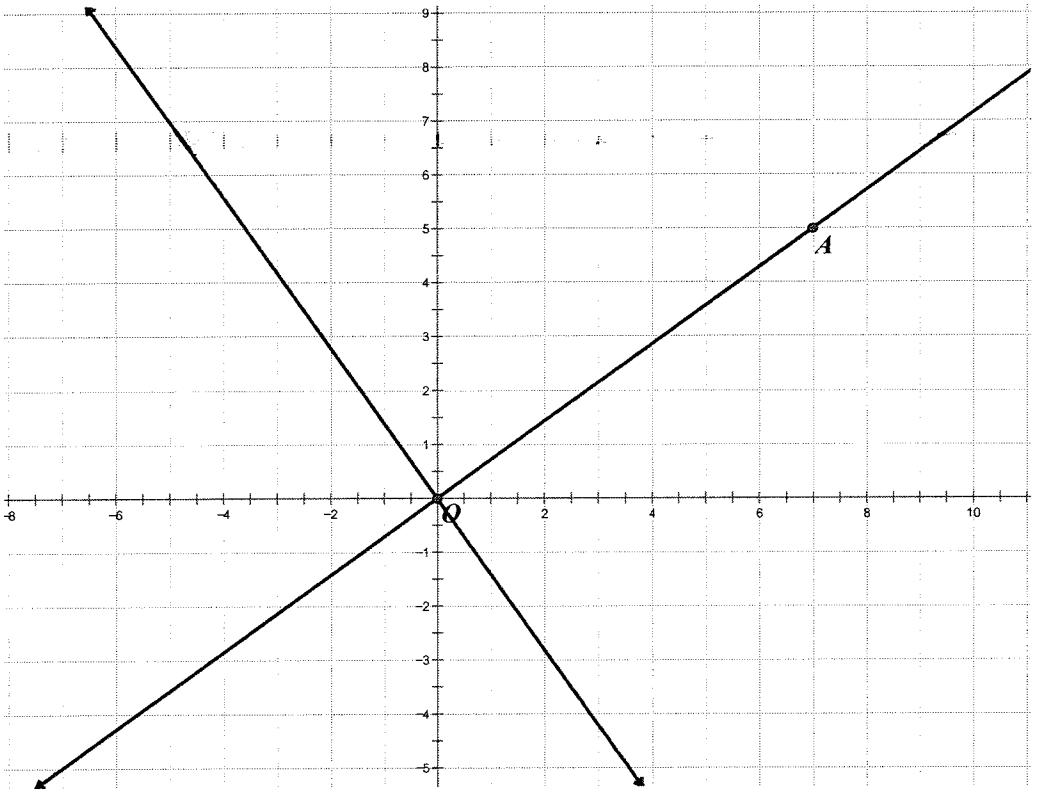
7.2 Slippery Slopes

A Solidify Understanding Task

While working on "Is It Right?" in the previous module you looked at several examples that lead to the conclusion that the slopes of perpendicular lines are negative reciprocals. Your work here is to formalize this work into a proof. Let's start by thinking about two perpendicular lines that intersect at the origin, like these:



1. Start by drawing a right triangle with the segment $\overline{OA}$ as the hypotenuse. These are often called slope triangles. Based on the slope triangle that you have drawn, what is the slope of $\overleftrightarrow{OA}$?

2. Now, rotate the slope triangle 90° about the origin. What are the coordinates of the image of point A?

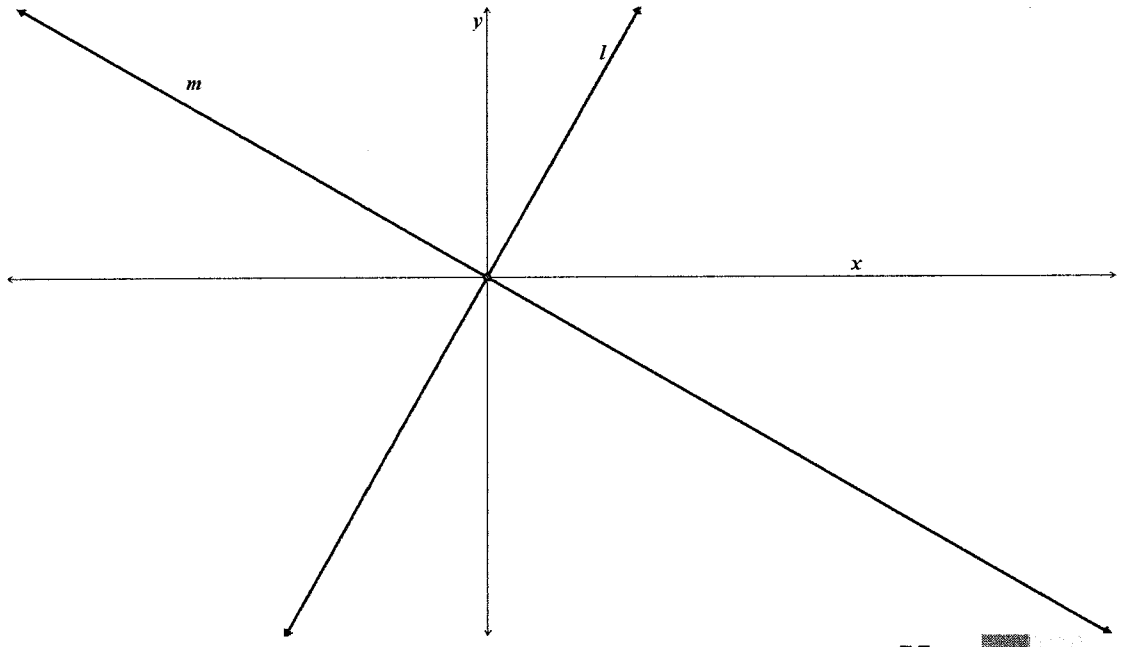
3. Using this new point, A', draw a slope triangle with hypotenuse $\overline{OA'}$. Based on the slope triangle, what is the slope of the line $\overleftrightarrow{OA'}$?

4. What is the relationship between these two slopes? How do you know?

5. Is the relationship changed if the two lines are translated so that the intersection is at (-5, 7)?

   How do you know?

To prove a theorem, we need to demonstrate that the property holds for any pair of perpendicular lines, not just a few specific examples. It is often done by drawing a very similar picture to the examples we have tried, but using variables instead of numbers. Using variables represents the idea that it doesn't matter which numbers we use, the relationship stays the same. Let's try that strategy with this theorem.



- Lines *l* and *m* are constructed to be perpendicular.
- Start by labeling a point P on the line *l*.
- Label the coordinates of P.
- Draw the slope triangle from point P.
- Label the lengths of the sides of the slope triangle.

6. What is the slope of line *l*?

   Rotate point P 90° about the origin, label it P' and mark it on line m. What are the coordinates of P'

7. Draw the slope triangle from point P'. What are the lengths of the sides of the slope triangle? How do you know?

8. What is the slope of line *m*?

9. What is the relationship between the slopes of line *l* and line *m*? How do you know?

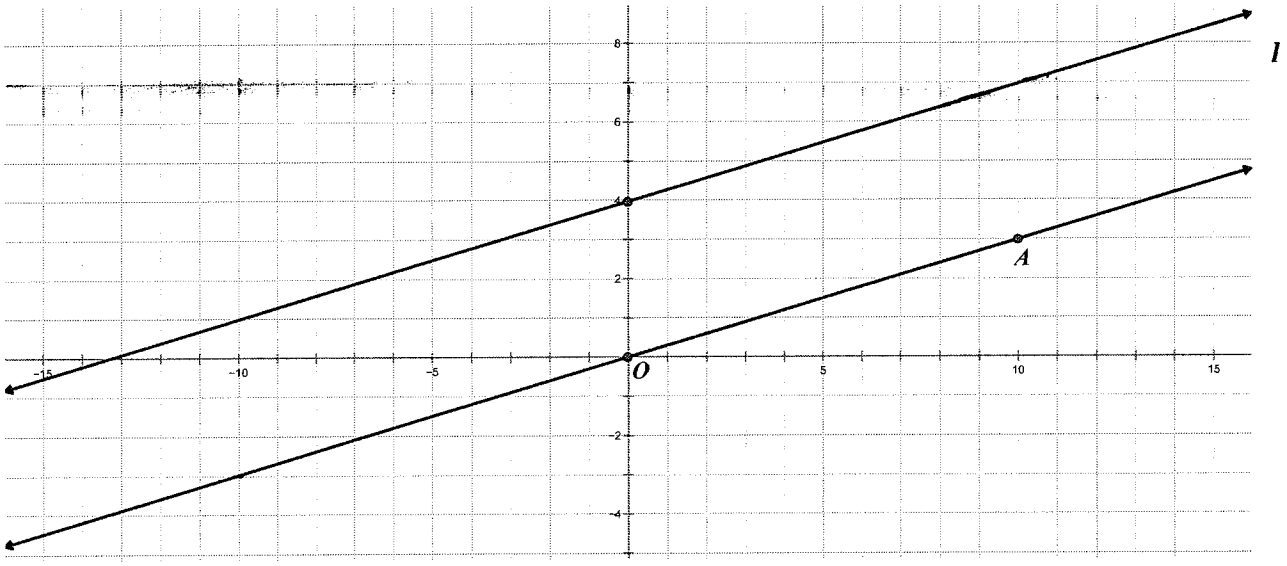
10. Is the relationship between the slopes changed if the intersection between line *l* and line *m* is translated to another location? How do you know?

11. Is the relationship between the slopes changed if lines *l* and *m* are rotated?

12. How do these steps demonstrate that the slopes of perpendicular lines are negative reciprocals for any pair of perpendicular lines?

Think now about parallel lines like the ones below.

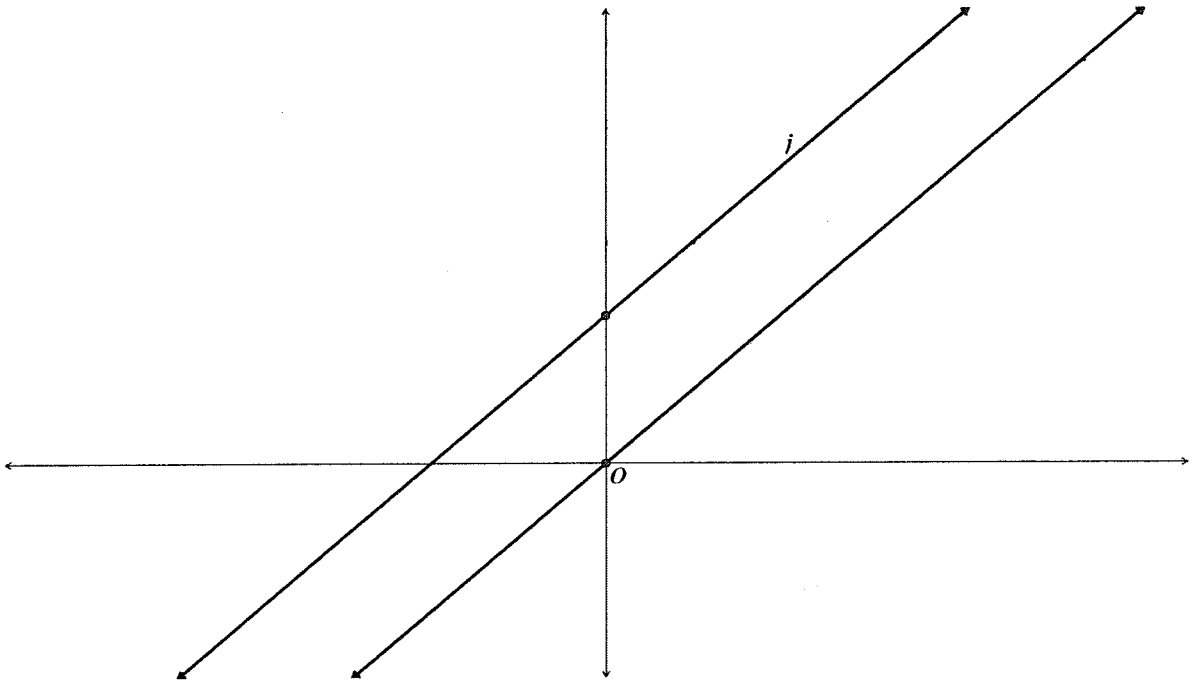
Draw the slope triangle from point A. What is the slope of $\overleftrightarrow{OA}$?



What translation(s) maps the slope triangle with hypotenuse $\overline{OA}$ onto line l?

What must be true about the slope of line l? Why?

Now you're going to try to use this example to develop a proof, like you did with the perpendicular lines. Here are two lines that have been constructed to be parallel.



Show how you know that these two parallel lines have the same slope and explain why this proves that all parallel lines have the same slope.