| 3 | |
| 4 | |
| 5 | |
| 8 | |
| 12 | |

## Problem Set

1. Joseph earns $15 for every lawn he mows. Is the amount of money he earns proportional to the number of lawns he mows? Make a table to help you identify the type of relationship.

| | | | | |
|---|---|---|---|---|
| Number of Lawns Mowed | | | | |
| Earnings ($) | | | | |

2. At the end of the summer, Caitlin had saved $120 from her summer job. This was her initial deposit into a new savings account at the bank. As the school year starts, Caitlin is going to deposit another $5 each week from her allowance. Is her account balance proportional to the number of weeks of deposits? Use the table below. Explain your reasoning.

| | | | | |
|---|---|---|---|---|
| Time (in weeks) | | | | |
| Account Balance ($) | | | | |

3. Lucas and Brianna read three books each last month. The table shows the number of pages in each book and the length of time it took to read the entire book.

| | | | |
|---|---|---|---|
| Pages Lucas Read | 208 | 156 | 234 |
| Time (hours) | 8 | 6 | 9 |

| | | | |
|---|---|---|---|
| Pages Brianna Read | 168 | 120 | 348 |
| Time (hours) | 6 | 4 | 12 |

a. Which of the tables, if any, shows a proportional relationship?

b. Both Lucas and Brianna had specific reading goals they needed to accomplish. What different strategies did each person employ in reaching those goals?

# Lesson 5: Identifying Proportional and Non-Proportional Relationships in Graphs

## Classwork

### Opening Exercise

Isaiah sold candy bars to help raise money for his scouting troop. The table shows the amount of candy he sold compared to the money he received.

| $x$<br>Candy Bars Sold | $y$<br>Money Received ($) |
|---|---|
| 2 | 3 |
| 4 | 5 |
| 8 | 9 |
| 12 | 12 |

Is the amount of candy bars sold proportional to the money Isaiah received? How do you know?

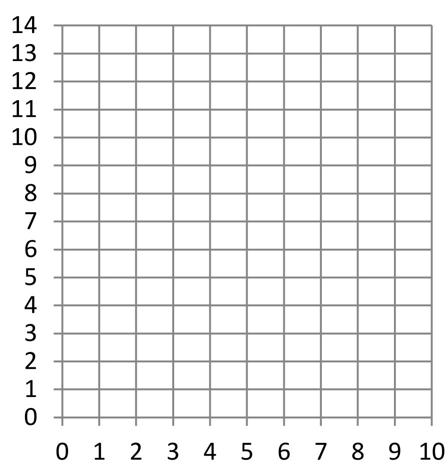
___________________________________________________________________________________________

___________________________________________________________________________________________

### Example 1: From a Table to Graph

Using the ratio provided, create a table that shows money received is proportional to the number of candy bars sold. Plot the points in your table on the grid.

| $x$<br>Candy Bars Sold | $y$<br>Money Received ($) |
|---|---|
| 2 | 3 |
| | |
| | |
| | |



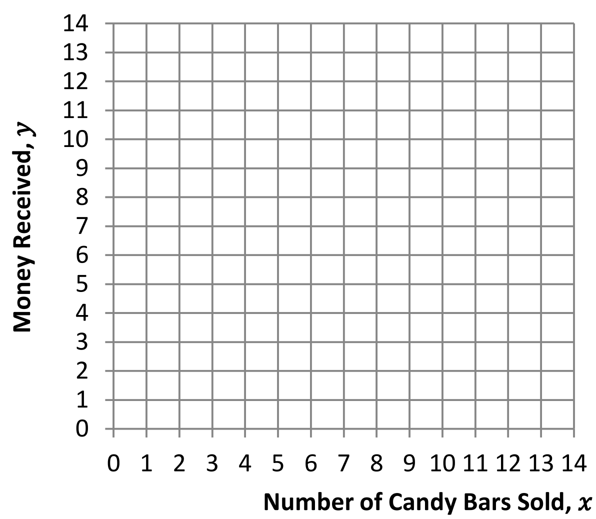
**Important Note:**

Characteristics of graphs of proportional relationships:

## Example 2

Graph the points from the Opening Exercise.

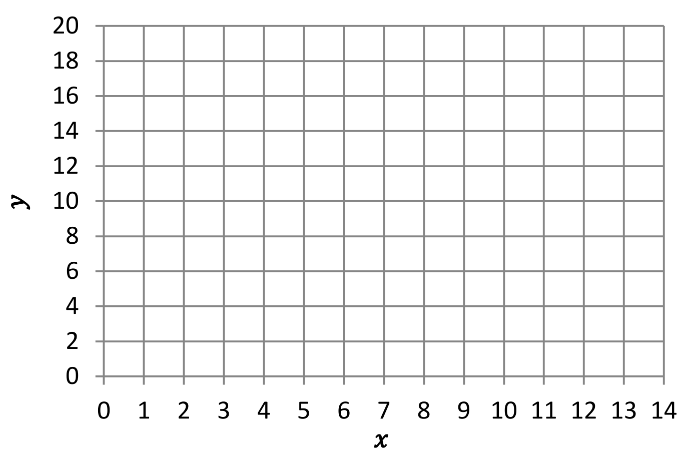
| $x$ Candy Bars Sold | $y$ Money Received ($) |
|---|---|
| 2 | 3 |
| 4 | 6 |
| 8 | 12 |
| 12 | 14 |




## Example 3

Graph the points provided in the table below and describe the similarities and differences when comparing your graph to the graph in Example 1.

| $x$ | $y$ |
|---|---|
| 0 | 6 |
| 3 | 9 |
| 6 | 12 |
| 9 | 15 |
| 12 | 18 |




Similarities with Example 1:

Differences from Example 1:

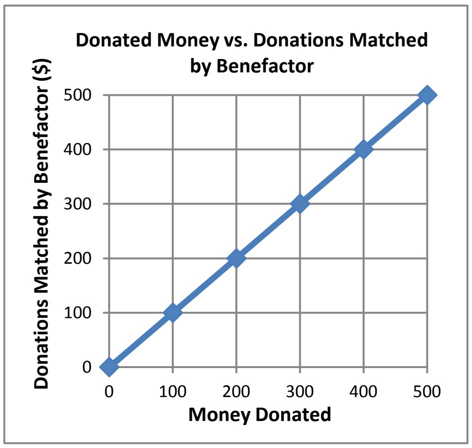
**Lesson Summary**

When two proportional quantities are graphed on a coordinate plane, the points appear on a line that passes through the origin.

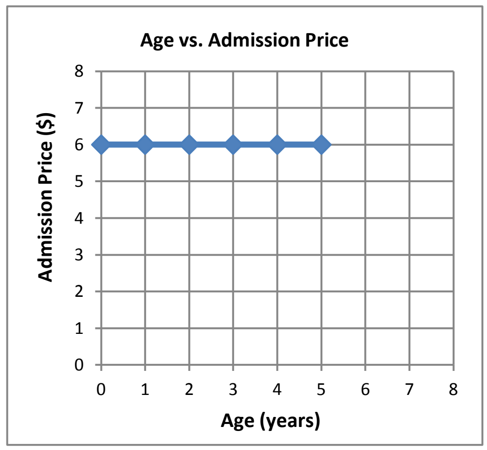
## Problem Set

1. Determine whether or not the following graphs represent two quantities that are proportional to each other. Explain your reasoning.

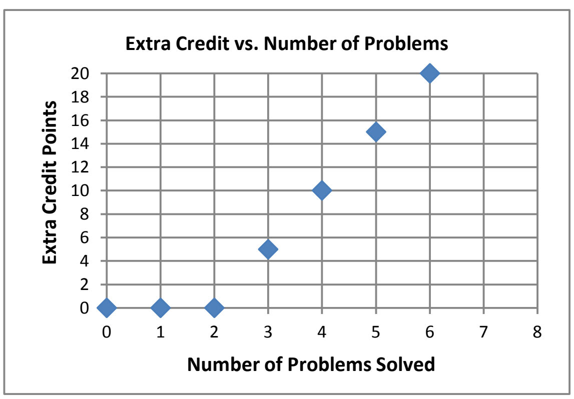
a.




b.

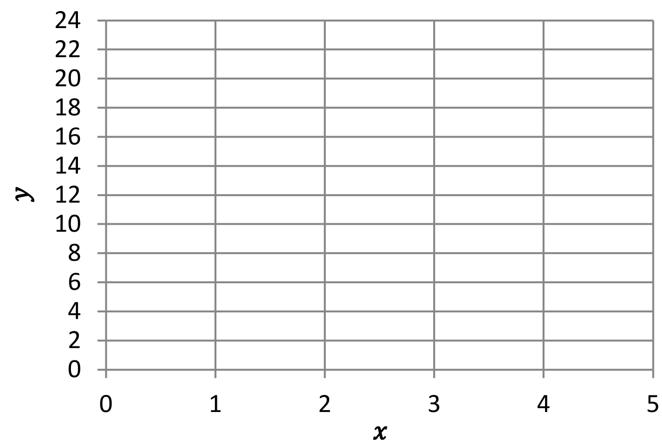



c.



2. Create a table and a graph for the ratios $2:22$, $3$ to $15$, and $1:11$. Does the graph show that the two quantities are proportional to each other? Explain why or why not.

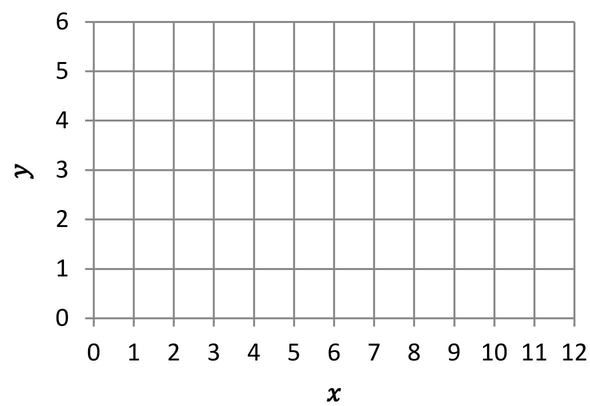
| $x$ | $y$ |
|---|---|
| | |
| | |
| | |




3. Graph the following tables and identify if the two quantities are proportional to each other on the graph. Explain why or why not.

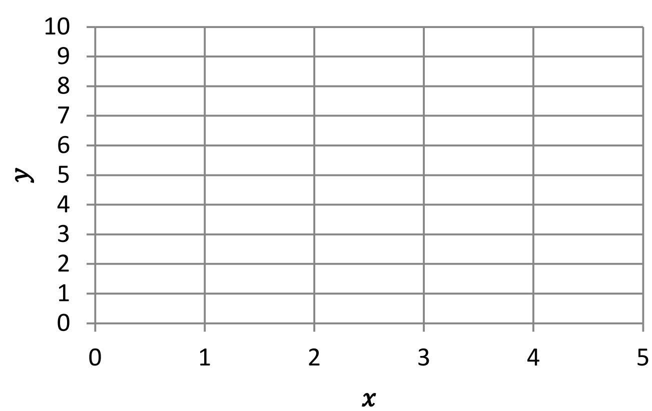
a.

| $x$ | $y$ |
|---|---|
| 3 | 1 |
| 6 | 2 |
| 9 | 3 |
| 12 | 4 |




b.

| $x$ | $y$ |
|---|---|
| 1 | 4 |
| 2 | 5 |
| 3 | 6 |
| 4 | 7 |



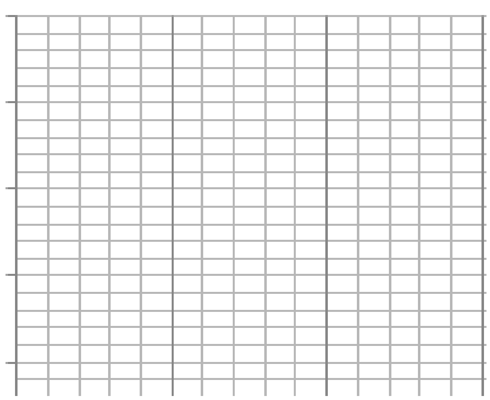
# Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

## Classwork

Today's Exploratory Challenge is an extension of Lesson 5. You will be working in groups to create a table and graph and identify whether the two quantities are proportional to each other.

### Poster Layout

Use for notes

| Problem: | Table: |
|---|---|
| Graph:  | Proportional or Not? Explanation: |

**Gallery Walk**

Take notes and answer the following questions:

- Were there any differences found in groups that had the same ratios?
- Did you notice any common mistakes? How might they be fixed?
- Were there any groups that stood out by representing their problem and findings exceptionally clearly?

Poster 1:

Poster 2:

Poster 3:

Poster 4:

Poster 5:

Poster 6:

Poster 7:

Poster 8:

**Note about Lesson Summary:**

**Lesson Summary**

**<u>Graphs of Proportional Relationships</u>**: The graph of two quantities that are proportional appear on a line that passes through the origin.

## Problem Set

Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping her money in the bank. The ratios below represent the number of years to the amount of money in the savings account.

- After one year, the interest accumulated, and the total in Sally's account was $\$312$.
- After three years, the total was $\$340$. After six years, the total was $\$380$.
- After nine years, the total was $\$430$. After $12$ years, the total amount in Sally's savings account was $\$480$.

Using the same four-fold method from class, create a table and a graph, and explain whether the amount of money accumulated and the time elapsed are proportional to each other. Use your table and graph to support your reasoning.

# Lesson 7: Unit Rate as the Constant of Proportionality

## Classwork

### Example 1: National Forest Deer Population in Danger?

Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were $144$ deer in a $16$ square mile area of the forest. In another part of the forest, conservationists counted $117$ deer in a $13$ square mile area. Yet a third conservationist counted $216$ deer in a $24$ square mile plot of the forest. Do conservationists need to be worried?

a. Why does it matter if the deer population is not constant in a certain area of the National Forest?

b. What is the population density of deer per square mile?

The unit rate of deer per $1$ square mile is _______.

Constant of Proportionality:

Explain the meaning of the constant of proportionality in this problem:

c. Use the unit rate of deer per square mile (or $\frac{y}{x}$) to determine how many deer are there for every $207$ square miles.

d. Use the unit rate to determine the number of square miles in which you would find $486$ deer?

**Vocabulary:**

A **constant** specifies a unique number.

A **variable** is a letter that represents a number.

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where $k$ is a positive constant, then $k$ is called the **constant of proportionality**. It is the value that describes the multiplicative relationship between two quantities, $x$ and $y$. The $(x, y)$ pairs represent all the pairs of values that make the equation true.

Note: In a given situation, it would be reasonable to assign any variable as a placeholder for the given quantities. For example, a set of ordered pairs $(t, d)$ would be all the points that satisfy the equation $d = rt$, where $r$ is the positive constant, or the constant of proportionality. This value for $r$ specifies a unique number for the given situation.

## Example 2: You Need WHAT???

Brandon came home from school and informed his mother that he had volunteered to make cookies for his entire grade level. He needs $3$ cookies for each of the $96$ students in $7$th grade. Unfortunately, he needs the cookies the very next day! Brandon and his mother determined that they can fit $36$ cookies on two cookie sheets.

a. Is the number of cookies proportional to the number of cookie sheets used in baking? Create a table that shows data for the number of sheets needed for the total number of cookies baked.

Table:

The unit rate of $\frac{y}{x}$ is ___________.

Constant of Proportionality:

Explain the meaning of the constant of proportionality in this problem:

b. It takes $2$ hours to bake $8$ sheets of cookies. If Brandon and his mother begin baking at $4{:}00$ p.m., when will they finish baking the cookies?

## Example 3: French Class Cooking

Suzette and Margo want to prepare crêpes for all of the students in their French class. A recipe makes 20 crêpes with a certain amount of flour, milk, and 2 eggs. The girls already know that they have plenty of flour and milk to make 50 crêpes, but they need to determine the number of eggs they will need for the recipe because they are not sure they have enough.

a. Considering the amount of eggs necessary to make the crêpes, what is the constant of proportionality?

b. What does the constant or proportionality mean in the context of this problem?

c. How many eggs are needed to make 50 crêpes?

**Lesson Summary**

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where $k$ is a positive constant, then $k$ is called the **constant of proportionality**.

## Problem Set

For each of the following problems, define the constant of proportionality to answer the follow-up question.

1. Bananas are $\$0.59$/pound.
   a. What is the constant of proportionality, $k$?
   b. How much will $25$ pounds of bananas cost?

2. The dry cleaning fee for $3$ pairs of pants is $\$18$.
   a. What is the constant of proportionality?
   b. How much will the dry cleaner charge for $11$ pairs of pants?

3. For every $\$5$ that Micah saves, his parents give him $\$10$.
   a. What is the constant of proportionality?
   b. If Micah saves $\$150$, how much money will his parents give him?

4. Each school year, the 7th graders who study Life Science participate in a special field trip to the city zoo. In $2010$, the school paid $\$1,260$ for $84$ students to enter the zoo. In $2011$, the school paid $\$1,050$ for $70$ students to enter the zoo. In $2012$, the school paid $\$1,395$ for $93$ students to enter the zoo.
   a. Is the price the school pays each year in entrance fees proportional to the number of students entering the zoo?
   b. Explain why or why not.
   c. Identify the constant of proportionality and explain what it means in the context of this situation.
   d. What would the school pay if $120$ students entered the zoo?
   e. How many students would enter the zoo if the school paid $\$1,425$?

# Lesson 8: Representing Proportional Relationships with Equations

## Classwork

Points to remember:

- Proportional relationships have a constant ratio, or unit rate.
- The constant ratio, or unit rate of $\frac{y}{x}$, can also be called the constant of proportionality.

### Discussion Notes

How could we use what we know about the constant of proportionality to write an equation?

## Example 1: Do We have Enough Gas to Make it to the Gas Station?

Your mother has accelerated onto the interstate beginning a long road trip and you notice that the low fuel light is on, indicating that there is a half a gallon left in the gas tank. The nearest gas station is 26 miles away. Your mother keeps a log where she records the mileage and the number of gallons purchased each time she fills up the tank. Use the information in the table below to determine whether you will make it to the gas station before the gas runs out. You know that if you can determine the amount of gas that her car consumes in a particular number of miles, then you can determine whether or not you can make it to the next gas station.

Mother's Gas Record

| Gallons | Miles driven |
|---|---|
| 8 | 224 |
| 10 | 280 |
| 4 | 112 |

a. Find the constant of proportionality and explain what it represents in this situation.

b. Write equation(s) that will relate the miles driven to the number of gallons of gas.

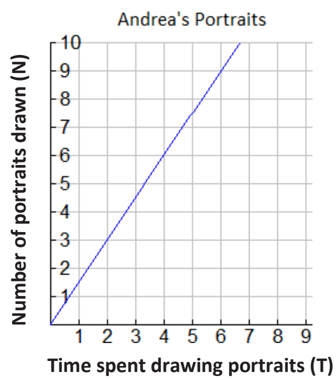
c. Knowing that there is a half-gallon left in the gas tank when the light comes on, will she make it to the nearest gas station? Explain why or why not.

d. Using the equation found in part (b), determine how far your mother can travel on 18 gallons of gas. Solve the problem in two ways: once using the constant of proportionality and once using an equation.

e. Using the constant of proportionality, and then using the equation found in part (b), determine how many gallons of gas would be needed to travel 750 miles.

## Example 2: Andrea's Portraits

Andrea is a street artist in New Orleans. She draws caricatures (cartoon-like portraits) of tourists. People have their portrait drawn and then come back later to pick it up from her. The graph below shows the relationship between the number of portraits she draws and the amount of time in hours she needs to draw the portraits.




a. Write several ordered pairs from the graph and explain what each ordered pair means in the context of this graph.

b. Write several equations that would relate the number of portraits drawn to the time spent drawing the portraits.

c. Determine the constant of proportionality and explain what it means in this situation.

### Lesson Summary

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where $k$ is a positive constant, then $k$ is called the **constant of proportionality**. The constant of proportionality expresses the multiplicative relationship between each $x$-value and its corresponding $y$-value.

## Problem Set

Write an equation that will model the proportional relationship given in each real-world situation.

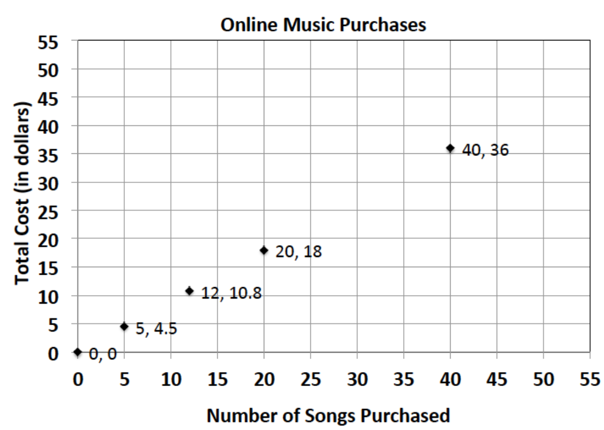
1. There are 3 cans that store 9 tennis balls. Consider the number of balls per can.
   a. Find the constant of proportionality for this situation.
   b. Write an equation to represent the relationship.

2. In 25 minutes Li can run 10 laps around the track. Determine the number of laps she can run per minute.
   a. Find the constant of proportionality in this situation.
   b. Write an equation to represent the relationship.

3. Jennifer is shopping with her mother. They pay $2 per pound for tomatoes at the vegetable stand.
   a. Find the constant of proportionality in this situation.
   b. Write an equation to represent the relationship.

4. It costs $15 to send 3 packages through a certain shipping company. Consider the number of packages per dollar.
   a. Find the constant of proportionality for this situation.
   b. Write an equation to represent the relationship.

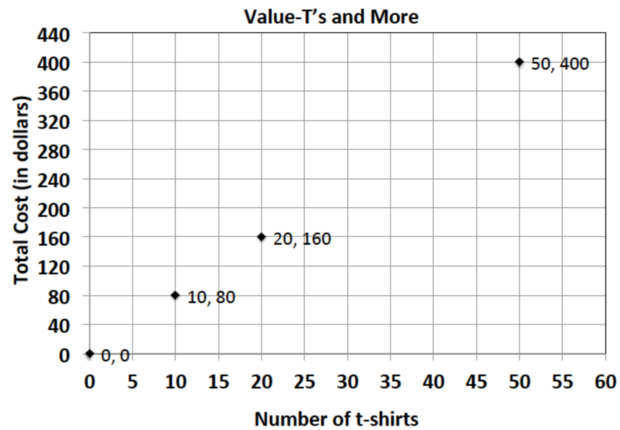
5. On average, Susan downloads 60 songs per month. An online music vendor sells package prices for songs that can be downloaded on to personal digital devices. The graph below shows the package prices for the most popular promotions. Susan wants to know if she should buy her music from this company or pay a flat fee of $58.00 per month offered by another company. Which is the better buy?
   a. Find the constant of proportionality for this situation.
   b. Write an equation to represent the relationship.
   c. Use your equation to find the answer to Susan's question above. Justify your answer with mathematical evidence and a written explanation.



6. Allison's middle school team has designed t-shirts containing their team name and color. Allison and her friend Nicole have volunteered to call local stores to get an estimate on the total cost of purchasing t-shirts. Print-o-Rama charges a set-up fee, as well as a fixed amount for each shirt ordered. The total cost is shown below for the given number of shirts. Value T's and More charges $8 per shirt. Which company should they use?

**Print-o-Rama**

| # shirts | Total cost |
|---|---|
| 10 | 95 |
| 25 | |
| 50 | 375 |
| 75 | |
| 100 | |




a. Does either pricing model represent a proportional relationship between the quantity of t-shirts and the total cost? Explain.

b. Write an equation relating cost and shirts for Value T's and More.

c. What is the constant of proportionality Value T's and More? What does it represent?

d. How much is Print-o-Rama's set-up fee?

e. Write a proposal to your teacher indicating which company the team should use. Be sure to support your choice. Determine the number of shirts that you need for your team.

# Lesson 9: Representing Proportional Relationships with Equations

## Classwork

### Example 1: Jackson's Birdhouses

Jackson and his grandfather constructed a model for a birdhouse. Many of their neighbors offered to buy the birdhouses. Jackson decided that building birdhouses could help him earn money for his summer camp, but he is not sure how long it will take him to finish all of the requests for birdhouses. If Jackson can build $7$ birdhouses in $5$ hours, write an equation that will allow Jackson to calculate the time it will take him to build any given number of birdhouses, assuming he works at a constant rate.

a. Write an equation that you could use to find out how long it will take him to build any number of birdhouses.

b. How many birdhouses can Jackson build in $40$ hours?

c. How long will it take Jackson to build $35$ birdhouses? Use the equation from part (a) to solve the problem.

d. How long will it take to build $71$ birdhouses? Use the equation from part (a) to solve the problem.

## Example 2: Al's Produce Stand

Al's Produce Stand sells $6$ ears of corn for $\$1.50$. Barbara's Produce Stand sells $13$ ears of corn for $\$3.12$. Write two equations, one for each produce stand, that models the relationship between the number of ears of corn sold and the cost. Then use each equation to help complete the tables below.

Al's Produce Stand

| | | | | |
|---|---|---|---|---|
| Ears | $6$ | $14$ | $21$ | |
| Cost | $\$1.50$ | | | $\$50.00$ |

Barbara's Produce Stand

| | | | | |
|---|---|---|---|---|
| Ears | $13$ | $14$ | $21$ | |
| Cost | $\$3.12$ | | | $\$49.92$ |

## Lesson Summary

How do you find the constant of proportionality? Divide to find the unit rate, $\frac{y}{x} = k$.

How do you write an equation for a proportional relationship? $y = kx$, substituting the value of the constant of proportionality in place of $k$.

What is the structure of proportional relationship equations and how do we use them? $x$ and $y$ values are always left as variables and when one of them is known, they are substituted into $y = kx$ to find the unknown using algebra.

## Problem Set

1. A person who weighs $100$ pounds on Earth weighs $16.6$ lb. on the moon.
   a. Which variable is the independent variable? Explain why.
   b. What is an equation that relates weight on Earth to weight on the moon?
   c. How much would a $185$ pound astronaut weigh on the moon? Use an equation to explain how you know.
   d. How much would a man that weighs $50$ pounds on the moon weigh on Earth?

2. Use this table to answer the following questions.

| Number of Gallons of Gas | Number of Miles Driven |
|---|---|
| $0$ | $0$ |
| $2$ | $62$ |
| $4$ | $124$ |
| $10$ | $310$ |

   a. Which variable is the dependent variable and why?
   b. Is the number of miles driven proportionally related to the number of gallons of gas consumed? If so, what is the equation that relates the number of miles driven to the number of gallons of gas?
   c. In any ratio relating the number of gallons of gas and the number of miles driven, will one of the values always be larger? If so, which one?
   d. If the number of gallons of gas is known, can you find the number of miles driven? Explain how this value would be calculated.
   e. If the number of miles driven is known, can you find the number of gallons of gas consumed? Explain how this value would be calculated.
   f. How many miles could be driven with $18$ gallons of gas?
   g. How many gallons are used when the car has been driven $18$ miles?
   h. How many miles have been driven when half of a gallon of gas is used?
   i. How many gallons of gas have been used when the car has been driven for a half mile?

3. Suppose that the cost of renting a snowmobile is $\$37.50$ for $5$ hours.
   a. If $c$ represents the cost and $h$ represents the hours, which variable is the dependent variable? Explain why?
   b. What would be the cost of renting $2$ snowmobiles for $5$ hours?

4. In Katya's car, the number of miles driven is proportional to the number of gallons of gas used. Find the missing value in the table.

| The Number of Gallons | The Number of Miles Driven |
|---|---|
| $0$ | $0$ |
| $4$ | $112$ |
| $6$ | $168$ |
| | $224$ |
| $10$ | $280$ |

   a. Write an equation that will relate the number of miles driven to the number of gallons of gas.
   b. What is the constant of proportionality?
   c. How many miles could Katya go if she filled her $22$-gallon tank?
   d. If Katya takes a trip of $600$ miles, how many gallons of gas would be needed to make the trip?
   e. If Katya drives $224$ miles during one week of commuting to school and work, how many gallons of gas would she use?

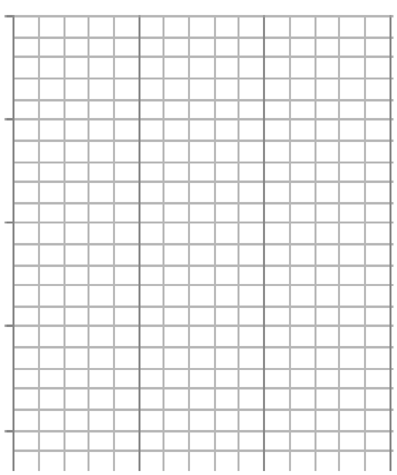
# Lesson 10: Interpreting Graphs of Proportional Relationships

## Classwork

### Example 1

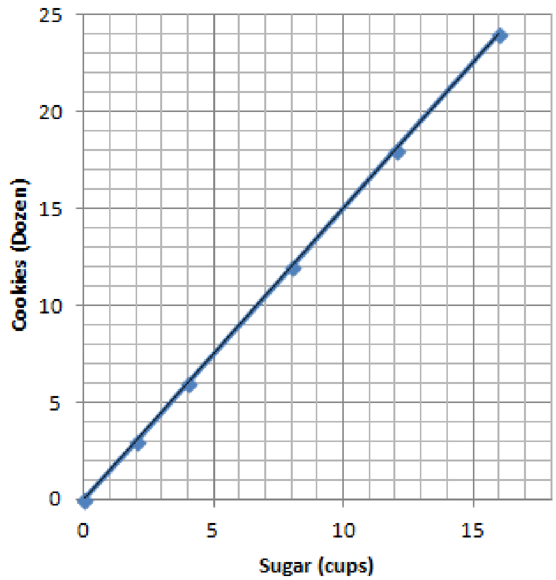
Grandma's Special Chocolate Chip Cookie recipe, which yields $4$ dozen cookies, calls for $3$ cups of flour.

Using this information, complete the chart:

| | | |
|---|---|---|
| Create a table comparing the amount of flour used to the amount of cookies. | Is the number of cookies proportional to the amount of flour used? Explain why or why not. | What is the unit rate of cookies to flour $\left(\frac{y}{x}\right)$ and what is the meaning in the context of the problem? |
| Model the relationship on a graph.  | Does the graph show the two quantities being proportional to each other? Explain | Write an equation that can be used to represent the relationship. |

## Example 2

Below is a graph modeling the amount of sugar required to make Grandma's Chocolate Chip Cookies.



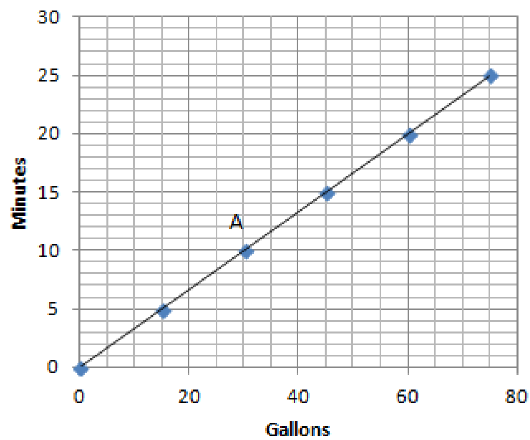

a. Record the coordinates from the graph in a table. What do these ordered pairs represent?

b. Grandma has 1 remaining cup of sugar. How many dozen cookies will she be able to make? Plot the point on the graph above.

c. How many dozen cookies can grandma make if she has no sugar? Can you graph this on the coordinate plane provided above? What do we call this point?

## Exercises

1. The graph below shows the amount of time a person can shower with a certain amount of water.




a. Can you determine by looking at the graph whether the length of the shower is proportional to the number of gallons of water? Explain how you know.

b. How long can a person shower with 15 gallons of water? How long can a person shower with $60$ gallons of water?

c. What are the coordinates of point $A$? Describe point $A$ in the context of the problem.

d. Can you use the graph to identify the unit rate?

e. Plot the unit rate on the graph. Is the point on the line of this relationship?

f. Write the equation to represent the relationship between the number of gallons of water used and the length of a shower.

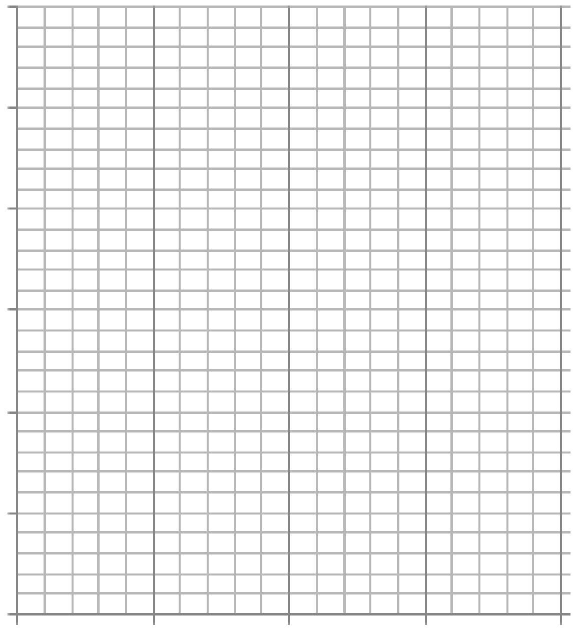
2. Your friend uses the equation $C = 50P$ to find the total cost, $C$, for the number of people, $P$, entering a local amusement park.

a. Create a table and record the cost of entering the amusement park for several different-sized groups of people.

b. Is the cost of admission proportional to the amount of people entering the amusement park? Explain why or why not.

c. What is the unit rate and what does it represent in the context of the situation?

d. Sketch a graph to represent this relationship.



e. What points must be on the graph of the line if the two quantities represented are proportional to each other? Explain why and describe these points in the context of the problem.

f. Would the point $(5, 250)$ be on the graph? What does this point represent in the context of the situation?

**Lesson Summary**

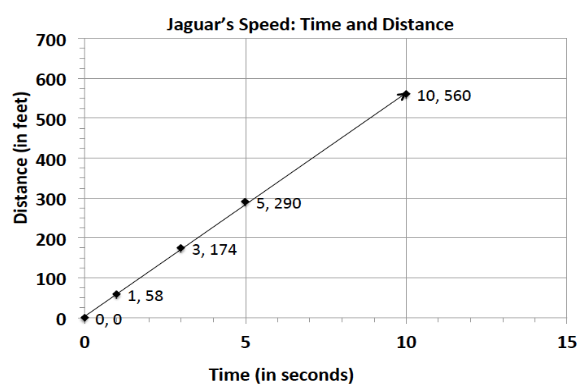
The points $(0,0)$ and $(1, r)$, where $r$ is the unit rate, will always appear on the line representing two quantities that are proportional to each other.

- The unit rate, $r$, in the point $(1, r)$ represents the amount of vertical increase for every horizontal increase of $1$ unit on the graph.
- The point $(0,0)$ indicates that when there is zero amount of one quantity, there will also be zero amount of the second quantity.

These two points may not always be given as part of the set of data for a given real-world or mathematical situation, but they will always appear on the line that passes through the given data points.

## Problem Set

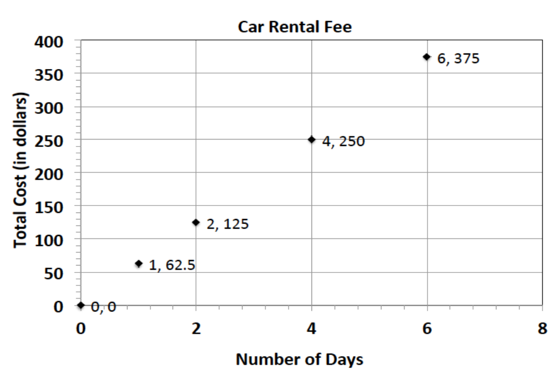
1. The graph to the right shows the relationship of the amount of time (in seconds) to the distance (in feet) run by a jaguar.




   a. What does the point $(5, 290)$ represent in the context of the situation?
   b. What does the point $(3, 174)$ represent in the context of the situation?
   c. Is the distance run by the jaguar proportional to the time? Explain why or why not.
   d. Write an equation to represent the distance run by the jaguar. Explain or model your reasoning.

2. Championship t-shirts sell for $22 each.
   a. What point(s) must be on the graph for the quantities to be proportional to each other?
   b. What does the ordered pair $(5, 110)$ represent in the context of this problem?
   c. How many t-shirts were sold if you spent a total of $88?

3. The graph represents the total cost of renting a car. The cost of renting a car is a fixed amount each day, regardless of how many miles the car is driven.




   a. What does the ordered pair $(4, 250)$ represent?
   b. What would be the cost to rent the car for a week? Explain or model your reasoning.

4. Jackie is making a snack mix for a party. She is using cashews and peanuts. The table below shows the relationship of the number of packages of cashews she needs to the number of cans of peanuts she needs to make the mix.

| Packages of Cashews | Cans of Peanuts |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |
| 4 | 8 |

a. What points must be on the graph for the number of cans of peanuts to be proportional to the number of packages of cashews? Explain why.

b. Write an equation to represent this relationship.

c. Describe the ordered pair $(12, 24)$ in the context of the problem.

5. The following table shows the amount of candy and price paid.

| Amount of Candy (in pounds) | 2 | 3 | 5 |
|---|---|---|---|
| Cost ( in dollars) | 5 | 7.5 | 12.5 |

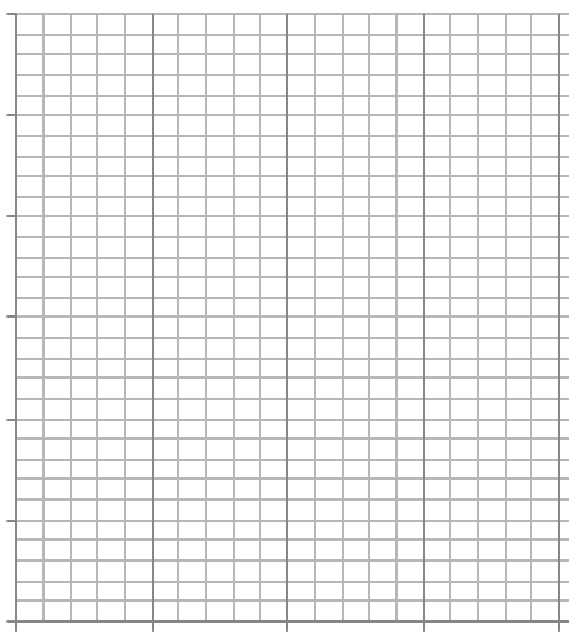
a. Is the cost of the candy proportional to the amount of candy?

b. Write an equation to illustrate the relationship between the amount of candy and the cost.

c. Using the equation, predict how much it will cost for $12$ pounds of candy.

d. What is the maximum amount of candy you can buy with $\$60$?

e. Graph the relationship.



# Lesson 11: Ratios of Fractions and Their Unit Rates

## Classwork

### Example 1: Who is Faster?

During their last workout, Izzy ran $2\frac{1}{4}$ miles in 15 minutes and her friend Julia ran $3\frac{3}{4}$ miles in 25 minutes. Each girl thought she was the faster runner. Based on their last run, which girl is correct? Use any approach to find the solution.

## Example 2: Is Meredith Correct?

A turtle walks $\frac{7}{8}$ of a mile in $50$ minutes. What is the unit rate expressed in miles per hour?

a. To find the turtle's unit rate, Meredith wrote the following complex fraction. Explain how the fraction $\frac{5}{6}$ was obtained.

$$\frac{\left(\frac{7}{8}\right)}{\left(\frac{5}{6}\right)}$$

b. Determine the unit rate, expressed in miles per hour.

## Exercises

1. For Anthony's birthday, his mother is making cupcakes for his $12$ friends at his daycare. The recipe calls for $3\frac{1}{3}$ cups of flour. This recipe makes $2\frac{1}{2}$ dozen cupcakes. Anthony's mother has only $1$ cup of flour. Is there enough flour for each of his friends to get a cupcake? Explain and show your work.

2. Sally is making a painting for which she is mixing red paint and blue paint. The table below shows the different mixtures being used.

| Red Paint (Quarts) | Blue Paint (Quarts) |
|---|---|
| $1\frac{1}{2}$ | $2\frac{1}{2}$ |
| $2\frac{2}{5}$ | $4$ |
| $3\frac{3}{4}$ | $6\frac{1}{4}$ |
| $4$ | $6\frac{2}{3}$ |
| $1.2$ | $2$ |
| $1.8$ | $3$ |

a. What is the unit rate for the values of the amount of blue paint to the amount of red paint?

b. Is the amount of blue paint proportional to the amount of red paint?

c. Describe, in words, what the unit rate means in the context of this problem.

**Lesson Summary**

A fraction whose numerator or denominator is itself a fraction is called a **complex fraction**.

Recall: A **unit rate** is a rate, which is expressed as $\frac{A}{B}$ units of the first quantity per $1$ unit of the second quantity for two quantities $A$ and $B$.

For example: If a person walks $2\frac{1}{2}$ miles in $1\frac{1}{4}$ hours at a constant speed, then the unit rate is

$\frac{2\frac{1}{2}}{1\frac{1}{4}} = \frac{\frac{5}{2}}{\frac{5}{4}} = \frac{5}{2} \cdot \frac{4}{5} = 2$. The person walks $2$ mph.

## Problem Set

1. Determine the quotient: $2\frac{4}{7} \div 1\frac{3}{6}$

2. One lap around a dirt track is $\frac{1}{3}$ mile. It takes Bryce $\frac{1}{9}$ hour to ride one lap. What is Bryce's unit rate, in miles, around the track?

3. Mr. Gengel wants to make a shelf with boards that are $1\frac{1}{3}$ feet long. If he has an $18$-foot board, how many pieces can he cut from the big board?

4. The local bakery uses $1.75$ cups of flour in each batch of cookies. The bakery used $5.25$ cups of flour this morning.
   a. How many batches of cookies did the bakery make?
   b. If there are $5$ dozen cookies in each batch, how many cookies did the bakery make?

5. Jason eats $10$ ounces of candy in $5$ days.
   a. How many <u>pounds</u> will he eat per day? (Recall: $16$ ounces $= 1$ pound)
   b. How long will it take Jason to eat $1$ pound of candy?

# Lesson 12: Ratios of Fractions and Their Unit Rates

## Classwork

During this lesson, you are remodeling a room at your house and need to figure out if you have enough money. You will work individually and with a partner to make a plan of what is needed to solve the problem. After your plan is complete, then you will solve the problem by determining if you have enough money.

### Example 1: Time to Remodel

You have decided to remodel your bathroom and install a tile floor. The bathroom is in the shape of a rectangle and the floor measures $14$ feet, $8$ inches long by $5$ feet, $6$ inches wide. The tiles you want to use cost $\$5$ each, and each tile covers $4\frac{2}{3}$ square feet. If you have $\$100$ to spend, do you have enough money to complete the project?

Make a Plan: Complete the chart to identify the necessary steps in the plan and find a solution.

| What I Know | What I Want to Find | How to Find it |
|---|---|---|
| | | |
| | | |
| | | |

Compare your plan with a partner. Using your plans, work together to determine how much money you will need to complete the project and if you have enough money.

**Exercises**

Which car can travel further on $1$ gallon of gas?

Blue Car: travels $18\frac{2}{5}$ miles using $0.8$ gallons of gas

Red Car: travels $17\frac{2}{5}$ miles using $0.75$ gallons of gas

## Problem Set

1. You are getting ready for a family vacation. You decide to download as many movies as possible before leaving for the road trip. If each movie takes $1\frac{2}{5}$ hours to download and you downloaded for $5\frac{1}{4}$ hours, how many movies did you download?

2. The area of a blackboard is $1\frac{1}{3}$ square yards. A poster's area is $\frac{8}{9}$ square yards. Find the unit rate and explain, in words, what the unit rate means in the context of this problem. Is there more than one unit rate that can be calculated? How do you know?

3. A toy jeep is $12\frac{1}{2}$ inches long while an actual jeep measures $18\frac{3}{4}$ feet long. What is the value of the ratio of the length of the toy jeep to length of the actual jeep? What does the ratio mean in this situation?

4. $\frac{1}{3}$ cup of flour is used to make $5$ dinner rolls.
   a. How much flour is needed to make one dinner roll?
   b. How many cups of flour are needed to make $3$ dozen dinner rolls?
   c. How many rolls can you make with $5\frac{2}{3}$ cups of flour?

# Lesson 13: Finding Equivalent Ratios Given the Total Quantity

## Classwork

### Example 1

A group of 6 hikers are preparing for a one-week trip. All of the group's supplies will be carried by the hikers in backpacks. The leader decides that each hiker will carry a backpack that is the same fraction of weight to all the other hikers' weights. This means that the heaviest hiker would carry the heaviest load. The table below shows the weight of each hiker and the weight of the backpack.

Complete the table. Find the missing amounts of weight by applying the same value of the ratio as the first two rows.

| Hiker's Weight | Backpack Weight | Total Weight (lb.) |
|---|---|---|
| 152 lb. 4 oz. | 14 lb. 8 oz. | |
| 107 lb. 10 oz. | 10 lb. 4 oz. | |
| 129 lb. 15 oz. | | |
| 68 lb. 4 oz. | | |
| | 8 lb. 12 oz. | |
| | 10 lb. | |
| | | |
| | | |

## Example 2

When a business buys a fast food franchise, it is buying the recipes used at every restaurant with the same name. For example, all Pizzeria Specialty House Restaurants have different owners, but they must all use the same recipes for their pizza, sauce, bread, etc. You are now working at your local Pizzeria Specialty House Restaurant, and listed below are the amounts of meat used on one meat-lovers pizza.

$\frac{1}{4}$ cup of sausage

$\frac{1}{3}$ cup of pepperoni

$\frac{1}{6}$ cup of bacon

$\frac{1}{8}$ cup of ham

$\frac{1}{8}$ cup of beef

What is the total amount of toppings used on a meat-lovers pizza? _________________________ cups

The meat must be mixed using this ratio to ensure that customers will receive the same great tasting meat-lovers pizza from every Pizzeria Specialty House Restaurant nationwide. The table below shows 3 different orders for meat-lovers pizzas on Super Bowl Sunday. Using the amounts and total for one pizza given above, fill in every row and column of the table so the mixture tastes the same.

| | Order 1 | Order 2 | Order 3 |
|---|---|---|---|
| Sausage (cups) | $1$ | | |
| Pepperoni (cups) | | | $3$ |
| Bacon (cups) | | $1$ | |
| Ham (cups) | $\frac{1}{2}$ | | |
| Beef (cups) | | | $1\frac{1}{8}$ |
| TOTAL (cups) | | | |

## Exercise

The table below shows 6 different-sized pans that could be used to make macaroni and cheese. If the ratio of ingredients stays the same, how might the recipe be altered to account for the different sized pans?

| Noodles (cups) | Cheese (cups) | Pan Size (number of cups) |
|---|---|---|
| | | $5$ |
| $3$ | $\frac{3}{4}$ | |
| | $\frac{1}{4}$ | |
| $\frac{2}{3}$ | | |
| $5\frac{1}{3}$ | | |
| | | $5\frac{5}{8}$ |

## Lesson Summary

To find missing quantities in a ratio table where a total is given, determine the unit rate from the ratio of two given quantities and use it to find the missing quantities in each equivalent ratio.

## Problem Set

1. Students in $6$ classes, displayed below, ate the same ratio of cheese pizza slices to pepperoni pizza slices. Complete the following table, which represents the number of slices of pizza students in each class ate.

| Slices of Cheese Pizza | Slices of Pepperoni Pizza | Total Pizza |
|---|---|---|
| | | $7$ |
| $6$ | $15$ | |
| $8$ | | |
| | $13\frac{3}{4}$ | |
| $3\frac{1}{3}$ | | |
| | | $2\frac{1}{10}$ |

2. To make green paint, students mixed yellow paint with blue paint. The table below shows how many yellow and blue drops from a dropper several students used to make the same shade of green paint.

   a. Complete the table.

| Yellow ($Y$) (ml) | Blue ($B$) (ml) | Total |
|---|---|---|
| $3\frac{1}{2}$ | $5\frac{1}{4}$ | |
| | | $5$ |
| | $6\frac{3}{4}$ | |
| $6\frac{1}{2}$ | | |

   b. Write an equation to represent the relationship between the amount of yellow paint and blue paint.

3. The ratio of the number of miles run to the number of miles biked is equivalent for each row in the table.

   a. Complete the table.

| Distance Run (miles) | Distance Biked (miles) | Total Amount of Exercise (miles) |
|---|---|---|
| | | $6$ |
| $3\frac{1}{2}$ | $7$ | |
| | $5\frac{1}{2}$ | |
| $2\frac{1}{8}$ | | |
| | $3\frac{1}{3}$ | |

   b. What is the relationship between distances biked and distances run?

4. The following table shows the number of cups of milk and flour that are needed to make biscuits. Complete the table.

| Milk (cups) | Flour (cups) | Total (cups) |
|---|---|---|
| $7.5$ | | |
| | $10.5$ | |
| $12.5$ | $15$ | |
| | | $11$ |

# Lesson 14: Multi-Step Ratio Problems

## Classwork

### Example 1: Bargains

Peter's Pants Palace advertises the following sale: Shirts are $\frac{1}{2}$ off the original price; pants are $\frac{1}{3}$ off the original price, and shoes are $\frac{1}{4}$ off the original price.

a. If a pair of shoes cost $\$40$, what is the sales price?

b. At Peter's Pants Palace, a pair of pants usually sells for $\$33.00$. What is the sale price of Peter's pants?

### Example 2: Big Al's Used Cars

A used car salesperson receives a commission of $\frac{1}{12}$ of the sales price of the car for each car he sells. What would the sales commission be on a car that sold for $\$21{,}999$?

## Example 3: Tax Time

As part of a marketing plan, some businesses mark up their prices before they advertise a sales event. Some companies use this practice as a way to entice customers into the store without sacrificing their profits.

A furniture store wants to host a sales event to improve its profit margin and to reduce its tax liability before its inventory is taxed at the end of the year.

How much profit will the business make on the sale of a couch that is marked-up by $\frac{1}{3}$ and then sold at a $\frac{1}{5}$ off discount if the original price is $\$2,400$?

## Example 4: Born to Ride

A motorcycle dealer paid a certain price for a motorcycle and marked it up by $\frac{1}{5}$ of the price he paid. Later he sold it for $\$14,000$. What is the original price?

**Lesson Summary**

- Discount price $=$ original price $-$ rate $\times$ original price <u>OR</u> $(1 - \text{rate}) \times$ original price
- Commission $=$ rate $\times$ total sales amount
- Markup price $=$ original price $+$ rate $\times$ original price <u>OR</u> $(1 + \text{rate}) \times$ original price

## Problem Set

1. A salesperson will earn a commission equal to $\frac{1}{32}$ of the total sales. What is the commission earned on sales totaling $\$24{,}000$?

2. DeMarkus says that a store overcharged him on the price of the video game he bought. He thought that the price was marked $\frac{1}{4}$ <u>of</u> the original price, but it was really $\frac{1}{4}$ <u>off</u> the original price. He misread the advertisement. If the original price of the game was $\$48$, what is the difference between the price that DeMarkus thought he should pay and the price that the store charged him?

3. What is the cost of a $\$1{,}200$ washing machine after a discount of $\frac{1}{5}$ the original price?

4. If a store advertised a sale that gave customers a $\frac{1}{4}$ discount, what is the fractional part of the original price that the customer will pay?

5. Mark bought an electronic tablet on sale for $\frac{1}{4}$ off the original price of $\$825.00$. He also wanted to use a coupon for $\frac{1}{5}$ off the sales price. Before taxes, how much did Mark pay for the tablet?

6. A car dealer paid a certain price for a car and marked it up by $\frac{7}{5}$ of the price he paid. Later he sold it for $\$24{,}000$. What is the original price?

7. Joanna ran a mile in physical education class. After resting for one hour, her heart rate was $60$ beats per minute. If her heart rate decreased by $\frac{2}{5}$, what was her heart rate immediately after she ran the mile?

# Lesson 15: Equations of Graphs of Proportional Relationships Involving Fractions

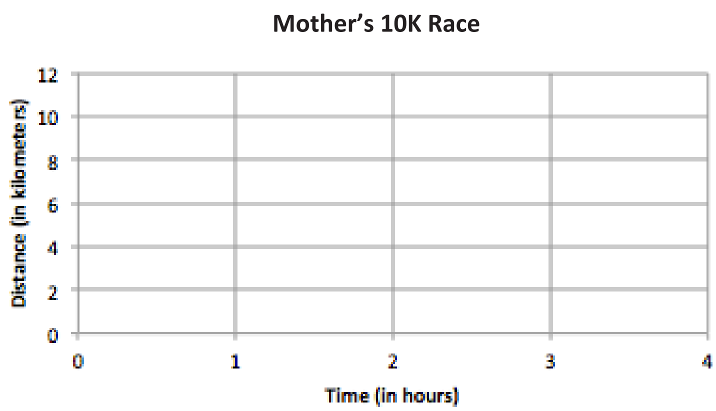
## Classwork

### Example 1: Mother's 10K Race

Sam's mother has entered a 10K race. Sam and his family want to show their support of their mother, but they need to figure out where they should go along the race course. They also need to determine how long it will take her to run the race so that they will know when to meet her at the finish line. Previously, his mother ran a 5K race with a time of $1\frac{1}{2}$ hours. Assume Sam's mother ran the same rate as the previous race in order to complete the chart.

Create a table that will show how far Sam's mother has run after each half hour from the start of the race, and graph it on the coordinate plane to the right.

| Time ($H$, in hours) | Distance Run ($D$, in km) |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |




a. What are some specific things you notice about this graph?

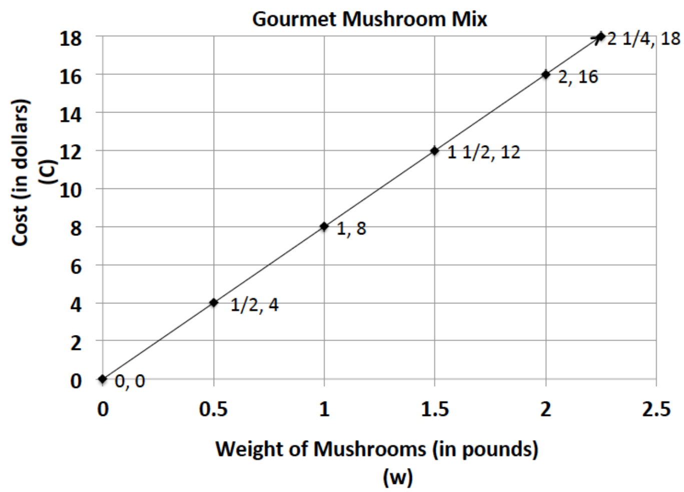
b. What is the connection between the table and the graph?

c. What does the ordered pair $\left(2, 6\frac{2}{3}\right)$ represent in the context of this problem?

## Example 2: Gourmet Cooking

After taking a cooking class, you decide to try out your new cooking skills by preparing a meal for your family. You have chosen a recipe that uses gourmet mushrooms as the main ingredient. Using the graph below, complete the table of values and answer the following questions.

| Weight (in pounds) | Cost (in dollars) |
|---|---|
| $0$ | $0$ |
| $\frac{1}{2}$ | $4$ |
| $1$ | |
| $1\frac{1}{2}$ | $12$ |
| | $16$ |
| $2\frac{1}{4}$ | $18$ |




a. Is this relationship proportional? How do you know from examining the graph?

b. What is the unit rate for cost per pound?

c. Write an equation to model this data.

d. What ordered pair represents the unit rate, and what does it mean?

e. What does the ordered pair $(2, 16)$ mean in the context of this problem?

f. If you could spend $\$10.00$ on mushrooms, how many pounds could you buy?

g. What would be the cost of $30$ pounds of mushrooms?

Lesson Summary

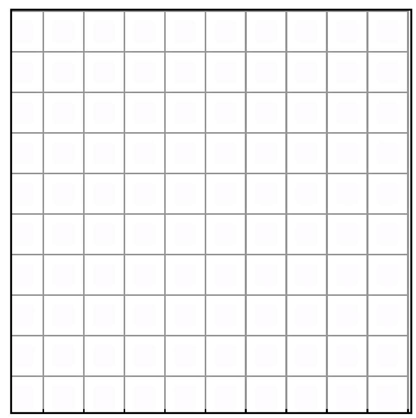
Proportional relationships can be represented through the use of graphs, tables, equations, diagrams, and verbal descriptions.

In a proportional relationship arising from ratios and rates involving fractions, *the graph* gives a visual display of *all values* of the proportional relationship, especially the quantities that fall between integer values.

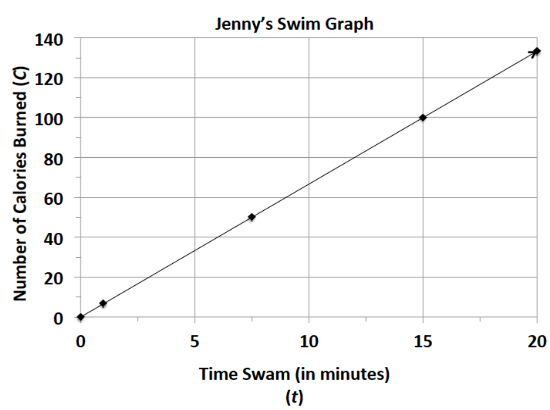
## Problem Set

1. Students are responsible for providing snacks and drinks for the Junior Beta Club Induction Reception. Susan and Myra were asked to provide the punch for the $100$ students and family members who will attend the event. The chart below will help Susan and Myra determine the proportion of cranberry juice to sparkling water that will be needed to make the punch. Complete the chart, graph the data, and write the equation that models this proportional relationship.

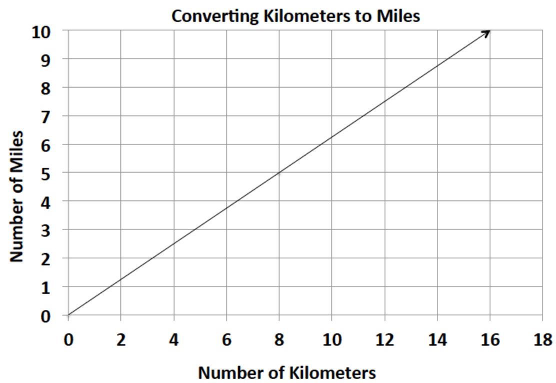
| Sparkling Water ($S$, in cups) | Cranberry Juice ($C$, in cups) |
|---|---|
| $1$ | $\frac{4}{5}$ |
| $5$ | $4$ |
| $8$ | |
| $12$ | $9\frac{3}{5}$ |
| | $40$ |
| $100$ | |



2. Jenny is a member of a summer swim team.
   a. Using the graph, determine how many calories she burns in one minute.
   b. Use the graph to determine the equation that models the number of calories Jenny burns within a certain number of minutes.
   c. How long will it take her to burn off a $480$-calorie smoothie that she had for breakfast?



3. Students in a world geography class want to determine the distances between cities in Europe. The map gives all distances in kilometers. The students want to determine the number of miles between towns so that they can compare distances with a unit of measure with which they are already familiar. The graph below shows the relationship between a given number of kilometers and the corresponding number of miles.




   a. Find the constant of proportionality or the rate of miles per kilometer for this problem and write the equation that models this relationship.
   b. What is the distance in kilometers between towns that are $5$ miles apart?
   c. Describe the steps you would take to determine the distance in miles between two towns that are $200$ kilometers apart?

4. During summer vacation, Lydie spent time with her grandmother picking blackberries. They decided to make blackberry jam for their family. Her grandmother said that you must cook the berries until they become juice and then combine the juice with the other ingredients to make the jam.
   a. Use the table below to determine the constant of proportionality of cups of juice to cups of blackberries.

| Cups of Blackberries | Cups of Juice |
|---|---|
| $0$ | $0$ |
| $4$ | $1\frac{1}{3}$ |
| $8$ | $2\frac{2}{3}$ |
| $12$ | |
| | $8$ |

   b. Write an equation that will model the relationship between the number of cups of blackberries and the number of cups of juice.
   c. How many cups of juice were made from $12$ cups of berries? How many cups of berries are needed to make $8$ cups of juice?

# Lesson 16: Relating Scale Drawings to Ratios and Rates

## Classwork

### Opening Exercise: Can You Guess the Image?

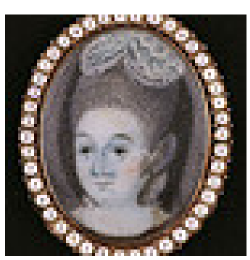
1. 

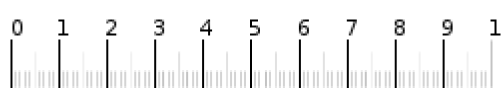

2. 

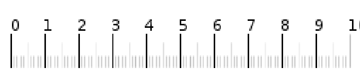
## Example 1

For the following problems, (a) is the actual picture and (b) is the drawing. Is the drawing an enlargement or a reduction of the actual picture?

1. a. 

   b. 

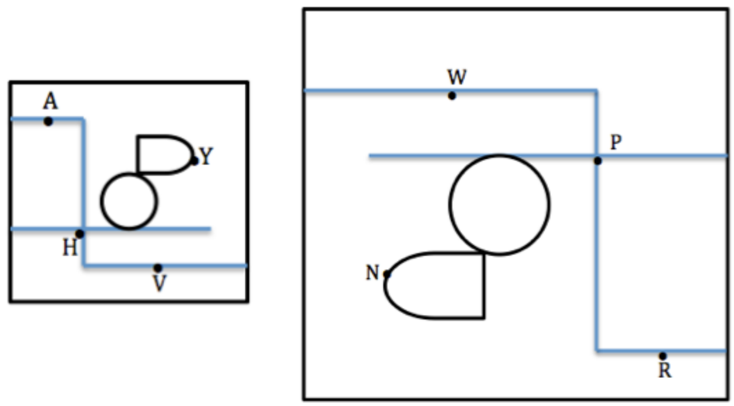
2. a. 


   b. 

**Scale Drawing**: a reduced or enlarged two-dimensional drawing of an original two-dimensional drawing.

## Example 2

Derek's family took a day trip to a modern public garden. Derek looked at his map of the park that was a reduction of the map located at the garden entrance. The dots represent the placement of rare plants. The diagram below is the top-view as Derek held his map while looking at the posted map.

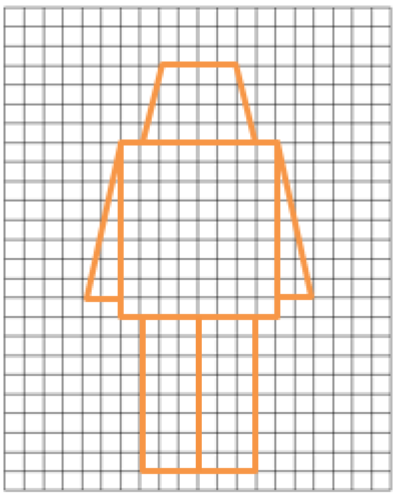


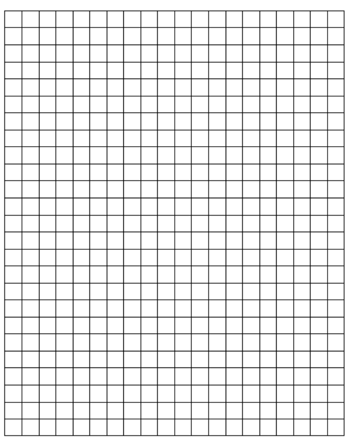

What are the corresponding points of the scale drawings of the maps?

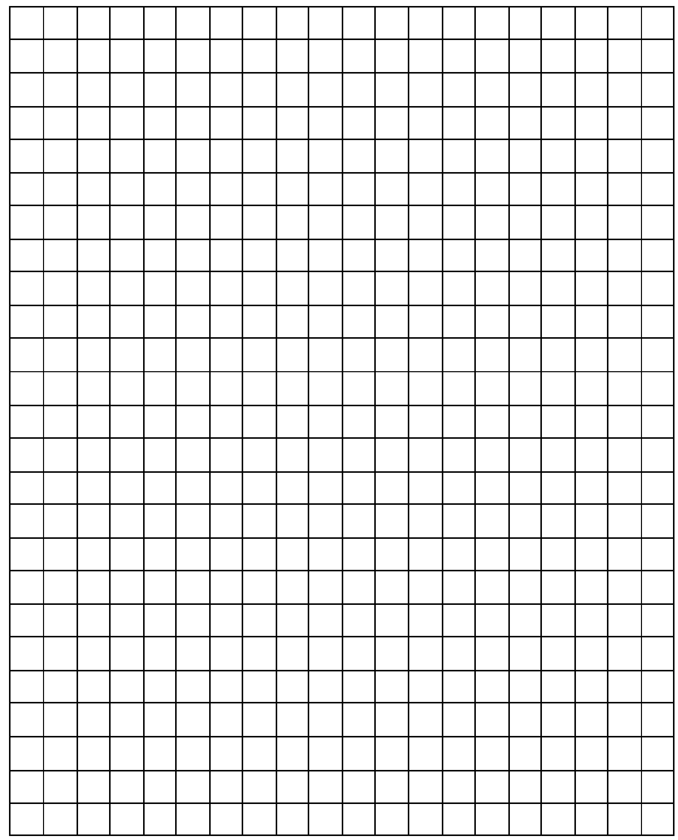
Point $A$ to _________ Point $V$ to ____________ Point $H$ to ____________ Point $Y$ to ____________

**Exploratory Challenge**

Create scale drawings of your own modern nesting robots using the grids provided.

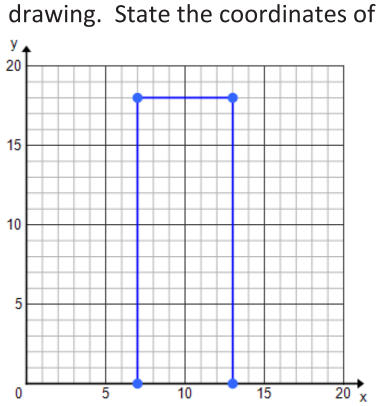


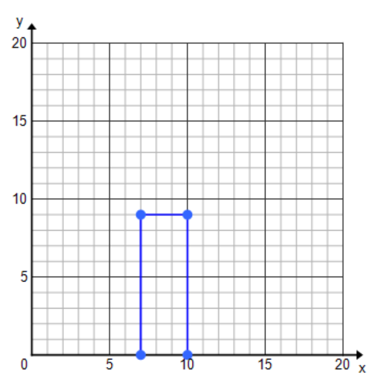




## Example 3

Celeste drew an outline of a building for a diagram she was making and then drew a second one mimicking her original drawing. State the coordinates of the vertices and fill in the table.



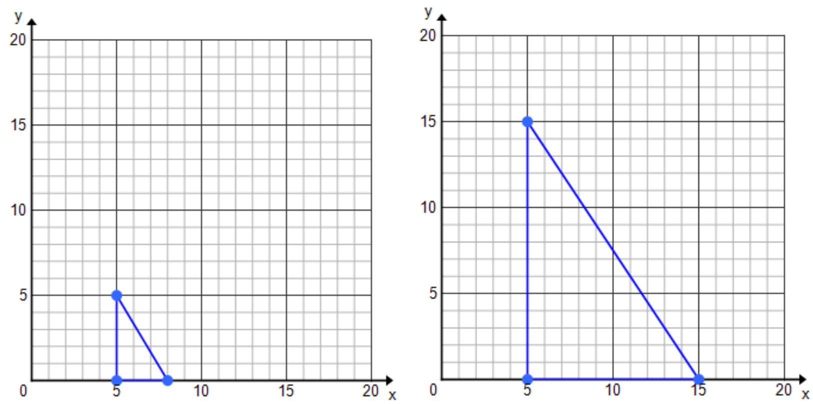




| | Height | Length |
|---|---|---|
| Original Drawing | | |
| Second Drawing | | |

Notes:

## Exercise

Luca drew and cut out a small right triangle for a mosaic piece he was creating for art class. His mother really took a liking to the mosaic piece and asked if he could create a larger one for their living room. Luca made a second template for his triangle pieces.




| | Height | Width |
|---|---|---|
| Original Image | | |
| Second Image | | |

a. Does a constant of proportionality exist? If so, what is it? If not, explain.

b. Is Luca's enlarged mosaic a scale drawing of the first image? Explain why or why not.

**Lesson Summary**

**Scale Drawing:** A drawing in which all lengths between points or figures in the drawing are reduced or enlarged proportional to the lengths in the actual picture. A constant of proportionality exists between corresponding lengths of the two images.

**Reduction**: The lengths in the scale drawing are smaller than those in the actual object or picture.

**Enlargement/Magnification**: The lengths in the scale drawing are larger than those in the actual object or picture.

**One-to-One Correspondence:** Each point in one figure corresponds to one and only one point in the second figure.

## Problem Set

For Problems 1–3, identify if the scale drawing is a reduction or an enlargement of the actual picture.

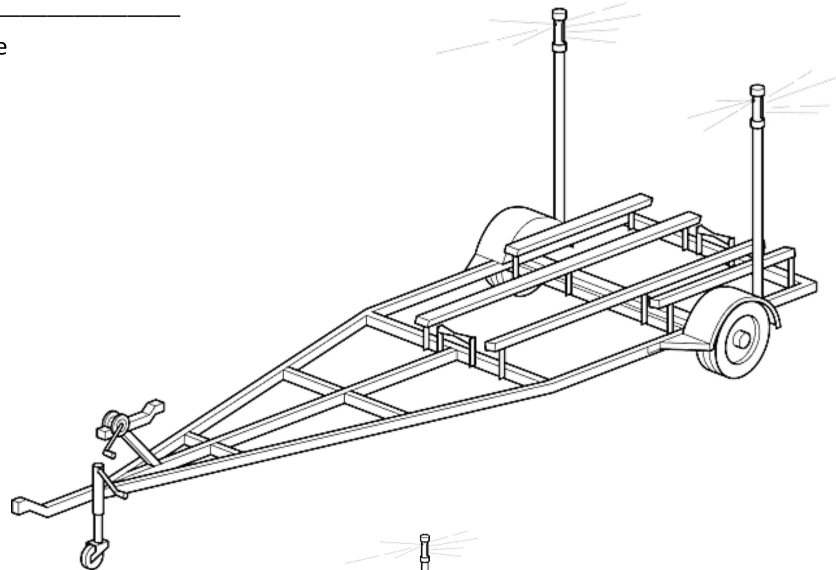
1. ____________________________

a. Actual Picture

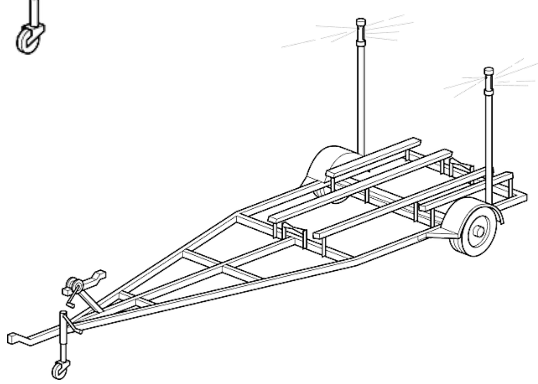
b. Scale Drawing

2. ______________________________

a. Actual Picture



b. Scale Drawing

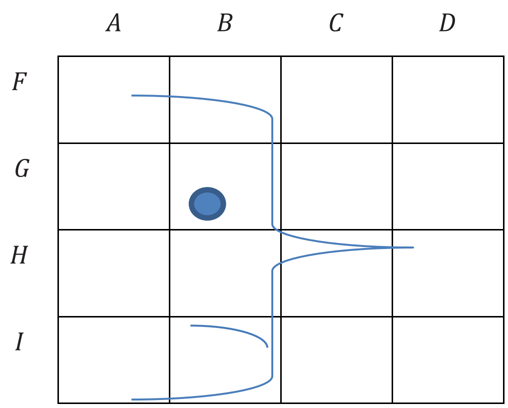


3. ____________________________

a. Actual Picture


b. Scale Drawing

4. Using the grid and the abstract picture of a face, answer the following questions:




   a. On the grid, where is the eye?

   b. What is located in $DH$?

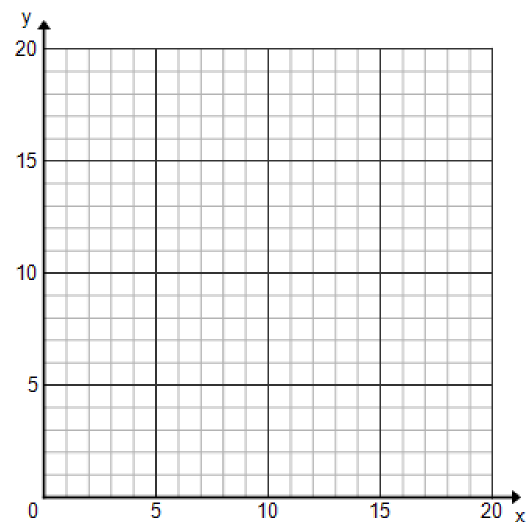
   c. In what part of the square $BI$ is the chin located?

5. Use the blank graph provided to plot the points and decide if the rectangular cakes are scale drawings of each other.

   Cake 1: $(5,3)$, $(5,5)$, $(11,3)$, $(11,5)$

   Cake 2: $(1,6)$, $(1,12)$,$(13,12)$, $(13,6)$

   How do you know?



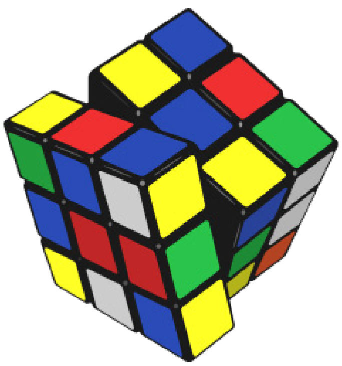
# Lesson 17: The Unit Rate as the Scale Factor

## Classwork

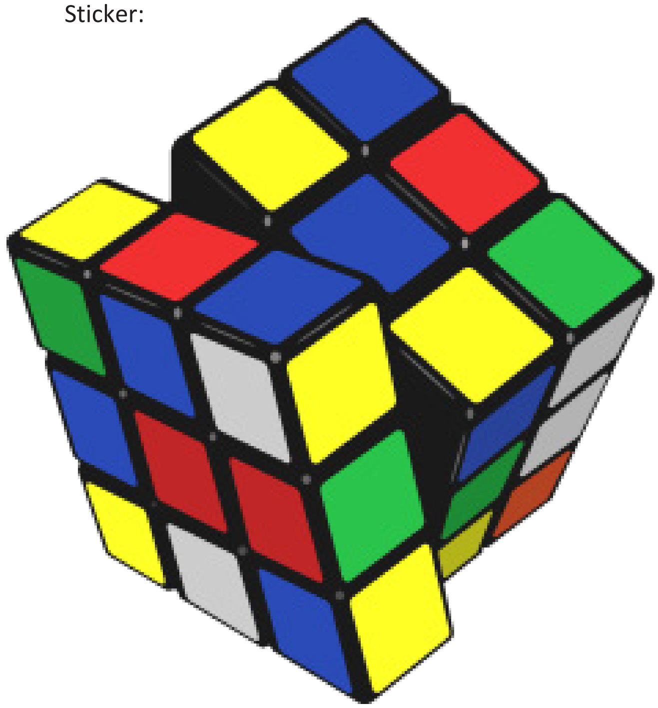
### Example 1: Rubin's Icon

Rubin created a simple game on his computer and shared it with his friends to play. They were instantly hooked, and the popularity of his game spread so quickly that Rubin wanted to create a distinctive icon so that players could easily identify his game. He drew a simple sketch. From the sketch, he created stickers to promote his game, but Rubin wasn't quite sure if the stickers were proportional to his original sketch.

Original Sketch:



Sticker:



Steps to check for proportionality for scale drawing and original object or picture:

1.

2.

3.

Key Idea:

The **scale factor** can be calculated from the ratio of any length in the scale drawing to its corresponding length in the actual picture. The scale factor corresponds to the unit rate and the constant of proportionality.

Scaling by factors *greater than* $1$ enlarge the segment, and scaling by factors *less than* $1$, reduce the segment.

**Exercise 1: App Icon**

## Example 2

Use a Scale Factor of 3 to create a scale drawing of the picture below.

Picture of the flag of Colombia**:**

### Exercise 2

Scale Factor = ___________________________

Picture of the flag of Colombia:

Sketch and notes:

## Example 3

Your family recently had a family portrait taken. Your aunt asks you to take a picture of the portrait using your phone and send it to her. If the original portrait is 3 feet by 3 feet, and the scale factor is $\frac{1}{18}$, draw the scale drawing that would be the size of the portrait on your phone.

Sketch and notes:

## Exercise 3

John is building his daughter a doll house that is a miniature model of their house. The front of their house has a circular window with a diameter of 5 feet. If the scale factor for the model house is $\frac{1}{30}$, make a sketch of the circular doll house window.

## Problem Set

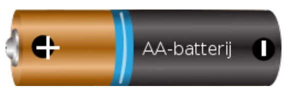
1. Giovanni went to Los Angeles, California for the summer to visit his cousins. He used a map of bus routes to get from the airport to his cousin's house. The distance from the airport to his cousin's house is 56 km. On his map, the distance is 4 cm. What is the scale factor?

2. Nicole is running for school president. Her best friend designed her campaign poster, which measured 3 feet by 2 feet. Nicole liked the poster so much, she reproduced the artwork on rectangular buttons that measured 2 inches by $1\frac{1}{3}$ inches. What is the scale factor?

3. Find the scale factor using the given scale drawings and measurements below.

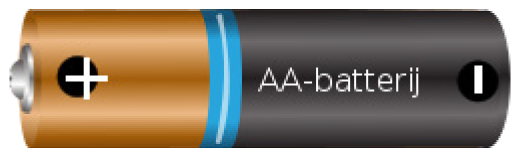
   Scale Factor: ___________________________

   <u>Actual Picture</u>

   3 cm

   <u>Scale Drawing</u>

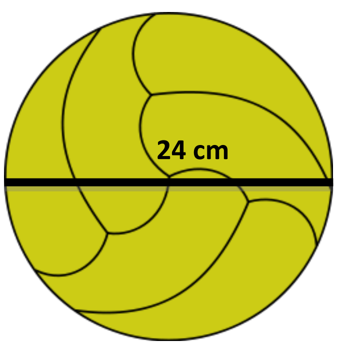
   
   

   5 cm

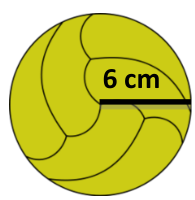
4. Find the scale factor using the given scale drawings and measurements below.

   Scale Factor: ___________________________

   <u>Actual Picture</u>

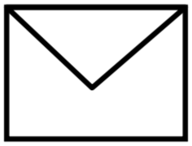
   
   

   <u>Scale Drawing</u>

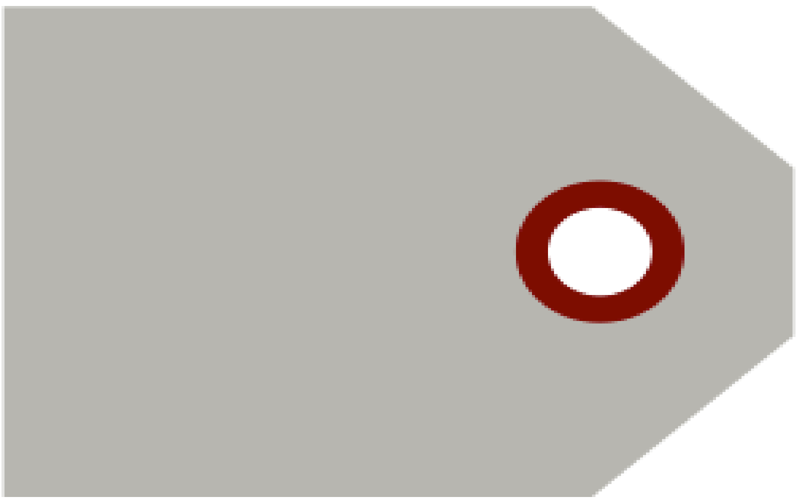
5. Using the given scale factor, create a scale drawing from the actual pictures in centimeters:

   a. Scale factor: $3$



$1$ in.

   b. Scale factor: $\frac{3}{4}$



6. Hayden likes building radio-controlled sailboats with her father. One of the sails, shaped like a right triangle, has side lengths measuring $6$ inches, $8$ inches and $10$ inches. To log her activity, Hayden creates and collects drawings of all the boats she and her father built together. Using the scale factor of $\frac{1}{4}$, create a scale drawing of the sail.

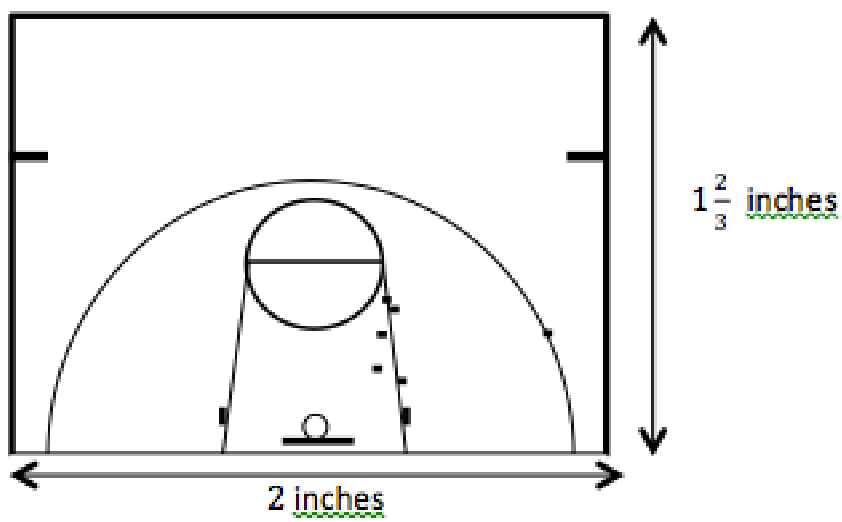
# Lesson 18: Computing Actual Lengths from a Scale Drawing

## Classwork

### Example 1: Basketball at Recess?

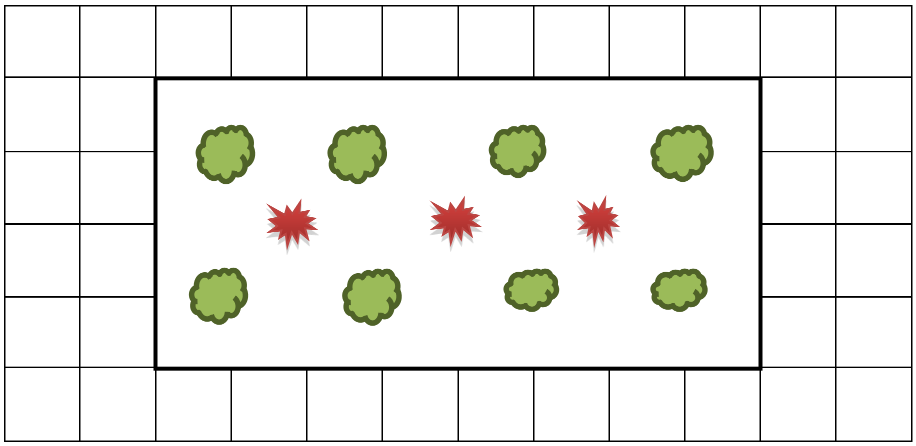
Vincent proposes an idea to the Student Government to install a basketball hoop along with a court marked with all the shooting lines and boundary lines at his school for students to use at recess. He presents a plan to install a half-court design as shown below. After checking with school administration, he is told it will be approved if it will fit on the empty lot that measures 25 feet by 75 feet on the school property. Will the lot be big enough for the court he planned? Explain.

Scale Drawing: 1 inch on the drawing corresponds to 15 feet of actual length.



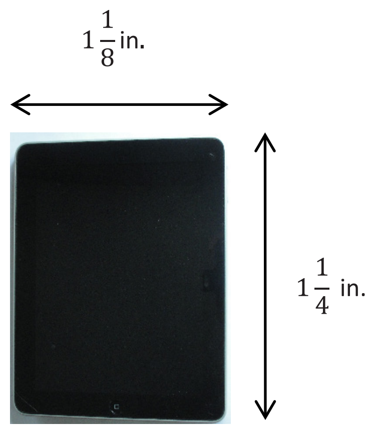
## Example 2

The diagram shown represents a garden. The scale is 1 centimeter for every 20 meters. Each square in the drawing measures 1 cm by 1 cm. Find the actual length and width of the garden based upon the given drawing.



## Example 3

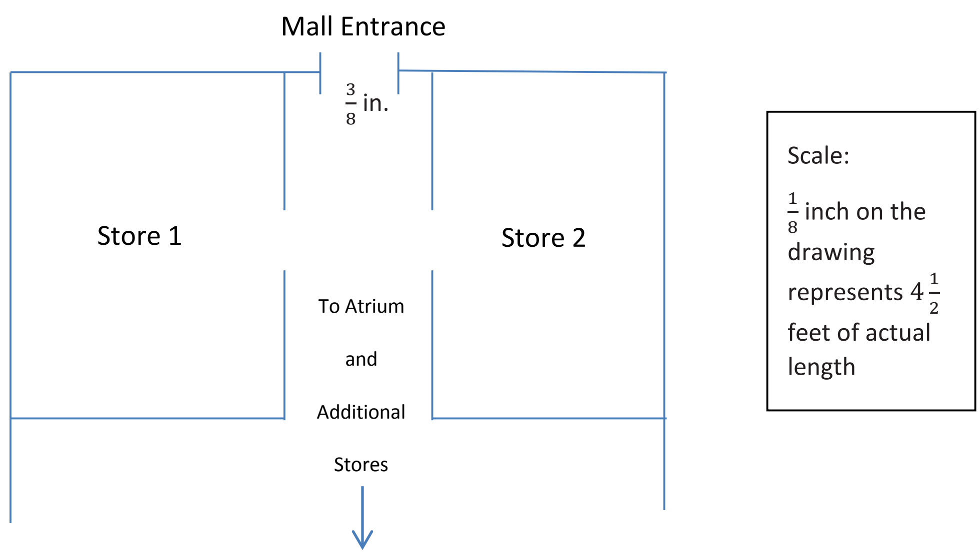
A graphic designer is creating an advertisement for a tablet. She needs to enlarge the picture given here so that $0.25$ inches on the scale picture will correspond to $1$ inch on the actual advertisement. What will be the length and width of the tablet on the advertisement?




Scale Picture of Tablet

## Exercises

1. Students from the high school are going to perform one of the acts from their upcoming musical at the atrium in the mall. The students want to bring some of the set with them so that the audience can get a better feel for the whole production. The backdrop that they want to bring has panels that measure $10$ feet by $10$ feet. The students are not sure if they will be able to fit these panels through the entrance of the mall since the panels need to be transported flat (horizontal). They obtain a copy of the mall floor plan, shown below, from the city planning office. Use this diagram to decide if the panels will fit through the entrance. Use a ruler to measure.



Answer the following questions.

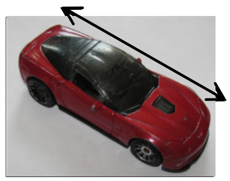
a. Find the actual distance of the mall entrance, and determine whether the set panels will fit.

b. What is the scale factor? What does it tell us?

## Problem Set

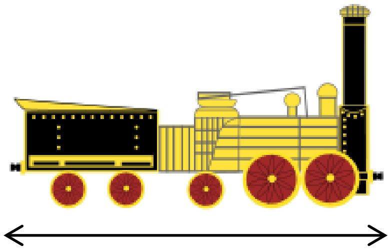
1. A toy company is redesigning their packaging for model cars. The graphic design team needs to take the old image shown below and resize it so that $\frac{1}{2}$ inch on the old packaging represents $\frac{1}{3}$ inch on the new package. Find the length of the image on the new package.

Car image length on old packaging measures 2 inches



2. The city of St. Louis is creating a welcome sign on a billboard for visitors to see as they enter the city. The following picture needs to be enlarged so that $\frac{1}{2}$ inch represents 7 feet on the actual billboard. Will it fit on a billboard that measures 14 feet in height?

3. Your mom is repainting your younger brother's room. She is going to project the image shown below onto his wall so that she can paint an enlarged version as a mural. Use a ruler to determine the length of the image of the train. Then determine how long the mural will be if the projector uses a scale where 1 inch of the image represents $2\frac{1}{2}$ feet on the wall.



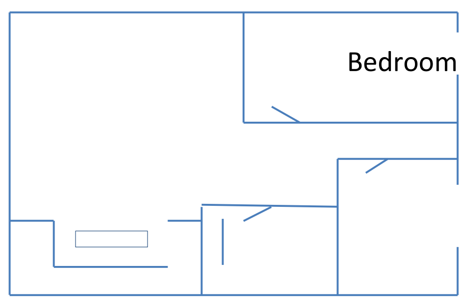
4. A model of a skyscraper is made so that 1 inch represents 75 feet. What is the height of the actual building if the height of the model is $18\frac{3}{5}$ inches?

5. The portrait company that takes little league baseball team photos is offering an option where a portrait of your baseball pose can be enlarged to be used as a wall decal (sticker). Your height in the portrait measures $3\frac{1}{2}$ inches. If the company uses a scale where $1$ inch on the portrait represents $20$ inches on the wall decal, find the height on the wall decal. Your actual height is $55$ inches. If you stand next to the wall decal, will it be larger or smaller than you?

6. The sponsor of a $5$K run/walk for charity wishes to create a stamp of its billboard to commemorate the event. If the sponsor uses a scale where $1$ inch represents $4$ feet, and the billboard is a rectangle with a width of $14$ feet and a length of $48$ feet, what will be the shape and size of the stamp?

7. Danielle is creating a scale drawing of her room. The rectangular room measures $20\frac{1}{2}$ feet by $25$ feet. If her drawing uses the scale where $1$ inch represents $2$ feet of the actual room, will her drawing fit on an $8\frac{1}{2}$ in. by $11$ in. piece of paper?

8. A model of an apartment is shown below where $\frac{1}{4}$ inch represents $4$ feet in the actual apartment. Use a ruler to measure the drawing and find the actual length and width of the bedroom.



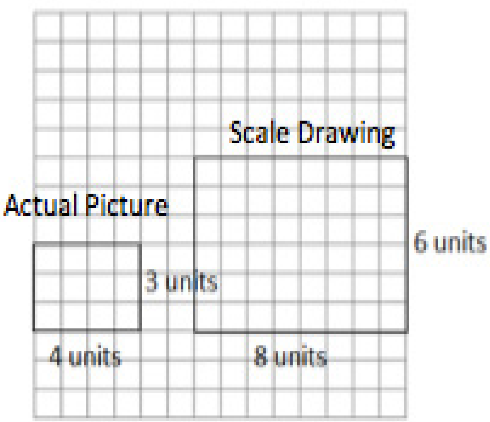
# Lesson 19: Computing Actual Areas from a Scale Drawing

## Classwork

### Examples: Exploring Area Relationships

Use the diagrams below to find the scale factor and then find the area of each figure.

**Example 1**



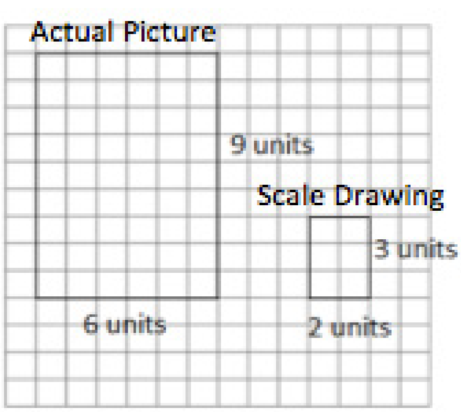

Scale factor: _________

Actual Area = ______________

Scale Drawing Area = ________________

Value of the Ratio of the Scale Drawing Area to the Actual Area: _________

**Example 2**



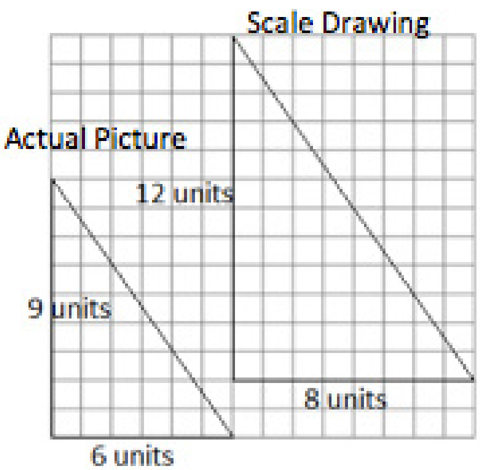

Scale factor: _________

Actual Area = ______________

Scale Drawing Area = ________________

Value of the Ratio of the Scale Drawing Area to the Actual Area: _________

**Example 3**




Scale factor: _________

Actual Area = ______________

Scale Drawing Area = ________________

Value of the Ratio of the Scale Drawing Area to the Actual Area: _________

**Results:** What do you notice about the ratio of the areas in Examples 1–3? Complete the statements below.

When the scale factor of the sides was 2, then the value of the ratio of the areas was _____________.

When the scale factor of the sides was $\frac{1}{3}$, then the value of the ratio of the areas was _____________.

When the scale factor of the sides was $\frac{4}{3}$, then the value of the ratio of the areas was _____________.

Based on these observations, what conclusion can you draw about scale factor and area?

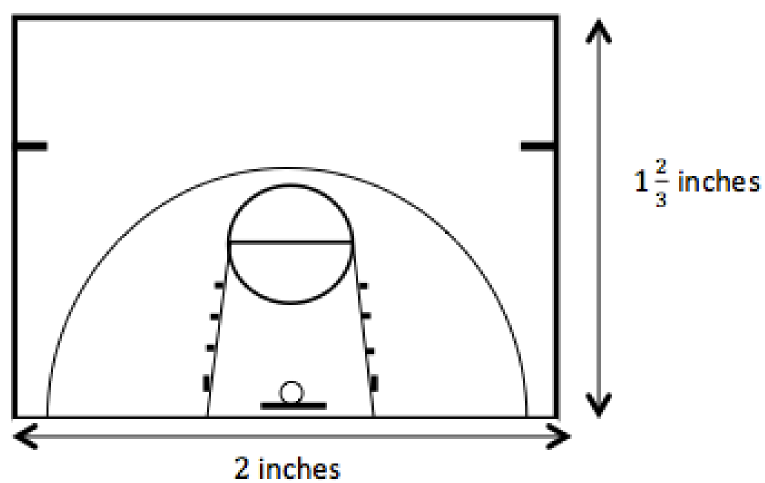
If the scale factor of the sides is $r$, then the ratio of the areas is ___________________.

## Example 4: They Said Yes!

The Student Government liked your half-court basketball plan. They have asked you to calculate the actual area of the court so that they can estimate the cost of the project.

Based on your drawing below, what will the area of the planned half-court be?

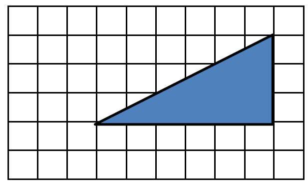
Scale Drawing: 1 inch on the drawing corresponds to 15 feet of actual length




Does the actual area you found reflect the results we found from Examples 1–3? Explain how you know.

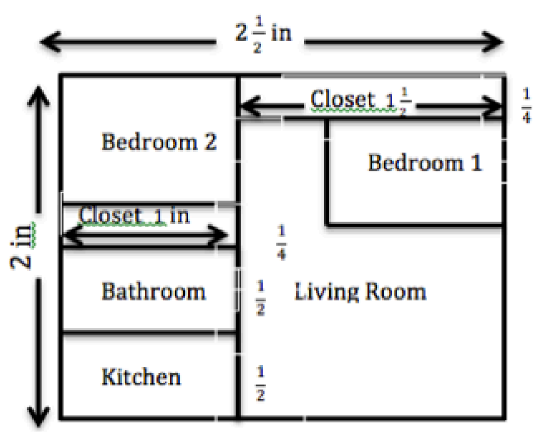
**Exercises**

1. The triangle depicted by the drawing has an actual area of 36 square units. What is the scale of the drawing? (Note: Each square on the grid has a length of 1 unit.)



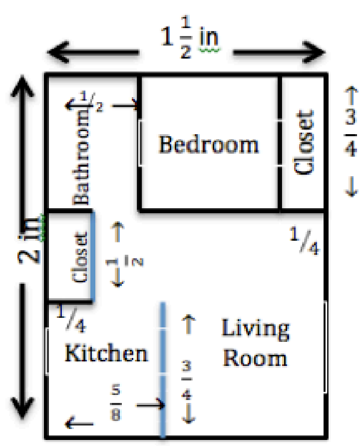
2. Use the scale drawings of two different apartments to answer the questions. Use a ruler to measure.

Suburban Apartment




**Scale:** 1 inch on scale drawing corresponds to 12 feet in the actual apartment.

City Apartment




**Scale:** 1 inch on scale drawing corresponds to 16 feet in actual apartment.

a. Find the scale drawing area for both apartments, and then use it to find the actual area of both apartments.

b. Which apartment has closets with more square footage? Justify your thinking.

c. Which apartment has the largest bathroom? Justify your thinking.

d. A one-year lease for the suburban apartment costs $\$750$ per month. A one-year lease for the city apartment costs $\$925$. Which apartment offers the greater value in terms of the cost per square foot?

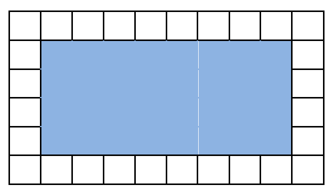
## Lesson Summary

Given the scale factor, $r$, representing the relationship between scale drawing length and actual length, the square of this scale factor, $r^2$, represents the relationship between the scale drawing area and the actual area.

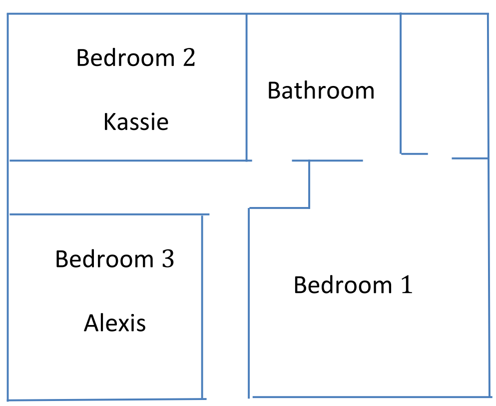
For example, if $1$ inch on the scale drawing represents $4$ inches of actual length, then the scale factor, $r$, is $\frac{1}{4}$. On this same drawing, $1$ square inch of scale drawing area would represent $16$ square inches of actual area since $r^2$ is $\frac{1}{16}$.

## Problem Set

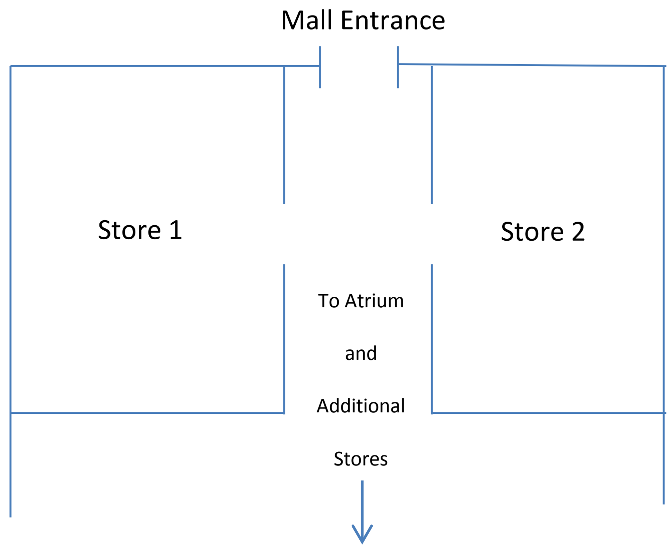
1. The shaded rectangle shown below is a scale drawing of a rectangle whose area is $288$ square feet. What is the scale factor of the drawing? (Note: Each square on grid has a length of $1$ unit.)



2. A floor plan for a home is shown below where $\frac{1}{2}$ inch corresponds to $6$ feet of the actual home. Bedroom $2$ belongs to $13$-year old Kassie, and Bedroom $3$ belongs to $9$-year old Alexis. Kassie claims that her younger sister, Alexis, got the bigger bedroom, is she right? Explain.



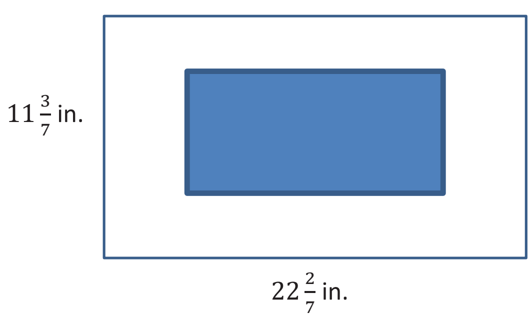
3. On the mall floor plan, $\frac{1}{4}$ inch represents 3 feet in the actual store.
   a. Find the actual area of Store 1 and Store 2.
   b. In the center of the atrium, there is a large circular water feature that has an area of $\left(\frac{9}{64}\right)\pi$ square inches on the drawing. Find the actual area in square feet.




4. The greenhouse club is purchasing seed for the lawn in the school courtyard. The club needs to determine how much to buy. Unfortunately, the club meets after school, and students are unable to find a custodian to unlock the door. Anthony suggests they just use his school map to calculate the area that will need to be covered in seed. He measures the rectangular area on the map and finds the length to be 10 inches and the width to be 6 inches. The map notes the scale of 1 inch representing 7 feet in the actual courtyard. What is the actual area in square feet?

5. The company installing the new in-ground pool in your backyard has provided you with the scale drawing shown below. If the drawing uses a scale of 1 inch to $1\frac{3}{4}$ feet, calculate the total amount of two-dimensional space needed for the pool and its surrounding patio.

Swimming Pool and Patio Drawing



# Lesson 20: An Exercise in Creating a Scale Drawing

## Classwork

Today you will be applying your knowledge from working with scale drawings to create a floor plan for your idea of the dream classroom.

### Exploratory Challenge: Your Dream Classroom

***Guidelines***

**Take measurements:** All students will work with the perimeter of the classroom as well as the doors and windows. Give students the dimensions of the room. Have students use the table provided to record the measurements.

**Create your dream classroom, and use the furniture catalog to pick out your furniture:** Students will discuss what their ideal classroom will look like with their partners and pick out furniture from the catalog. Students should record the actual measurements on the given table.

**Determine the scale and calculate scale drawing lengths and widths:** Each pair of students will determine its own scale. The calculation of the scale drawing lengths, widths, and areas is to be included.

**Scale Drawing:** Using a ruler and referring back to the calculated scale length, students will draw the scale drawing including the doors, windows, and furniture.

**Measurements**

| | Classroom Perimeter | Windows | Door | Additional Furniture | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Actual Length: | | | | | | | | | |
| Width: | | | | | | | | | |
| Scale Drawing Length: | | | | | | | | | |
| Width: | | | | | | | | | |

Scale: __________________

**Initial Sketch**

Use this space to sketch the classroom perimeter, draw out your ideas and play with the placement of the furniture.

**Area**

| | Classroom | | | | | | |
|---|---|---|---|---|---|---|---|
| Actual Area: | | | | | | | |
| Scale Drawing Area: | | | | | | | |

## Lesson Summary

Scale Drawing Process:

1. Measure lengths and widths carefully with a ruler or tape measure. Record the measurements in an organized table.
2. Calculate the scale drawing lengths, widths, and areas using what was learned in previous lessons.
3. Calculate the actual areas.
4. Begin by drawing the perimeter, windows and doorways.
5. Continue to draw the pieces of furniture making note of placement of objects (distance from nearest wall).
6. Check for reasonableness of measurements and calculations.

## Problem Set

### Interior Designer

You won a spot on a famous interior designing TV show! The designers will work with you and your existing furniture to redesign a room of your choice. Your job is to create a top-view scale drawing of your room and the furniture within it.

- With the scale factor being $\frac{1}{24}$, create a scale drawing of your room or other favorite room in your home on a sheet of $8.5 \times 11$ inch graph paper.
- Include the perimeter of the room, windows, doorways, and three or more furniture pieces (such as tables, desks, dressers, chairs, bed, sofa, ottoman, etc.).
- Use the table to record lengths and include calculations of areas.
- Make your furniture "moveable" by duplicating your scale drawing and cutting out the furniture.
- Create a "before" and "after" to help you decide how to rearrange your furniture. Take a photo of your "before."
- What changed in your furniture plans?
- Why do you like the "after" better than the "before"?

| | Entire Room | Windows | Doors | Desk/ Tables | Seating | Storage | Bed | | |
|---|---|---|---|---|---|---|---|---|---|
| Actual Length: | | | | | | | | | |
| Actual Width: | | | | | | | | | |
| Scale Drawing Length: | | | | | | | | | |
| Scale Drawing Width: | | | | | | | | | |

| | Entire Room Length | Desk/Tables | Seating | Storage | Bed | | |
|---|---|---|---|---|---|---|---|
| Actual Area: | | | | | | | |
| Scale Drawing Area: | | | | | | | |

# Lesson 21: An Exercise in Changing Scales

## Classwork

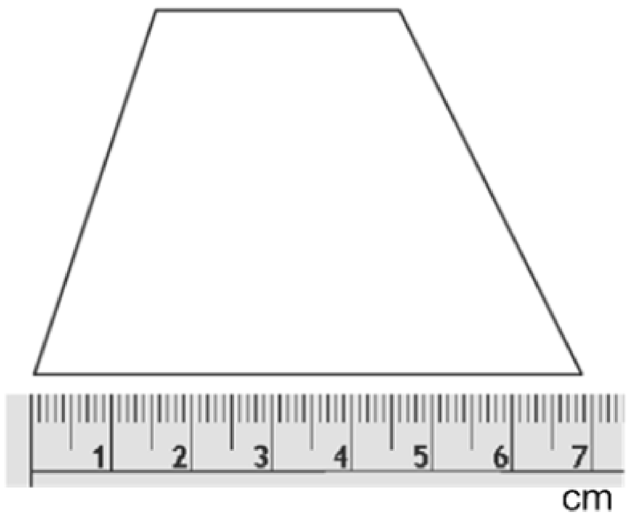
How does your scale drawing change when a new scale factor is presented?

### Exploratory Challenge: A New Scale Factor

The school plans to publish your work on the dream classroom in the next newsletter. Unfortunately, in order to fit the drawing on the page in the magazine, it must be $\frac{1}{4}$ its current length. Create a new drawing ($SD2$) in which all of the lengths are $\frac{1}{4}$ those in the original scale drawing ($SD1$) from Lesson $20$.

**Exercise**

The picture shows an enlargement or reduction of a scale drawing of a trapezoid.




Using the scale factor written on the card you chose, draw your new scale drawing with correctly calculated measurements.

a. What is the scale factor between the original scale drawing and the one you drew?

b. The longest base length of the actual trapezoid is 10 cm. What is the scale factor between original scale drawing and the actual trapezoid?

c. What is the scale factor between the new scale drawing you drew and the actual trapezoid?

**Changing Scale Factors:**

- To produce a scale drawing at a different scale, you must determine the new scale factor. The new scale factor is found by dividing the different (new drawing) scale factor by the original scale factor.
- To find each new length, you can multiply each length in the original scale drawing by this new scale factor.

**Steps:**

- Find each scale factor.
- Divide the new scale factor by the original scale factor.
- Divide the given length by the new scale factor (the quotient from the prior step).

## Lesson Summary

Variations of Scale Drawings with different scale factors are scale drawings of an original scale drawing.

From a scale drawing at a different scale, the scale factor for the original scale drawing can be computed without information of the actual object, figure, or picture.

- For example, if *scale drawing one* has a scale factor of $\frac{1}{24}$ and *scale drawing two* has a scale factor of $\frac{1}{72}$, then the scale factor relating *scale drawing two* to *scale drawing one* is

$$\frac{1}{72} \text{ to } \frac{1}{24} = \frac{\frac{1}{72}}{\frac{1}{24}} = \frac{1}{72} \cdot \frac{24}{1} = \frac{1}{3}.$$

*Scale drawing two* has lengths that are $\frac{1}{3}$ the size of the lengths of *scale drawing one.*

## Problem Set

1. Jake reads the following problem: If the original scale factor for a scale drawing of a square swimming pool is $\frac{1}{90}$, and the length of the original drawing measured to be $8$ inches, what is the length on the new scale drawing if the scale factor of the new scale drawing length to actual length is $\frac{1}{144}$?

   He works out the problem like so:

   $8 \div \frac{1}{90}$ = $720$ inches.

   $720 \times \frac{1}{144}$ = $5$ inches.

   Is he correct? Explain why or why not.

2. What is the scale factor of the new scale drawing to the original scale drawing ($SD2$ to $SD1$)?

3. Using the scale, if the length of the pool measures $10$ cm on the new scale drawing:
   a. Using the scale factor from Problem 1, $\frac{1}{144}$, find the actual length of the pool in meters?
   b. What is the surface area of the floor of the actual pool? Rounded to the nearest tenth.
   c. If the pool has a constant depth of $1.5$ meters, what is the volume of the pool? Rounded to the nearest tenth.
   d. If $1$ cubic meter of water is equal to $264.2$ gallons, how much water will the pool contain when completely filled? Rounded to the nearest unit.

4. Complete a new scale drawing of your dream room from the Problem Set in Lesson 20 by either reducing by $\frac{1}{4}$ or enlarging it by $4$.

# Lesson 22: An Exercise in Changing Scales

## Classwork

Using the new scale drawing of your dream room, list the similarities and differences between this drawing and the original drawing completed for Lesson 20.

<u>Similarities</u> <u>Differences</u>

Original Scale Factor: ____________________ New Scale Factor: ________________________

What is the relationship between these scale factors?

> Key Idea:
>
> Two different scale drawings of the same top-view of a room are also scale drawings of each other. In other words, a scale drawing of a different scale can also be considered a scale drawing of the original scale drawing.

### Example 1: Building a Bench

To surprise her mother, Taylor helped her father build a bench for the front porch. Taylor's father had the instructions with drawings but Taylor wanted to have her own copy. She enlarged her copy to make it easier to read. Using the following diagram, fill in the missing information. To complete the first row of the table, write the scale factor of the bench to the bench, the bench to the original diagram, and the bench to Taylor's diagram. Complete the remaining rows similarly.

The pictures below show the diagram of the bench shown on the original instructions and the diagram of the bench shown on Taylor's enlarged copy of the instruction.

Original Drawing of Bench (top view) Taylor's Drawing (top view) Scale factor to bench: $\frac{1}{12}$

2 inches

6 inches

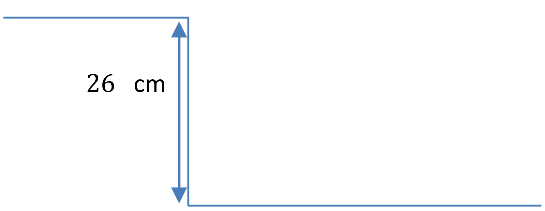
Scale Factors

| | Bench | Original Diagram | Taylor's Diagram |
|---|---|---|---|
| Bench | 1 | | |
| Original Diagram | | 1 | |
| Taylor's Diagram | | | 1 |

### Exercise 1

Carmen and Jackie were driving separately to a concert. Jackie printed a map of the directions on a piece of paper before the drive, and Carmen took a picture of Jackie's map on her phone. Carmen's map had a scale factor to the actual distance of $\frac{1}{563{,}270}$. Using the pictures, what is the scale of Carmen's map to Jackie's map? What was the scale factor of Jackie's printed map to the actual distance?

Jackie's Map




Carmen's Map

4 cm

**Exercise 2**

Ronald received a special toy train set for his birthday. In the picture of the train on the package, the boxcar has the following dimensions: length is $4\frac{5}{16}$ inches; width is $1\frac{1}{8}$ inches; and height is $1\frac{5}{8}$ inches. The toy boxcar that Ronald received has dimensions $l$ is $17.25$ inches; $w$ is $4.5$ inches; and $h$ is $6.5$ inches. If the actual boxcar is $50$ feet long:

a. Find the scale factor of the picture on the package to the toy set.

b. Find the scale factor of the picture on the package to the actual boxcar.

c. Use these two scale factors to find the scale factor between the toy set and the actual boxcar.

d. What are the width and height of the actual boxcar?

### Lesson Summary

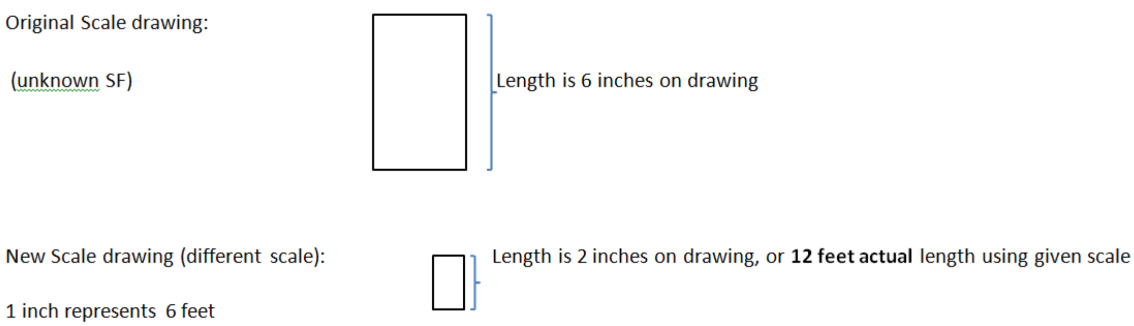
The scale drawing of a different scale is a scale drawing of the original scale drawing.

To find the scale factor for the original drawing, write a ratio to compare the drawing length from the original drawing to its corresponding actual length from the second scale drawing.

Refer to the example below where we compare the drawing length from the Original Scale drawing to its corresponding actual length from the New Scale drawing:

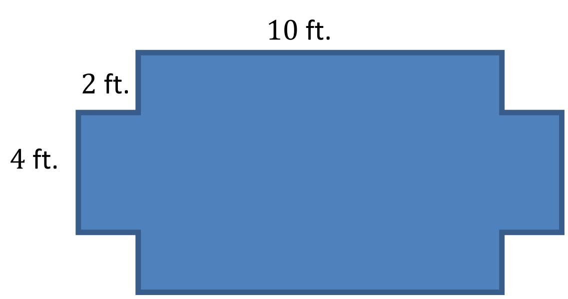
$6$ inches represents $12$ feet or $0.5$ feet represent $12$ feet

This gives an equivalent ratio of $\frac{1}{24}$ for the scale factor of the original drawing.




## Problem Set

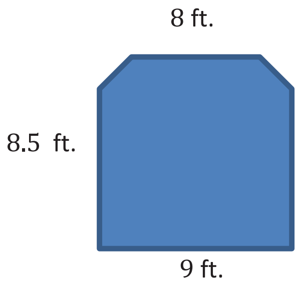
1. For the scale drawing, the actual lengths are labeled onto the scale drawing. Measure the lengths, in centimeters, of the scale drawing with a ruler, and draw a new scale drawing with a scale factor ($SD2$ to $SD1$) of $\frac{1}{2}$.

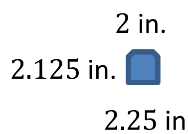


2. Compute the scale factor of the new scale drawing ($SD2$) to the first scale drawing ($SD1$) using the information from the given scale drawings.

a. Original Scale Factor: $\frac{6}{35}$

New Scale Factor: $\frac{1}{280}$

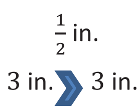


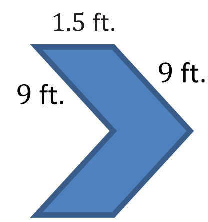




Scale Factor: ______________

b. Original Scale Factor: $\frac{1}{12}$

New Scale Factor: 3

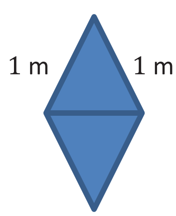


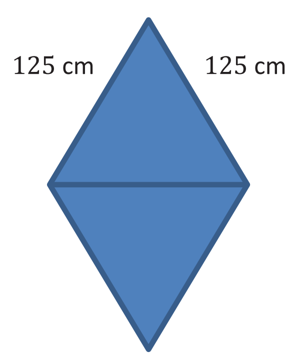




Scale Factor: ________________

c. Original Scale Factor: 20

New Scale Factor: 25




Scale Factor: ________________