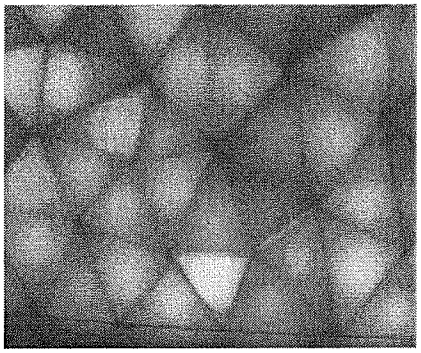
6.9 Congruent Triangles

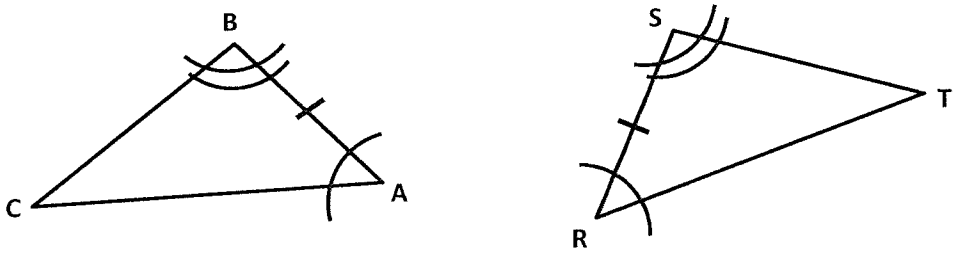
A Solidify Understanding Task



Zac and Sione are trying to decide how much information they need to know about two triangles before they can convince themselves that the two triangles are congruent.

They are wondering if knowing that two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle—a set of criteria their teacher refers to as ASA—is enough to know that the two triangles are congruent. They are trying to justify that this would be so.

To start reasoning about the congruence of the two triangles, Zac and Sione have created the following diagram in which they have marked an ASA relationship between the triangles.



1. Based on the diagram, which angles have Zac and Sione indicated are congruent? Which sides?

2. To convince themselves that the two triangles are congruent, what else would Zac and Sione need to know?

Zac's Argument

"I know what to do," said Zac. "We can translate point A until it coincides with point R, then rotate $\overline{AB}$ about point R until it coincides with $\overline{RS}$. Finally, we can reflect ΔABC across $\overleftrightarrow{RS}$ and then everything coincides so the triangles are congruent." [Zac and Sione's teacher has suggested they use the word "coincides" when they want to say that two points or line segments occupy the same position on the plane. They like the word, so they plan to use it a lot.]

What do you think about Zac's argument? Does it convince you that the two triangles are congruent? Does it leave out any essential ideas that you think need to be included?

3. Write a paragraph explaining your reaction to Zac's argument:

Sione isn't sure that Zac's argument is really convincing. He asks Zac, "How do you know point C coincides with point T after you reflect the triangle?"

4. How do you think Zac might answer Sione's question?

While Zac is trying to think of an answer to Sione's question he adds this comment, "And you really didn't use all of the information about the corresponding congruent parts of the two triangles."

"What do you mean?" asked Zac.

Sione replied, "You started using the fact that $\angle A \cong \angle R$ when you translated ΔABC so that vertex A coincides with vertex R. And you used the fact that $\overline{AB} \cong \overline{RS}$ when you rotated $\overline{AB}$ to coincide with $\overline{RS}$, but where did you use the fact that $\angle B \cong \angle S$?"

"Yeah, and what does it really mean to say that two angles are congruent?" Zac added. "Angles are more than just their vertex points."

5. How might thinking about Zac and Sione's questions help improve Zac's argument?

Sione's Argument

"I would start the same way you did, by translating point A until it coincides with point R, rotating $\overline{AB}$ about point R until it coincides with $\overline{RS}$, and then reflecting ΔABC across $\overleftrightarrow{RS}$," Sione said. "But then I would want to convince myself that points C and T coincide. I know that an angle is made up of two rays that share a common endpoint. Since I know that $\overrightarrow{AB}$ coincides with $\overrightarrow{RS}$ and $\angle A \cong \angle R$, that means that $\overrightarrow{AC}$ coincides with $\overrightarrow{RT}$. Likewise, I know that $\overrightarrow{BA}$ coincides with $\overrightarrow{SR}$ and $\angle B \cong \angle S$, so $\overrightarrow{BC}$ must coincide with $\overrightarrow{ST}$. Since $\overrightarrow{AC}$ and $\overrightarrow{BC}$ intersect at point C, and $\overrightarrow{RT}$ and $\overrightarrow{ST}$ intersect at

point T, points C and T must also coincide because the corresponding rays coincide. Therefore, $\overline{BC} \cong \overline{ST}$, $\overline{CA} \cong \overline{TR}$, and $\angle C \cong \angle T$ because both angles are made up of rays that coincide!"

At first Zac was confused by Sione's argument, but he drew diagrams and carefully marked and sketched out each of his statements until it started to slowly make sense.

6. Do the same kind of work that Zac did to make sense of Sione's argument. What parts of his argument are unclear to you? What ideas did sketching out the words of his proof help you to clarify?

Sione's argument suggests that ASA is sufficient criteria for determining if two triangles are congruent. Now Zac and Sione are wondering about other criteria, such as SAS or SSS, or perhaps even AAA (which Zac immediately rejects because he thinks two triangles can have the same angle measures but be different sizes).

7. Draw two triangles that have SAS congruence. Be sure to mark you triangles to show which sides and which angles are congruent.

8. Write out a sequence of transformations to show that the two triangles potentially coincide.

9. If Sione were to examine your work in #8, what questions would he wonder about?

10. How can you use the given congruence criteria (SAS) to resolve Simone's wonderings?

Repeat 7-10 for SSS congruence.