6.10 Congruent Triangles to the Rescue

A Practice Understanding Task

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Part 1

Zac and Sione are exploring isosceles triangles—triangles in which two sides are congruent.

Zac: I think every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up of the two congruent sides, and the midpoint of the third side.

Sione: That's a pretty big claim—to say you know something about *every* isosceles triangle. Maybe you just haven't thought about the ones for which it isn't true.

Zac: But I've folded lots of isosceles triangles in half, and it always seems to work.

Sione: *Lots* of isosceles triangles are not *all* isosceles triangles, so I'm still not sure.

1. What do you think about Zac's claim? Do you think every isosceles triangle has a line of symmetry? If so, what convinces you this is true? If not, what concerns do you have about his statement?

2. What else would Zac need to know about the line through the vertex point of the angle made up of the two congruent sides and the midpoint of the third side in order to know that it is a line of symmetry? (Hint: Think about the definition of a line of reflection.)

3. Sione thinks Zac's "crease line" (the line formed by folding the isosceles triangle in half) creates two congruent triangles inside the isosceles triangle. Which criteria—ASA, SAS or SSS—could she use to support this claim? Describe the sides and/or angles you think are congruent, and explain how you know they are congruent.

4. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "base angles" of an isosceles triangle (the two angles that are not formed by the two congruent sides)?

5. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "crease line"? (You might be able to make a couple of claims about this line—one claim comes from focusing on the line where it meets the third, non-congruent side of the triangle; a second claim comes from focusing on where the line intersects the vertex angle formed by the two congruent sides.)

Part 2

Like Zac, you have done some experimenting with lines of symmetry, as well as rotational symmetry. In the tasks *Symmetries of Quadrilaterals* and *Quadrilaterals—Beyond Definition* you made some observations about sides, angles and diagonals of various types of quadrilaterals based on your experiments and knowledge about transformations. Many of these observations can be further justified based on looking for congruent triangles and their corresponding parts, just as Zac and Sione did in their work with isosceles triangles.

Pick one of the following quadrilaterals to explore:

- A **rectangle** is a quadrilateral that contains four right angles.
- A **rhombus** is a quadrilateral in which all sides are congruent.
- A **square** is both a rectangle and a rhombus, that is, it contains four right angles and all sides are congruent

1. Draw an example of your selected quadrilateral, with its diagonals. Label the vertices of the quadrilateral *A*, *B*, *C*, and *D*, and label the point of intersection of the two diagonals as point *N*.

2. Based on (1) your drawing, (2) the given definition of your quadrilateral, and (3) information about sides and angles that you can gather based on lines of reflection and rotational symmetry, list as many pairs of congruent triangles as you can find.

For each pair of congruent triangles you list, state the criteria you used—ASA, SAS or SSS—to determine that the two triangles are congruent, and explain how you know that the angles and/or sides required by the criteria are congruent.

Congruent Triangles	Criteria Used (ASA, SAS, SSS)	How I know the sides and/or angles required by the criteria are congruent
If I say $\Delta RST \cong \Delta XYZ$	based on SSS	then I need to explain: • how I know that $\overline{RS} \cong \overline{XY}$, and • how I know that $\overline{ST} \cong \overline{YZ}$, and • how I know that $\overline{TR} \cong \overline{ZX}$ so I can use SSS criteria to say $\Delta RST \cong \Delta XYZ$

3. Now that you have identified some congruent triangles in your diagram, can you use the congruent triangles to justify something else about the quadrilateral, such as:

- the diagonals bisect each other
- the diagonals are congruent
- the diagonals are perpendicular to each other
- the diagonals bisect the angles of the quadrilateral

Pick one of the bulleted statements you think is true about your quadrilateral and try to write an argument that would convince Zac and Sione that the statement is true.